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Formulation of a fault governing law at high sliding speeds: Inferences from dynamic rupture models

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ABSTRACT

Understanding the behavior of natural faults at cosesimic slip velocities ($v \sim 1$–10 m/s or more) has become a challenging achievement for experimentalists and modelers of earthquake instabilities. The rate- and state-dependent friction laws, originally obtained in slow slip rate conditions, have been widely adopted in dynamic rupture models by assuming their validity well above the experimental range of observations. In this paper we consider a modification at high speeds, in which the steady state friction becomes independent of v above a transitional value v_T. Our results show that this modification has dramatic effects on the dynamic propagation; as long as v_T decreases the breakdown stress drop decreases, as well as the slip-weakening distance and the fracture energy density. Moreover, we found that the subshear regime is favored as v_T decreases; we found that for the strength parameter S greater than 1.482 the supershear rupture propagation is inhibited. Finally, we demonstrate that the exponential weakening, often observed in laboratory experiments, can be theoretically explained in the framework of the rate and state laws.

1. Introduction

The behavior of rocks and tribochemical products of faulting has been investigated for a long time with the aim to model, in a deterministic framework, the dissipative processes occurring during a crustal earthquake. This basically consists in the formulation of analytical laws, the constitutive equations, which describe the temporal evolution of the frictional resistance of the seismogenic structure. The lively debate about the more appropriate expression of such a governing model has pervaded the scientific community and it has involved theoretical, numerical and laboratory investigations (see Bizzarri (2011b) for a comprehensive review). Basically, this debate is centered on the distinction between two apparently opposite worlds (Bizzarri and Cocco, 2006), the first being dominated by the slip-dependent friction laws and the second being focused on the rate- and state-dependent friction laws (RS thereinafter).

After the pioneering experimental studies by Dieterich and Ruina (Ruina (1983) and references cited therein) the RS models have been widely employed to numerically simulate repeated instability events (i.e., seismic cycles), dynamic rupture processes, seismic wave excitations, aftershocks generations, earthquake triggering, etc. More recently, the continuous development of experimental machines poses the problem of the applicability of the above-mentioned friction laws at seismic regimes (1–10 m/s or more). Indeed, the canonical formulations of the RS constitutive models have been often criticized, in that they have been originally retrieved for sliding velocity several orders of magnitude below the seismic range. At a more fundamental level there is a theoretical issue concerning the problem of scaling, i.e., how to relate the laboratory results to the real-world seismicity (e.g., Scholz, 1988). This holds both for the analytical expression of the inferred governing model and for the values of the parameters retrieved through best fitting procedures.

In this study we focus on the behavior of the frictional resistance at high fault slip velocities. We consider a modification to the classical Ruina–Dieterich equation, originally proposed by Weeks (1993) and we explore the response of the fault obeying such a rheological law.

2. Rheology of faults at high speeds

In this paper we adopt the widely-used Ruina–Dieterich (RD henceforth) model (Ruina, 1983), written in the following form:

$$\begin{cases} \tau = \left[\mu_* + a \ln\left(\dfrac{v}{v_*}\right) + \Theta\right]\sigma_n^{eff} \\ \dfrac{d}{dt}\Theta = -\dfrac{v}{L}\left[\Theta + b \ln\left(\dfrac{v}{v_*}\right)\right] \end{cases} \quad (1)$$

where τ expresses the frictional resistance, v is the fault slip velocity and the constitutive parameters a and b describe the slip-hardening

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phase (i.e., the so-defined direct effect) and the evolution effect of traction, respectively. Physically they account for the thermally-activated exponential creep occurring at the microscopic level. Both a and b are material properties, in that they can depend on pressure, temperature, etc. In this paper we restrict the analysis to the simple case of temporal constants a and b; the effects of their temporal evolution have been discussed elsewhere (e.g., Bizzarri, 2011a). The constant L in Eq. (1) is the characteristic length scale for the evolution of the dimensionless state variable Θ, σ_n^{eff} is the effective normal stress and μ_* and v_* are reference values for the frictional coefficient and the slip velocity, respectively.

The condition $d\Theta/dt=0$ defines the steady state of the system; in such a case we have

$$\Theta^{ss}(v) = b\ \ln\left(\frac{v_*}{v}\right) \tag{2}$$

and correspondingly

$$\tau^{ss}(v) = \mu^{ss}(v)\sigma_n^{eff} = \left[\mu_* + (b-a)\ln\left(\frac{v_*}{v}\right)\right]\sigma_n^{eff} \tag{3}$$

The governing model (1) has been proved to reproduce all the main features expected during an earthquake instability (e.g., Cocco and Bizzarri, 2002). More recently, Bizzarri (2011b) showed that the ground motions originated from a fault governed by Eq. (1) are practically indistinguishable, also in terms of frequency content, from those obtained by assuming the linear slip-weakening (SW thereinafter) friction law (Ida, 1972), provided that both the models have the same fracture energy density (or work per unit area; see Eq. (18) in Bizzarri (2011b)).

Based on the laboratory inferences of Scholz and Engelder (1976) and Dieterich (1978), Weeks (1993) suggested a modification of the constitutive model (1) by assuming that the steady state friction becomes independent of v at high speeds; namely, for $v \geq v_T$ we have

$$\begin{cases} \tau = \left[\mu_* + a\ \ln\left(\dfrac{v_T}{v_*}\right) + \Theta\right]\sigma_n^{eff} \\ \dfrac{d}{dt}\Theta = -\dfrac{v}{L}\left[\Theta + b\ \ln\left(\dfrac{v_T}{v_*}\right)\right] \end{cases} \tag{4}$$

so that we obtain

$$\Theta^{ss}(v) = b\ \ln\left(\frac{v_*}{v_T}\right) \tag{5}$$

and

$$\tau^{ss}(v) = \mu^{ss}(v)\sigma_n^{eff} = \left[\mu_* + (b-a)\ln\left(\frac{v_*}{v_T}\right)\right]\sigma_n^{eff} \tag{6}$$

In Eqs. (4)–(6) v_T represents a high speed velocity cut off which discriminates between the classical formulation (Eq. (1)) and the so-called frozen Ruina–Dieterich (FRD henceforth) model (Eq. (4)). The implicit time dependence of the friction in Eq. (4) is ensured by that of the state variable, which still nonlinearly depends on the sliding speed v. Eqs. (3) and (6) can be unified in a single function, both as $\tau^{ss}(v) = [\mu_* + (b-a)\ln(v_*/v + v_*/v_T)]\sigma_n^{eff}$ (see Weeks (1993); his Eq. (5)), and as $\tau^{ss}(v) = [\mu_* + (b-a)\ln(v_*/v + e^{-n})]\sigma_n^{eff}$ (where n is a positive number, see Tse and Rice (1986); their Eq. (5)) which are valid for any arbitrary value of v.

Remarkably, for $v \geq v_T$ we can find a closed-form analytical solution for the state variable; in the special case of $v_T = v_*$, for times $t \geq t_T$ (where t_T is such that $v(t_T) = v_T$) we have $\Theta(t) = Ce^{-u/L}$, C being a constant ($C = Q_T e^{u_T/L}$ where $\Theta_T \equiv \Theta(t_T)$ and $u_T \equiv u(t_T)$) and u being the fault slip. Therefore, for $t \geq t_T$ we can simply write

$$\tau = \left[\mu_* + Ce^{-u/L}\right]\sigma_n^{eff} \tag{7}$$

(Note that τ in Eq. (7) is the general expression for the frictional resistance and not the steady state, which in this special case simply equals $\mu_*\sigma_n^{eff}$ (see Eq. (6)). Note also that the quantity C is constant, for a given v_T. Moreover, we also emphasize that a result similar to Eq. (7) can be found in the more general case, when $v_T \neq v_*$.)

From laboratory experiments performed on a rotary shear apparatus at low normal stresses (up to 2 MPa) and at relatively high sliding speeds ($v \sim 1$ m/s) (Mizoguchi et al., 2007; see their Eq. (1)) infer the following constitutive equation (see also Bizzarri (2010a)):

$$\tau = \left[\mu_f + (\mu_u - \mu_f)\ e^{-u/(d/\ln(20))}\right]\sigma_n^{eff} \tag{8}$$

where μ_u and μ_f are the upper and final level of the friction coefficient, respectively, and d is the characteristic length scale of the model. Compared to Eq. (7), model (8) makes the following associations straightforward:

$$\begin{aligned} &\mu_* \leftrightarrow \mu_f \\ &C \leftrightarrow \mu_u - \mu_f \\ &L \leftrightarrow \frac{d}{\ln(20)} \end{aligned} \tag{9}$$

Interestingly, the constitutive model (8) which has been further generalized by Sone and Shimamoto (2009), who also included an initial slip-hardening phase has been proposed simply as a fit of laboratory data, without giving any physical foundation or theoretical justification. Indeed, Eq. (9) suggests that the model (8) can be explained in the framework of the RS friction law; namely, it descends from the FRD model (5).

It is important to point out that while L in the framework of the RS laws has a precise physical meaning (as reported above), d simply is a length scale of the friction law (8). Eq. (9) explicitly indicates that the two characteristic length scales L and d are different ($L \sim 0.3d$). The same occurs when we compare the equivalent slip-weakening distance d_0^{eq} emerging from the traction versus slip curve in the framework of RS models (see Cocco and Bizzarri, 2002). Indeed, in the case of the classical RD law we have $L \sim 0.2 d_0^{eq}$ (see Fig. 11 in Bizzarri and Cocco (2003)).

As pointed out by Cocco and Bizzarri (2002), in RS models we do not have a prior-imposed value of the final level of friction μ_f (formally μ_f^{eq}); indeed, we can only exploit some analytical relations (e.g., Eqs. (13), (B1) and (B2) of Bizzarri and Cocco (2003)) to estimate μ_f^{eq}. On the contrary, when $v_T = v_*$ we can a priori know that $\mu_f = \mu_*$ (in other words μ_* can be physically interpreted as the level of friction attained after the breakdown process).

3. Numerical results

The elastodynamic equation for faults is solved numerically (Bizzarri and Cocco, 2005), by neglecting body forces and by assuming the governing equations discussed in Section 2. The nucleation procedure is the same as in Bizzarri (2009); the fault starts from the steady state $\Theta^{ss}(v_0)$ ($v_0 \equiv v(t=0)$), except for a circular nucleation patch I_{nucl} (having a radius of 88 fault nodes) where the state variable is lower than Θ^{ss}; in the hypocenter H we have $\Theta_H = 1 \times 10^{-4}$ and Θ is sinus-tapered to $\Theta^{ss}(v_0)$ over the nucleation region. Since $\tau_0 = \tau^{ss}(v_0)$ everywhere over the fault surface, the shape of the state variable at $t=0$ corresponds to the introduction of an imposed stress drop within I_{nucl}. This drop is maximum in the hypocenter, where $\Delta\tau = \tau^{ss}(v_0) - \tau(v = v_0, \Theta = \Theta_H) = 16.57$ MPa for the adopted parameters, and gradually reduces to zero at the borders of I_{nucl}.

We consider a vertical, strike slip fault, with spatially homogeneous properties, having the same geometry as shown in Fig. S1 of the auxiliary material of Bizzarri (2009) and expanding bilaterally from H. The parameters are listed in Table 1.

Table 1
Parameters adopted in the present study.

Parameter	Value
Medium and discretization parameters	
Lamé constants, $\lambda = G$	24.3 GPa
S-wave velocity, v_S	3 km/s
P-wave velocity, v_P	5.196 km/s
Cubic mass density, ρ	2700 kg/m^3
Fault length, L^f	12 km
Fault width, W^f	13 km
Spatial grid size, $\Delta x_1 = \Delta x_2 = \Delta x_3 \equiv \Delta x$	12 m[a]
Time step, Δt	6.66×10^{-4} s [a]
Coordinates of the hypocenter, $H \equiv (x_1^H, x_3^H)$	(6, 6.996) km
Fault constitutive parameters	
Effective normal stress, σ_n^{eff}	120 MPa
Logarithmic direct effect parameter, a	0.012
Evolution effect parameter, b	0.02
Scale length for state variable evolution, L	2×10^{-2} m
Reference value of the friction coefficient, μ_*	0.56
Reference value of the fault slip velocity, v_*	0.1 mm/s
Initial fault slip velocity, v_0	0.1 μm/s
Initial value of the state variable, Θ_0	13.82 $(=\Theta^{ss}(v_0))$
Magnitude of the initial shear stress, τ_0	73.83 MPa $(=\tau^{ss}(v_0))$

[a] For the adopted parameters the Courant–Friedrichs–Lewy ratio, $w_{CFL} = v_S \Delta t/\Delta x$, equals 0.1665 and the estimate of the critical frequency for spatial grid dispersion, $f_{acc}^{(s)} = v_S/(6\Delta x)$, equals 42 Hz.

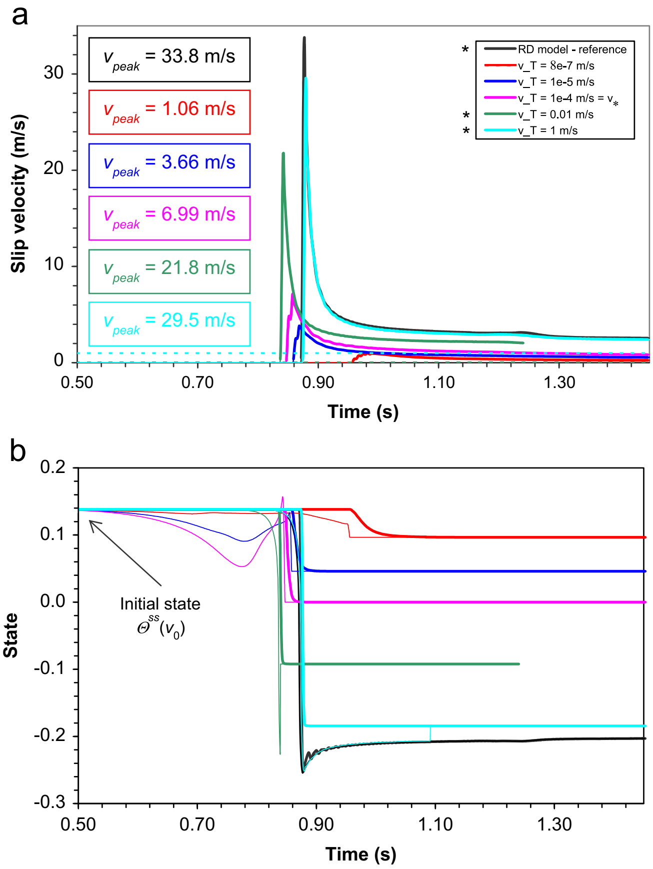

Fig. 1. Results obtained for different transitional velocities v_T (the values are indicated in the legend). The black lines correspond to the reference RD case without modification at high speeds (governing model (1)). (a) Time history of the fault slip velocity. The asterisks denote the supershear ruptures, the peak fault slip velocity are reported for each simulation and dotted lines denote the values of v_T. (b) Time history of the state variable. Thin lines represent the steady state curves (Eqs. (2) and (3) or (5) and (6) for $v \geq v_T$). The solutions pertain to a fault node at the hypocentral depth and at a distance of 3 km from the hypocenter. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

In Figs. 1 and 2 we report the results obtained by assuming different values of the transitional velocity v_T; we have considered values from the literature (Weeks (1993) uses $v_T = 10$ μm/s, blue curves in Figs. 1 and 2; Dieterich (1978) uses $v_T = 0.1$ mm/s, magenta curves in Figs. 1 and 2) and higher values in order to cover the whole range of the slip velocities attained in a reference case without modifications at high speeds (i.e., governing model (1), black curves in Fig. 1). A compendious summary of the results from the simulations is reported in Table 2.

From Fig. 1a it emerges that as long as v_T decreases the peak slip velocity decreases too, even reducing more than a factor of 10. The shape of v is similar and remarkably we do not observe the healing of slip, even by assuming v_T close to the initial velocity; v asymptotically reaches a final value after the completion of the breakdown processes (stress release). This corresponds to a new steady state of friction (confirming previous findings of Bizzarri and Cocco (2003)), as we can from Fig. 1b, where the steady state curves $\Theta^{ss}(v)$ are superimposed to the time evolutions of the state variable. A decreasing value of v_T progressively reduces the importance of the so-called direct effect; from Fig. 2a it is clear that the slip-hardening phase is less significant for low values of v_T. Correspondingly, the upper yield stress values $(\tau_u^{eq} = \mu_u^{eq} \sigma_n^{eff})$ are reduced (see also Table 2). The opposite holds for the final level of friction $(\tau_f^{eq} = \mu_f^{eq} \sigma_n^{eff})$; as long as v_T decreases, τ_f^{eq} increases. In particular, we can see that when $v_T = v_*$ (magenta curve in Fig. 2a) $\mu_f = \mu_*$, as expected theoretically (see the discussion at the end of Section 2). The changes of τ_u^{eq} and τ_f^{eq} lead to a net reduction of the dynamic stress drop $\Delta\tau_b = \tau_u^{eq} - \tau_f^{eq}$; from Table 2 the reference value of 41.3 MPa for the RD case is reduced nearly by a factor of 10 in the FRD model with $v_T = 0.8$ μm/s.

Another interesting outcome of our numerical experiments is the slip distance over which the stress is released, d_0^{eq} is reduced for increasing values of the threshold velocity (see Fig. 2a). Correspondingly, the ratio d_0^{eq}/L decreases from the reference value of 5.2 for the canonical RD model to about 3 when $v_T = 0.8$ μm/s. The significant variations of the traction evolution explain why the fracture energy density E_G (Bizzarri, 2010b; his Eq. (1)) is reduced for decreasing values of v_T (see Table 2).

The analysis of Fig. 2b further confirms what we have previously observed; the final level of the traction is in a new steady state and the direct effect becomes less significant when v_T decreases. We can see that the hardening phase (during which the friction coefficient increases for increasing slip and slip velocity) is practically indistinguishable in the early stages of the rupture for all the numerical experiments. However, when the term $\ln(v/v_*)$ is frozen, the evolutionary term Θ becomes paramount and it causes a fast acceleration phase at a nearly constant traction. We recall here that in RS framework the instant of failure (namely, the time when the traction starts to decrease) is the net result of two competing mechanisms, the first being the hardening phase (mainly controlled by the slight increase of the slip velocity) and the second being the evolutionary stage (essentially controlled by the time evolution of the state variable).

4. Discussion

The governing model considered in the present study (see Eqs. (1) and (4)) in some sense complements other type of regularizations performed at low speeds to the classical RS laws, as discussed in details in Bizzarri (2011b). Here the modification is performed at high slip velocities, as in the framework of the flash heating of asperity contacts (see Bizzarri, 2011b; his Eq. (46)), where at high speeds the evolution of the state variable is modified with respect to the classical equation. In the latter model the transitional velocity (v_{fh}) typically has a value of the

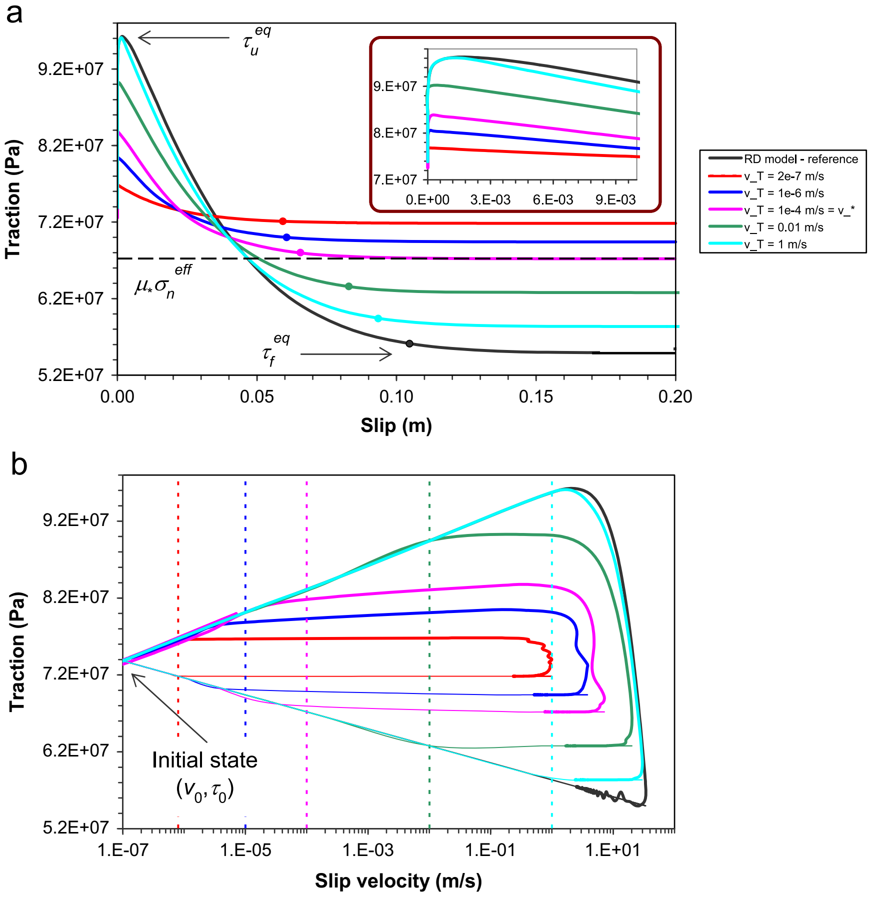

Fig. 2. Results for different v_T. (a) Slip-weakening curve with inset reporting the initial hardening phase. The full circles mark d_0^{eq} (see Table 2). (b) Phase portrait (i.e., traction versus slip velocity; note the logarithmic scale in the abscissa). (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

Table 2
Synopsis of the results from the ensemble of the numerical simulations. The asterisk marks the simulations reported in Figs. 1 and 2.

v_T (m/s)	μ_u^{eq}	μ_f^{eq}	d_0^{eq} [a] (cm)	d_0^{eq}/L	S [b]	$\Delta\tau_b$ [c] (MPa)	E_G [d] (MJ/m^2)	Regime
* 8×10^{-7}	0.640	0.599	5.93	2.965	1.499	4.99	0.0823	Subshear
9×10^{-6}	0.669	0.579	6.05	3.025	1.498	10.8	0.1794	Subshear
* 1×10^{-5}	0.670	0.578	6.07	3.035	1.498	11.0	0.1847	Subshear
* 1×10^{-4}	0.698	0.560	6.56	3.280	1.495	16.5	0.2913	Subshear
1×10^{-3}	0.725	0.542	7.11	3.555	1.489	22.0	0.4085	Subshear
2.5×10^{-3}	0.736	0.534	7.59	3.795	1.485	24.2	0.4403	Subshear
* 1×10^{-2}	0.752	0.523	8.29	4.145	1.482	27.4	0.5916	Supershear
* 1	0.801	0.486	9.35	4.675	1.438	37.7	0.8762	Supershear
10	0.802	0.465	10.34	5.170	1.243	40.5	1.195	Supershear
* Arbitrarily large[e]	0.802	0.458	10.47	5.235	1.188	41.3	3.036	Supershear

[a] The equivalent characteristic slip-weakening distance d_0^{eq} is determined as the fault slip cumulated up to the time when the 95% of the dynamic stress drop is accomplished.
[b] The strength parameter S is computed from Eq. (10).
[c] The dynamic stress drop is computed as $\Delta\tau_b = \tau_u^{eq} - \tau_f^{eq}$.
[d] The fracture energy density E_G is computed from Eq. (1) of Bizzarri (2010b) by assuming $d = d_0^{eq}$ and τ_{res} as the value of the traction attained when $u = d_0^{eq}$ (note that $\tau_{res} \neq \tau_f^{eq}$ owing to the definition of d_0^{eq}).
[e] This corresponds to the classical RD law without the modification at high speeds (constitutive model (1)).

order of few cm/s, while the values used by Weeks (1993) are of the order of few μm/s.

The introduction of the cut off velocity v_T essentially results in a transition from the classical behavior of RD law at low speeds to a pure slip-dependent weakening at high speed, as analytically stated by Eq. (7). Sone and Shimamoto (2009) introduce a further velocity-dependence in the slip-dependent law originally proposed by Mizoguchi et al. (2007). Analytically, this velocity-dependence is missed in the present model, but there is no doubt that the most important dependence in the model proposed by Sone and Shimamoto (2009) remains in the developed slip, as also demonstrated by the numerical models of Bizzarri (2010a). Moreover, based on laboratory experiments conducted on both unfractured and precut granite samples (i.e., considering not only friction experiments as those allowed with rotary shear machines), Ohnaka (2003) it can be concluded that the slip-dependence of the frictional

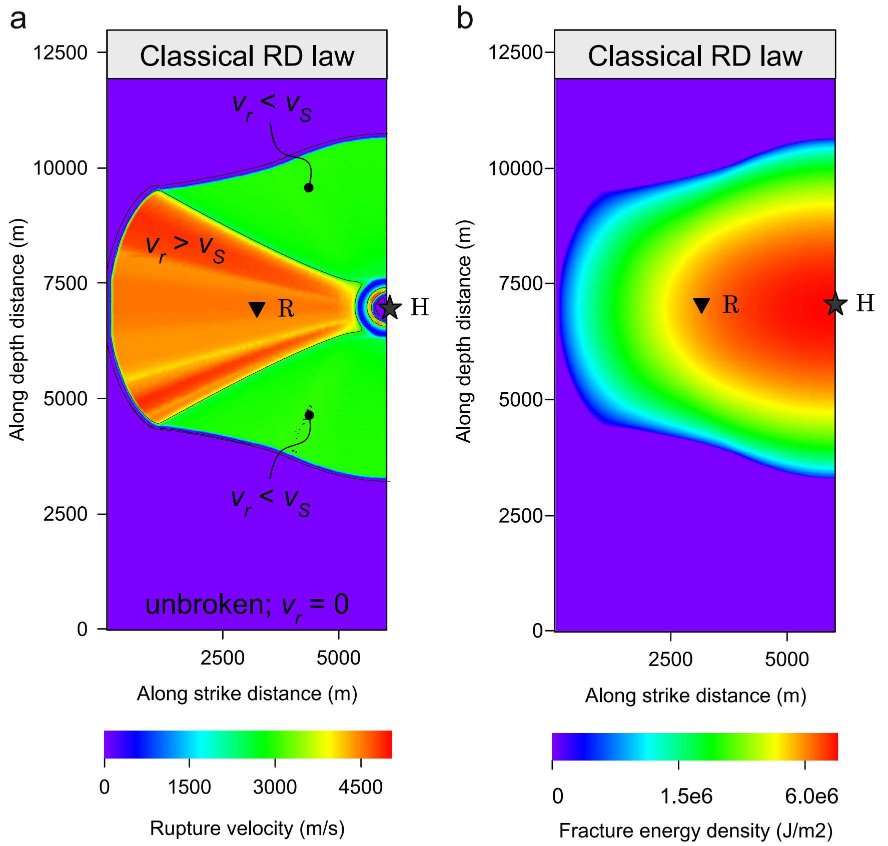

Fig. 3. Distribution of the fault plane of the rupture speed (panel (a)) and of the fracture energy density (panel (b)) in the case of RD law. The hypocenter H is denoted by a star; the receiver R where the solutions of Figs. 1 and 2 are plotted is denoted by a triangle. Only one half of the fault in the strike direction is plotted, since the rupture expands bilaterally from H.

resistance is paramount (see also Ohnaka and Yamashita (1989)). On the other hand, it should also be mentioned that some authors report both a slip and velocity dependence on the basis of laboratory data from friction experiments (e.g., Goldsby and Tullis, 2011; Reches and Lockner, 2010). Slip- and rate-dependent friction can be also found in the theoretical model of the mechanical lubrication, recently proposed by Bizzarri (2012b), but it has an ancient origin (see for instance Cochard and Madariaga (1996) and Fukuyama and Madariaga (1998)). On the other hand, we mention that Rice et al. (2001) (see his Section 3.2) suggest the possibility of ill-posedness in some slip- and velocity-dependent friction problems.

The FRD governing model significantly reduces the recurrence time (i.e., seismic cycle), as already shown by Bizzarri et al. (2011). The results presented here indicate that as long as v_T decreases, the direct effect becomes less important and consequently the hardening phase preceding the breakdown processes is shorter (Fig. 2a and b). Moreover, the maximum level of traction (namely τ_u^{eq}) decreases and the opposite holds for the residual level (τ_f^{eq}); see Fig. 2a. This contributes to significant reduction (up to a factor of 10; see Table 2) in the dynamic stress drop $\Delta\tau_b = \tau_u^{eq} - \tau_f^{eq}$. We see that, for a given set of governing parameters, the stress drop is reduced as long as v_T decreases. Indeed, $\Delta\tau_b$ is accomplished over a decreasing slip distance when v_T decreases; we found that the equivalent slip-weakening distance d_0^{eq} is reduced from roughly 5 times L (as theoretically expected for the canonical RD; see Bizzarri and Cocco, 2003) down to about 2 times L (see Fig. 2a and Table 2). We also found that the slip velocity—which is directly correlated to the degree of instability of a developing rupture; Bizzarri (2012a)—is also affected by the different choice of the transitional velocity; the peak of 33.8 m/s obtained in the RD case is reduced roughly by a factor of 30 when the FRD law is assumed and $v_T = 0.8$ μm/s (see Fig. 1a).

A prominent outcome of the present study is that the modification of the frictional behavior at high speeds can inhibit the supershear propagation which we have in the reference case of the canonical formulation of the RD law (see Figs. 3a and 4a). This can be interpreted in terms of the variations of τ_u^{eq} and τ_f^{eq} caused by the modifications of the constitutive models at high speeds. In turn these changes lead to a different value of the strength parameter

$$S = \frac{\tau_u^{eq} - \tau_0}{\tau_0 - \tau_f^{eq}} \quad (10)$$

as listed in Table 2. We found that for $v_T < 0.01$ m/s the rupture is no longer supershear, but exhibits rupture velocities everywhere lower than the S wave speed. This corresponds to values of S greater than 1.482. This number is slightly greater than the value of 1.19 found by Dunham (2007) in the case of 3-D ruptures spontaneously propagating on unbounded faults with homogeneous properties and obeying the linear SW friction law, but still lower than the value of 1.77 found by Andrews (1976) in 2-D bilateral ruptures. A lower value for the 3-D case is not unexpected, given that the convexity of the rupture front defocuses the stress wave radiation from slip within the crack (Dunham, 2007).

The above result does not imply that the FRD model is unable to simulate supershear earthquake propagation; indeed, by choosing adequate values of constitutive parameters we can still obtain supershear ruptures. For example, in the case of $v_T = v_*$ by selecting $a = 0.009$ (instead of $a = 0.012$) we can observe the generation of supershear speeds. At the same time, by selecting values of a smaller that 0.012 we can also observe significant dynamic stress drops also

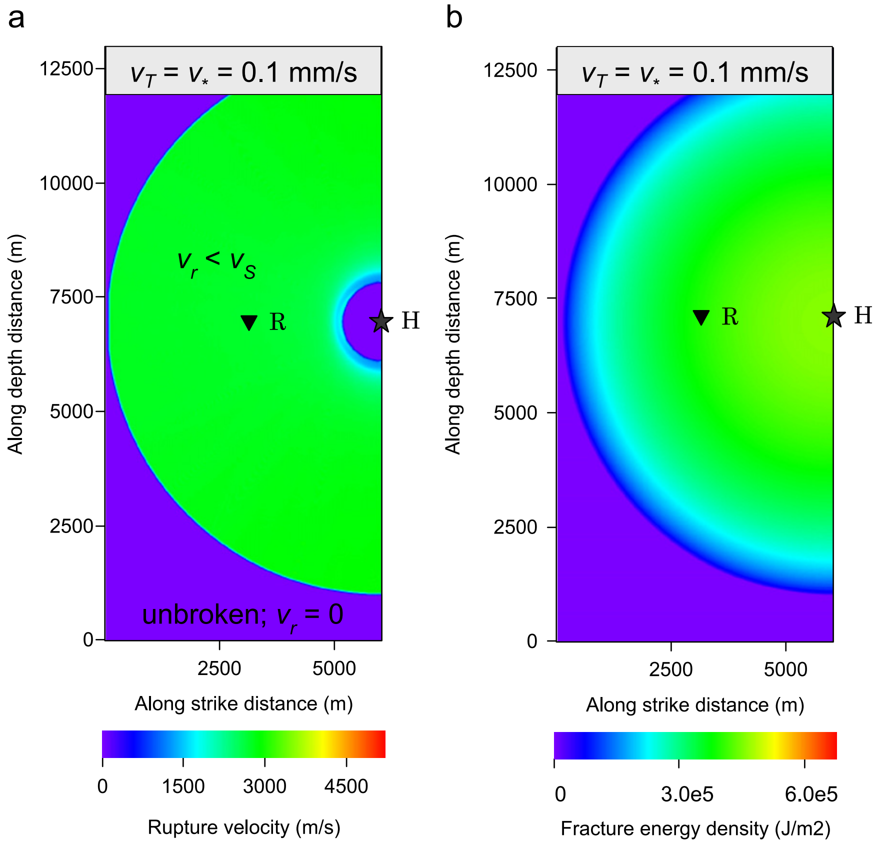

Fig. 4. Same as Fig. 3, but for the case of v_T=0.1 mm/s.

with the FRD law (even if they are systematically lower than those predicted by the classical RD model).

Remarkably, we can see that the peak of the fault slip velocity is attained when $\tau = \tau_f^{eq}$ in the reference case, which is supershear, but it is reached before that instant in other simulations, which are subshear (see Fig. 2b). This result confirms previous findings in 2-D by Tinti et al. (2004) and reinforces the limitation of the possibility to retrieve the slip-weakening distance from the time integral of the fault slip velocity history up to its peaks, as originally suggested by Mikumo et al. (2003).

As a consequence of the variations of the traction evolution, different values of v_T cause the fracture energy density to change significantly (see Figs. 3b and 4b). We emphasize that the governing parameters are exactly the same in all the numerical experiments and only the analytical formulation of the RD law changes. A reduction roughly of a factor of 35 is obtained from the canonical RD law to the FRD law with v_T=0.8 μm/s (see Table 2).

Our results indicate that high-end values of v_T (i.e., of the order of seismic slip rates) actually have little effect on the results compared to the standard RS model, as most of slip occurs while the slip velocity is less than v_T. On the other hand, for low-end values of v_T (i.e., of the order of micrometers per second) the effect is dramatic reductions in the dynamic stress drop and peak slip velocity. The presence of the parameter v_T can be regarded as an useful way to reconcile the results pertaining to low speed friction to those of high speeds. This additional parameter can implicitly accounts for various chemico-physical processes occurring during faulting. From an experimental point of view the presence of this additional parameter in the governing model can represent an advantage, in that the experimentalists have to determine the value of v_T in different configurations in order to be able to infer implications for the ongoing failure. For example, our results indicate that if the value of v_T is low in a specific configuration (namely $v_T < 0.01$ m/s for the parameters adopted here) then the rupture is expected to be the only subshear. Of course some caveats are needed, because the specific value mentioned above holds for the simplified geometry of our model and should be generalized to other rheological regimes.

5. Concluding remarks

In this paper we have considered the response of a spontaneous, fully dynamic 3-D rupture developing on a vertical strike slip fault, obeying a modified version of the canonical Ruina–Dieterich (RD) governing equation. This modification, originally proposed by Weeks (1993), assumes that the steady state friction becomes independent of the sliding speeds when the fault slip velocity exceeds a threshold, or transitional, value (v_T). This constitutive model (the frozen Ruina–Dieterich model, FRD) has been (and continues to be) employed in several theoretical studies (Tse and Rice, 1986; Belardinelli et al., 2003; Mitsui and Cocco, 2010; see also Horowitz and Ruina, 1989).

In the recent years a very large numbers of high speed friction experiments have been conducted and some of them interpret resulting data with an exponential decay of friction with the developed slip. The resulting friction laws are intrinsically different (both analytically and conceptually; see also the discussion in Bizzarri (2011b)) with respect to the canonical formulations of the RS friction laws that have been largely used in the numerical models. It is important to emphasize that these studies (e.g., Mizoguchi et al., 2007) simply fit the laboratory data, without any physical interpretation or theoretical basis. In this paper, we have demonstrated that the FRD model (Eq. (4)) provides a theoretical explanation of the exponential slip-dependent law proposed by Mizoguchi et al., 2007. This equivalence is analytically verified in the case of $v_T = v_*$, but is also verified (numerically) for different values of the threshold velocity (see Fig. 2a). This result is important, in that the exponential weakening has been postulated or inferred in many theoretical models (Lachenbruch, 1980; Matsu'ura et al., 1992; Rice, 2006 among others; see also Table 1 of Bizzarri (2011b) for a compendious summary) and in laboratory friction experiments (Wibberley and Shimamoto, 2005; De Paola et al., 2011), but always without any connection to a theoretical and physical framework (such that of the RS models).

We have also seen that the modification of the frictional behavior at high speeds tends to inhibit the supershear rupture propagation and thus can influence the frequency content of the resulting ground motions (Bizzarri et al., 2010).

The behavior of friction at high speed (and, in particular, the most important dependence of the friction on different physical observables) certainly is a central issue in the physics of the earthquake source (e.g., Goldsby and Tullis, 2011; Bizzarri and Cocco, 2006). Although relevant in absolute terms, it has been demonstrated (Bizzarri, 2011b) that the differences between various friction models do not become dramatic (both in terms of on-fault response and of ground motions and spectral contents) if the ruptures obeying the different governing models are energetically equivalent (namely, if they have the same fracture energy density). One of the outcomes of the present study is that it contributes to reconcile (as also previously done by Bizzarri and Cocco (2003) and Bizzarri (2011b)) the frameworks of the rate- and state-dependent friction laws and the exponential slip-dependent laws, often retrieved from laboratory experiments.

Despite the epistemic problem discussed above, the modeling of the fault dynamics with the modified version of the RS law considered in the present study, characterized by the additional tuning parameter v_T, remains a possibility. The present formulation, however, suffers from a potential limitation. Indeed, some laboratory experiments (Tsutsumi and Shimamoto, 1997; Tullis and Goldsby, 2003a, 2003b; Prakash and Yuan, 2004; Hirose and Shimamoto, 2005; Beeler et al., 2008; Han et al., 2010; Reches and Lockner, 2010; Goldsby and Tullis, 2011) indicate that the friction can drop significantly at high speeds. However, it is well known that governing models incorporating a strong velocity weakening, such as the flash-heating of asperity contacts, the mechanical lubrication, the viscous rheology accounting for melting processes, when applied to spontaneous earthquake models with realistic conditions, have the relevant problem of producing very high, often unrealistic slip velocities (Bizzarri, 2009; Bizzarri, 2012a). Moreover, the laboratory experiments mentioned above also suggest dramatic stress drops associated with the strong velocity weakening; unfortunately, there are no seismological evidences of nearly complete stress drops, in that typical values are in the range of 1–10 MPa (Kanamori and Anderson, 1975; Hanks, 1977; Abercrombie and Leary, 1993) and the same result has been confirmed by kinematic inferences (e.g., Tinti et al., 2005). Future friction and fracture experiments conducted at higher normal stresses and with more realistic loading velocity histories will tell us whether this dramatic weakening is a universal behavior for crustal faulting. In that case we have to admit that RS, also within the frozen formulation, breaks down and other constitutive models should be employed to simulate earthquake events.

To conclude, we emphasize again the canon: the choice of the analytical formulation of a constitutive model can have relevant consequences on the behavior of simulated earthquakes, both in terms of the rupture speed and energetic balance, and it should be corroborated by theoretical arguments and observations.

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