Pulse-like dynamic earthquake rupture propagation under rate-, state- and temperature-dependent friction

Andrea Bizzarri

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[1] Healing of faults is an important process in earthquake source physics since it accounts for a rapid restrengthening of the fault traction and for a consequent short slip duration, as indicated by slip inversions of seismic data. In this paper we show that a laboratory-derived constitutive model, with an explicit dependence on the temperature developed by frictional heat, can provide a suitable explanation for the generation of self-healing slip pulses. The model requires neither special modifications at low or high speeds nor the introduction of heterogeneities in the material properties, as previously proposed. We also demonstrate through numerical experiments of 3-D ruptures that the temperature evolution can discriminate between crack-like and slip pulses mode of propagation. In particular, we find that for a moderate level of strain localization (slipping zone width larger than 20 mm) ruptures behave as classical enlarging cracks. Citation: Bizzarri, A. (2010), Pulse-like dynamic earthquake rupture propagation under rate-, state- and temperature-dependent friction, Geophys. Res. Lett., 37, L18307, doi:10.1029/2010GL044541.

1. Introduction

[2] In contrast to an infinitely expanding crack-like earthquake rupture, for which the slip accumulated on the fault surface continues to increase through time, a pulse-like rupture exhibits a maximum (saturation) slip and a corresponding short slip duration (i.e., a short time interval, the dislocation rise time, when fault slip velocity is nonzero). In the latter mode of propagation, often referred to as self-healing, the slipping region occupies a small width behind the rupture front. There is a strong observational support suggesting that most earthquakes for which the slip inversion of high frequency seismic records is possible propagate as slip-pulses [Heaton, 1990; Beroza and Mikumo, 1996; see also Kamamori and Anderson, 1975]. The adoption of a short slip duration (compared to the duration of the event) is one of the most common assumptions made in kinematic models of ground motions [see, e.g., Schmedes et al., 2010].

[3] The interest in self-healing slip pulses has an early origin in theoretical studies of fracture mechanics [Yoffe, 1951; Broberg, 1978; Freund, 1979]. More recently, Nielsen and Madariaga [2003] analytically found a solution for a self-similar, pulse-like, 2-D, antiplane crack. In the literature some different mechanisms have been proposed to reproduce in dynamic models of earthquake ruptures the pulse-like mode of propagation: i) the introduction of rheological heterogeneities on the fault [e.g., Bizzarri et al., 2001; Tinti et al., 2005], ii) analytical regularizations of the constitutive law through the introduction of a threshold velocity [Perrin et al., 1995; Zheng and Rice, 1998], iii) ad hoc modifications of the state variable evolution equation [Nielsen et al., 2000; see also Cocco et al., 2004] or governing model [Cochard and Madariaga, 1996], and iv) a contrast in material properties (inducing in turn different normal stress polarities) across the sliding interface of an 2-D, inplane crack [Weertman, 1980; Andrews and Ben-Zion, 1997; Adams, 1998]. Very recently, numerical simulations in 3-D by Bizzarri [2009a] and in 2-D by Noda et al. [2009] show that the incorporation of the flash heating of micro-asperity contacts (essentially consisting of a modification of the Ruina’s law at high slip velocities) can lead to a sustained pulse-like rupture propagation. In this study we will show through numerical experiments of 3-D dynamic earthquake ruptures, spontaneously spreading over a fault of finite width, that a laboratory-based friction law with an explicit dependence on the temperature caused by frictional heat can produce the generation of sustained pulse-like propagation, even in homogeneous conditions.

2. Fault Governing Equation

[5] A large number of theoretical and experimental efforts in recent years have attempted to clarify the competing dependences of the fault friction on the different physical observables, describing the various dissipative processes taking place during an earthquake rupture [see Bizzarri, 2009b, and references therein]. Although the most prominent dependence is still matter of a lively discussion [Bizzarri and Cocco, 2006c], many numerical studies have been conducted in the framework of the laboratory-derived rate- and state-dependent friction laws [e.g., Dieterich, 1979; Ruina, 1983]. These constitutive equations state that the frictional resistance is a function of the slip history and not only of the instantaneous slip velocity. The previous slipping episodes are described by the temporal evolution of some state variables that are related to the microstructural conditions of the sliding surfaces that in turn change according to the evolution of slip.

[6] An important improvement in the formulation of such a governing model is represented by the introduction of an explicit dependence on temperature in both fault friction expression and state variable(s) evolution, first proposed by Chester and Higgs [1992]. In their low-velocity laboratory experiments these authors found that an abrupt increase in temperature induces direct and evolution effects similar to, but in the opposite sense as, those caused by a step increase
Table 1. Model Discretization and Reference Constitutive Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lamé constants, (\lambda = G)</td>
<td>27 GPa</td>
</tr>
<tr>
<td>S wave velocity, (v_s)</td>
<td>3 km/s</td>
</tr>
<tr>
<td>P wave velocity, (v_p)</td>
<td>5.196 km/s</td>
</tr>
<tr>
<td>Cubic mass density, (\rho)</td>
<td>3000 kg/m³</td>
</tr>
<tr>
<td>Fault length, (L)</td>
<td>12 km</td>
</tr>
<tr>
<td>Fault width, (W_f)</td>
<td>11.6 km</td>
</tr>
<tr>
<td>Spatial grid size, (\Delta x)</td>
<td>8 m</td>
</tr>
<tr>
<td>Time step, (\Delta t)</td>
<td>4.44 × 10⁻⁴ s</td>
</tr>
</tbody>
</table>

**Fault Constitutive Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective normal stress, (\sigma_n^{eff})</td>
<td>120 MPa</td>
</tr>
<tr>
<td>Logarithmic direct effect parameter, (a)</td>
<td>0.016</td>
</tr>
<tr>
<td>Evolution effect parameter, (b)</td>
<td>0.02</td>
</tr>
<tr>
<td>Characteristic scale length, (L)</td>
<td>2 × 10⁻² m</td>
</tr>
<tr>
<td>Reference value of friction coefficient, (\mu_\star)</td>
<td>0.56</td>
</tr>
<tr>
<td>Initial shear stress, (\tau_0)</td>
<td>70.52 MPa</td>
</tr>
<tr>
<td>Initial temperature in the center of the slipping zone, (T_0)</td>
<td>483.15 K</td>
</tr>
<tr>
<td>Activation energies, (Q_a) and (Q_b)</td>
<td>1 × 10⁷ J/mol</td>
</tr>
<tr>
<td>Heat capacity for unit volume of the bulk composite, (c)</td>
<td>3 × 10⁴ J/(mol·K)</td>
</tr>
<tr>
<td>Thermal diffusivity, (\chi)</td>
<td>1 × 10⁻⁶ m²/s</td>
</tr>
<tr>
<td>Slipping zone thickness, (2w)</td>
<td>2 mm</td>
</tr>
</tbody>
</table>

*This corresponds to 210°C, a representative temperature for the hypothermal depths of typical crustal earthquakes.

**Values from laboratory experiments of Chester [1994] and Blanpied et al. [1998]. The same values were assumed by Kato [2001].

In slip velocity. Moreover, they found that the characteristic distance controlling the evolution effect (i.e., the decrease of traction) is roughly the same for both temperature and rate variations. This results in the following governing model [see Chester, 1994]:

\[
\tau = \mu_\star + a \ln \left( \frac{v}{v_\star} \right) + \Theta + b \left( \frac{1}{T - T_\star} \right) \sigma_n^{eff} - \frac{d}{dt} \Theta = -v \frac{b}{L} \left[ \ln \left( \frac{v}{v_\star} \right) + \Theta + \frac{bQ_a}{R} \left( \frac{1}{T - T_\star} \right) \right]
\]  

(1)

In the previous equations \(\tau\) is the magnitude of the shear traction (\(\tau = \mu_\star \sigma_{n,\star}^{eff}\) being \(\sigma_{n,\star}^{eff}\) the effective normal stress), \(a\) and \(b\) are dimensionless constitutive parameters, \(L\) is a characteristic length, \(\mu_\star\) and \(v_\star\) are reference parameters for friction coefficient \(\mu\) and fault slip velocity modulus \(v\), respectively, and \(\Theta\) is the state variable. \(R\) is the universal gas constant (\(R = 8.314472 \text{ J/(K mol)}\)), \(T^f\) and \(T^s\) are actual and reference absolute temperatures, respectively, and \(Q_a\) and \(Q_b\) are apparent activation energies controlling the rate-limiting processes pertaining to the direct and evolution effect, respectively. Although \(\sigma_n^{eff}\) is assumed to be constant through time, we note that \(\mu\) itself implicitly depends on \(\sigma_n^{eff}\) because of its explicit dependence on \(T^f\) (which in turn depends on the normal stress via the fault traction; see next equation (2)).

[7] The basic physical foundation of the rate-, state- and temperature-dependent model (1) is that it is attributed to the slip rate an Arrhenious relationship of the form

\[
\frac{d\ln(v)}{d[T]} = -\frac{Q_a}{R}
\]

in order to describe the thermally activated micro–mechanisms during the coseismic slip. On the other hand, Nakatani [2001] interpreted the dependence of \(\mu\) on \(T^f\) through a linear variation of the parameter \(a\) for the increasing temperature at contacts. We notice that if we neglect temperature changes (i.e., if \(T^f\) always equals \(T^s\)) then the model (1) reduces to the classical Ruina-Dieterich law (RD thereinafter), in that the last terms vanish both in the analytical expression of \(\mu\) and in the evolution equation for \(\Theta\).

[8] In our dynamic earthquake rupture models the temperature \(T^f\) expresses the solution of Fourier's heat conduction equation, which reads [Bizzarri and Cocco, 2006a]:

\[
T^f(x, t) = T'_0 + \frac{1}{2c_w} \int_0^{t-c} \text{erf} \left( \frac{w}{2\sqrt{\chi(t-t')}} \right) \cdot \tau(x_1, x_3, t') dx_1 dx_3 dx_3
\]

(2)

where the doublet \((x_1, x_3)\) defines a node on the vertical strike-slip fault plane, \(T'_0\) is the initial (i.e., at \(t = 0\)) temperature, \(w\) is the half-thickness of the slipping zone where the slip is concentrated (see Ben-Zion and Sammis [2003] for a general description of the fault structure), \(c\) is the heat capacity for unit volume, \(\chi\) is the thermal diffusivity, \(\text{erf}(\cdot)\) is the error function and is an arbitrarily small, positive, real number (see Bizzarri and Cocco [2006a] for analytical details).

3. Pulse-Like Rupture Propagation

[9] The solution of the fundamental elastodynamic equation neglecting body forces is accomplished numerically via the finite difference code described in detail by Bizzarri and Cocco [2005]. The adopted fault boundary condition is expressed by the coupled equations (1) and (2). If not otherwise mentioned, we assume the parameter values tabulated in Table 1. The fault geometry is the same that of Bizzarri [2010b, Figure 2].

[10] In Figure 1 we plot the comparison between the solution obtained with model (1) (red curves) and that obtained with the classical RD law (black curves). We can clearly see that the RD law predicts, as expected, a crack-like mode of propagation; the final value of the sliding velocity is roughly of the order of \(2\sqrt{\Delta \tau_0} G \), where \(\Delta \tau_0\) is the \(S\) wave speed, \(\Delta \tau_0\) is the dynamic stress drop (expressing the difference between the initial stress and the stress level after the dynamic stress release) and \(G\) is the rigidity of the medium surrounding the fault (see Nielsen and Carlson, 2000; Bizzarri and Cocco, 2003). On the contrary, model (1) produces a pulse-like rupture (Figure 1a), in that the fault slip velocity has a compact support. We emphasize that the additional terms in model (1), containing the dependence on the temperature evolution, circumvent the problems that the direct effect term prevents slip healing from occurring. The time evolution of the temperature is reported in the inset of Figure 1a. The temperature change is significant in both cases; with the RS law \(T^f\) continues to increase because slip velocity is non zero in the whole time window here considered, while it exhibits a maximum in the case of model (1), as a consequence of the compactness of the support of \(v\) in this case.

[11] The prominent difference between two solutions is that in model (1) the state variable significantly increases after the breakdown process, definitely more than in the
case of the RD law (Figure 1c). This basically represents a temperature-hardening effect, which has been experimentally observed by Blanpied et al. [1998], or alternatively an inverse dependence on velocity (inset in Figure 1d). Correspondingly, we can notice a fast restrengthening in the case of model (1), consisting in a rapid increase in traction after the stress release. On the contrary, in the RD case the final level of friction is maintained during the enlarging crack-like propagation (Figure 1d). We emphasize that the increase in the fault resistance is the ultimate cause of the healing of slip and of the difference between the two solutions.

It is apparent from Figure 1b that for the RD law, after the dynamic stress drop, the traction tends to reach a new steady state, i.e., \( \frac{d}{dt} \Theta = 0 \) (as expected due to the well-known self-damping behavior of \( \Theta \) e.g., Bizzarri and Cocco, 2003). In the governing model (1) the final friction is significantly lower than its steady state value \( \mu^\infty(v, T_f) = \mu_\infty - (b - a) \ln \left( \frac{v_{\text{peak}}}{v} \right) - \frac{b Q_a}{R} \left( \frac{1}{T_f} - \frac{1}{T_e} \right) \) (see also Figure 1c).

Figure 1. Comparison between the solutions obtained with the constitutive model (1) and the RD law, calculated at a fault point located at hypocentral depth and at a distance of 3 km from the hypocenter. (a) Time history of slip velocity \( v \) (with the value of the slip duration, where \( v \neq 0 \), in case of model (1)). In the inset we plot the two evolutions of temperature change (\( \Delta T_f = T_f - T_0 \)). (b) Time history of friction coefficient \( \mu \). (c) Evolution of the state variable \( \Theta \). (d) Slip-weakening curve, with inset reporting the phase diagram (i.e., traction vs. slip velocity). \( d_0^\text{eq} \) denotes the slip accumulated up to the end of the stress drop, when \( \tau = \tau^\text{motion} \). In Figures 1b and 1c the steady state curves (\( \mu^\infty \) and \( \Theta^\infty \), respectively) are superimposed.

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[13] In the auxiliary material we discuss additional numerical simulations where we adopt different constitutive parameters \( (a, b, \sigma_{\text{off}}^\infty \text{ and } T_0^\infty) \); these experiments confirm and generalize (see Figure AM.1 of Text S1) the results previously discussed.1

[14] Although both the ruptures behave as subsonic earthquakes, in that both models exhibit rupture velocities lower than \( v_S \), the rupture obeying the RD law is slightly less unstable; from Figure 1a we have that the rupture front arrives later in the target fault receiver. Moreover, model (1) predicts a faster stress release, accomplished for smaller amounts of slip (i.e., the equivalent slip-weakening distance, \( d_0^\text{eq} \), is smaller (Figure 1d)). This is not surprising, given that the strength parameter (the value of which is inversely proportional to the degree of instability of a fault, \( S = (\tau^\text{eq} - \tau_\Theta)/(\tau_0 - \tau^\text{eq}) \), where \( \tau_0 \) is the initial shear stress, \( \tau^\text{eq} \) is the peak traction and \( \tau^\text{eq} \) its values after the stress drop) is 3.28 for the RD model and 2.66 for governing model (1). In

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1Auxiliary materials are available in the HTML. doi:10.1029/2010GL044541.
agreement with the results of Tinti et al. [2004, Figure 2a], the peak in $v$ is attained before the stress drop is completed (see Figure AM.2 of Text S1) or in other words for a slip $u < d_0^e$, further demonstrating that the retrieval of the slip-weakening distance from a slip rate history is problematic.

It is apparent from Figure 1d that the inclusion of temperature in the constitutive model decreases the value of the fracture energy density ($E_G = 0.13$ MJ/m$^2$ for the model (1) instead of $E_G = 0.31$ MJ/m$^2$ for the RD law), which is known to control the rupture dynamics [Bizzarri, 2010b; Schmedes et al., 2010].

4. Self-Healing or Enlarging Crack?

The distinction between the classical enlarging crack-like rupture mode and the self-healing mode basically depends on the presence in model (1) of the additional terms containing the temperature due to the frictional heat. These terms are controlled by the values of the activation energies and by $b - a$, expressing the degree of instability of a fault [Tinti et al., 2005, and references therein]. Moreover, they depend on $T'$, which in turn is essentially controlled by the degree of the strain localization (i.e., by slipping zone thickness $2w$ [see Bizzarri and Cocco, 2006b]).

In Figure 2 we report the results obtained for different temperature evolutions, corresponding to various thicknesses of the slipping zone (from 2 mm to 1 m). We can note systematic trends as far as $2w$ increases: i) $\Delta T'$ decreases (inset in Figure 2a), as previously found [Bizzarri and Cocco, 2006b]), ii) after the breakdown process the traction tends to approach its steady state value (Figure 2b), as in the RD case (black line in Figure 1c), iii) the restrengthening process is less evident (Figure 2c), and consequently iv) the mode of propagation switches from self-healing to enlarging crack (Figure 2a). In particular, we have that for $w > 10$ mm (with other parameters as in Table 1) the rupture behaves essentially as a classical enlarging crack; in these cases the temperature evolution is not able to dominate the direct effect. Finally, we can note that v) the breakdown process becomes longer and therefore $d_0^e$ tends to increase; the ratio $d_0^e/L$ roughly changes from 2 to 5, by changing $w$ from 1 mm to 0.5 m.

5. Discussion and Conclusions

The process of fault healing is of prominent importance in the mechanics of earthquake ruptures, because it predicts a rapid restrengthening (during which the fault recovers its frictional strength) and correspondingly it implies a short duration of slip (compared to the event duration), as suggested by inversions of recorded seismic data [Heaton, 1990; Beroza and Mikumo, 1996].

In this paper, through the numerical solution of the fundamental elasto-dynamic equation for planar faults, we have shown that a laboratory-derived constitutive model, with an explicit dependence on temperature developed by frictional heat, is a suitable mechanism to model the self-healing mode of propagation, basically consisting in slip pulses (in that there is a cessation of slip at an interior point on the rupture surface).

One of the outcomes of the present study is that the governing model (1) has an important advantage, since it requires neither an arbitrary regularization of the friction laws at low slip rates [Perrin et al., 1995; Zheng and Rice, 1998] nor ad hoc modifications of the constitutive relations [Cochard and Madariaga, 1996; Nielsen et al., 2000]. These approaches can be essentially regarded as mathematical manipulations of the original constitutive equations that do not have either an experimental support or a strong physical basis. Indeed, the constitutive model (1) is empirical in its
origin and its validity has been also supported by the laboratory experiments of Blanpied et al. [1998]. This model has been also recently used by Kato [2001] to simulate the time occurrence of first unperturbed instability (the transition from a slow, quasi-static phase to a dynamic stage) of a simple 1-D spring slider approximation of a fault zone having null thickness.

[22] An important limitation of the model (1) is that it has been obtained for velocities smaller than those typically realized in crustal earthquakes and those simulated in our synthetic ruptures. In this paper we have conservatively assumed it is valid also at seismogenic depths and for higher speeds; we believe that new laboratory experiments would eventually clarify if this assumption is correct. They would also test whether the temperature-hardening that we model will persist for seismic slip velocities.

[23] We have also found here that the pulse width (or time duration; \( t_{\text{pulse}} \) in Figure 1a) is not fixed a priori as a characteristic input parameter, as in previous models [e.g., Nielson, 2000], but is depends on the fault dynamics, even in homogeneous conditions.

[24] Although not so dramatic as those predicted by other constitutive models [e.g., Noda et al., 2009; Bizzarri, 2010a], the governing model (1) describes the dynamic weakening of faults and produces dynamic stress drops within the range typically inferred for earthquakes observations [e.g., Kanamori and Anderson, 1975]. At the same time, the peaks of \( v \) are more realistic than those obtained with flash heating [Noda et al., 2009].

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References


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