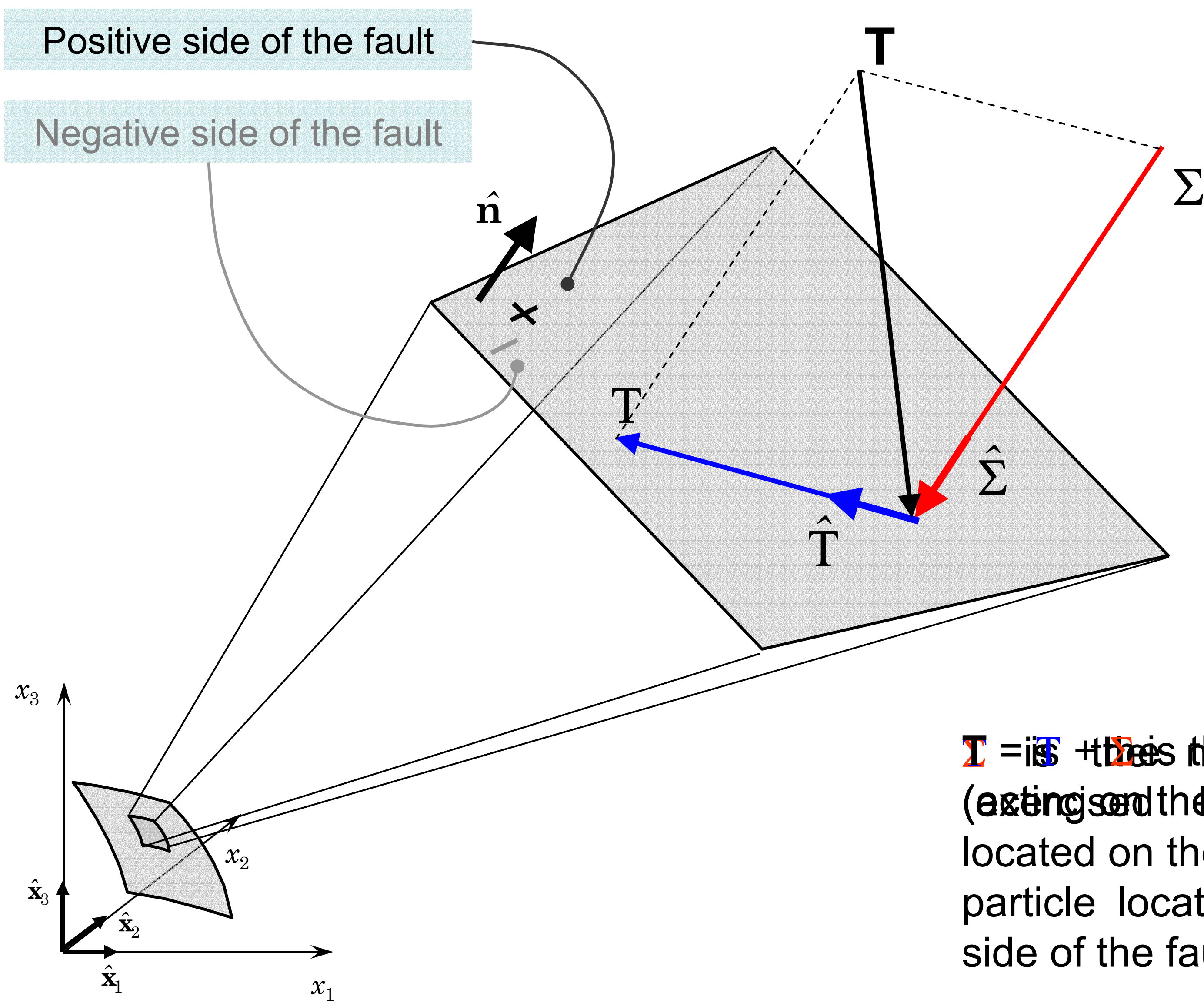
Fault governing laws
(constitutive equations)

Notations and symbols



T is the total traction (exerted by a particle located on the +ve side on a particle located on the –ve side of the fault surface)

$\mathbf{T} = \mathrm{T} + \Sigma$ total traction (acting on the fault surface).

$T_j = n_i \sigma_{ij}^{eff}$ *Cauchy*' s formula, where $\mathbf{T} = (T_1, T_2, T_3)$, $\mathbf{n} = (n_1, n_2, n_3)$ and

$$\sigma_{ij}^{eff} = \sigma_{ij} - p_{fluid}\,\delta_{ij} = \begin{bmatrix} \sigma_{11} - p_{fluid} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} - p_{fluid} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} - p_{fluid} \end{bmatrix}$$

$\mathcal{T}_j = n_i \sigma_{ij}^{eff} - n_j (n_i \sigma_{ij}^{eff} n_j^{\mathrm{T}})$ shear traction

$\Sigma_j = n_j (n_i \sigma_{ij}^{eff} n_j^{\mathrm{T}})$ normal traction

Fracture Criteria & Constitutive Laws

1. FRACTURE CRITERION

- Condition that specify, at a given fault point and at a given time, if there is a rupture or not.
- It can be expressed in terms of **energy**, in terms of **maximum frictional resistence**, and so on.
- It is based on (*i*) the *Benioff (1951)* hypothesis: The fracture occours when the stress in a volume reaches the rock strength
  or, analogoulsy,
  (*ii*) the *Reid (1910)* statement: The fracture takes place when the stress attains a value greater than the rock can endure.

2. CONSTITUTIVE LAW

- **Analytical relation existing between the components of the stress tensor and physical observable(s), like the slip, the slip velocity, the state variable, etc..**

- From a mathematical point of view it is a **Fault Boundary Condition** (**FBC**) that controls earthquake dynamics and its complexity in space and in time.

- Its simplest form consider only **two frictional levels**, τ_u and τ_f; it accounts for stress drop (or stress realease), but the process is instantaneous: there is a singularity at crack tip.

- **Cohesive zone models**: *Barenblatt (1959a, 1959b)*, *Ida (1972)*, *Andrews (1976a, 1976b)*. In these models the singularity is removed and the sress release occours over a breakdown zone distance X_b and in a breakdown zone time T_b.

- Friction laws (Rate and State dependent f. l.): *Dieterich (1976)*, *Ruina (1980, 1983)*. They accounts for fault spontaneous nucleation, re - strengthening, healing, etc.

CONSTITUTIVE LAW (continues)

- “ The central issue is *whether* faults obey simple friction laws, and if so, what is the friction coefficient associated with fault slip “ (*Scholz and Hanks, 2004*).

CONSTITUTIVE LAW (continues)

- In full of generality we can express the constitutive (or governing) as:

$$\tau = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\ \sigma_n^{eff}(\sigma_n, p_f)$$

where:

1st – order dependencies

u is the Slip (i. e. displ. disc.) modulus,

v is the Slip Velocity modulus (its time der.),

$\Psi = (\Psi_1, \ldots, \Psi_N)$ is the State Variable vector,

T is the Temperature (accounting for Ductility, Plastic Flow, Melting and Vaporization),

H is the Humidity,

λ_c is the Characteristic Length of surface (accounting for Roughness and Topography of asperity contacts),

h is the Hardness,

g is the Gouge (accounting for Surface Consumption and Gouge formation),

C_e is the Chemical Environment

Strength & Constitutive Laws

1. THE STRENGTH PARAMETER

- **Hystorically introduced by *Das and Aki (1977a, 1977b)* to have a quantitative extimate of the ability to fracture for a fault**
- Its expression can be generalized as:

  $$S = (\mu_u \sigma_n^{eff} - \tau_0) / (\tau_0 - \mu_f \sigma_n^{eff})$$

  where μ are the friction coefficient.

- We can also define

2. THE FAULT STRENGTH

- **Is the parameter that quantify the Strenght in the more general case, in which a fault is described by a rhealistic friction laws**

$$S^{fault} = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\ \sigma_n^{eff}(\sigma_n, p_{fluid})$$

Slip - Dependent Friction Laws

1. LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f)\dfrac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

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ilaw = 21
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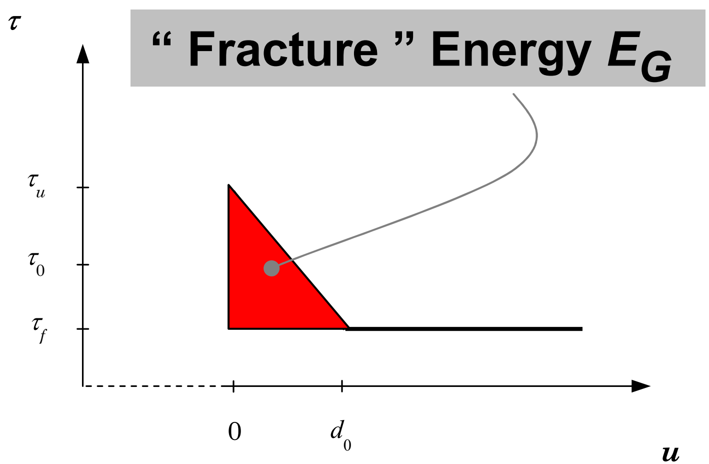
SW



Barenblatt (1959a, 1959b), Ida (1972), Andrews (1976a, 1976b), and many authors thereinafter

d_0 is the characteristic slip – weakening distance

2. NON LINEAR SLIP – WEAKEING LAW WITH SLIP – HARDENING

$$\tau = \left\{ \left[\left(\tau_0 - \mu_f\right)\left(1 + \alpha_{OW} \ln\left(1 + \frac{u}{\beta_{OW}}\right)\right)\right] e^{-\frac{u}{d_0}} + \mu_f \right\} \sigma_n^{eff}$$

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ilaw = 23
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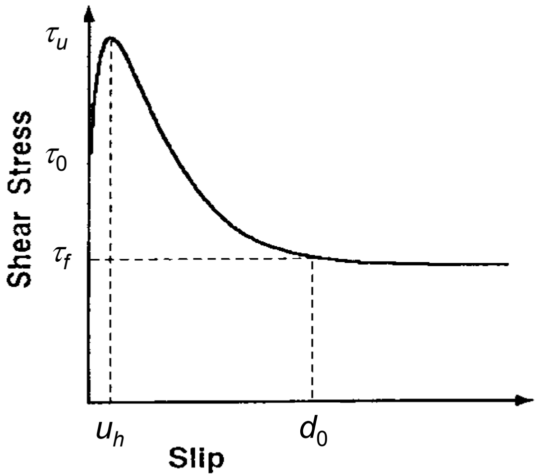
OW

$$u_h : \left.\frac{d\tau}{du}\right|_{u_h} = 0 ; \qquad \begin{cases} u_h = r d_0 & (\text{e.g. } r = 0.1) \\ \tau(u_h) = \tau_u \end{cases}$$



Ohnaka and Yamashita (1989) and the following papers by Ohnaka and coworkers

u_h is associated with the preparatory phase of the imminent macroscopic failure in the cohesive zone. It accounts for micro–cracking

Rate - and State - Dependent Friction Laws

1. DIETERICH IN REDUCED FORMULATION

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi v_*}{L} \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi v}{L} \end{cases}$$

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ilaw = 31
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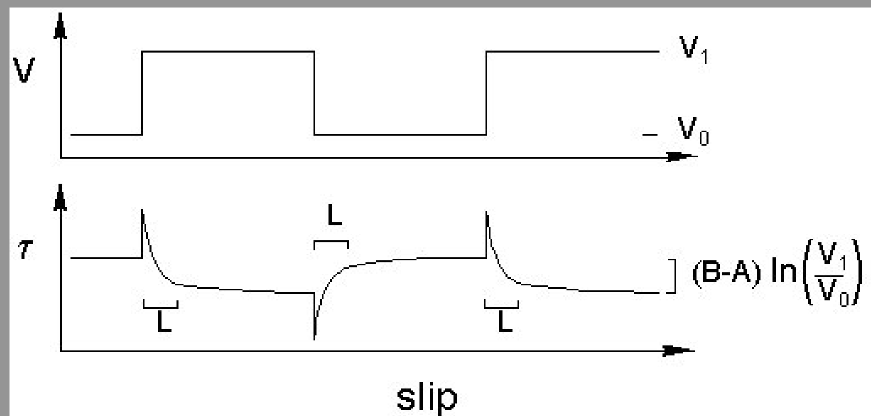
DR

However, while in velocity stepping experiments the traction response following the velocity variation is directly controlled by the parameter *L*, its effects are much less evident during the dynamic rupture propagation.

Bizzarri and Cocco (2005)

Response to an abrupt jump in load



2. RUINA – DIETERICH

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v}\right) + b \ln\left(\dfrac{\Psi\, v_*}{L}\right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi = -\dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L}\right) \end{cases}$$

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ilaw = 32
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RD

Ruina (1980, 1983), Beeler et al. (1984), Roy and Marone (1996)

3. DIETERICH – RUINA WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{\ eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi\, v}{L} - \left(\dfrac{\alpha_{LD} \Psi}{b\, \sigma_n^{\ eff}} \right) \dfrac{\mathrm{d}}{\mathrm{d}t} \sigma_n^{\ eff} \end{cases}$$

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ilaw = 31

decis10=T
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DR

Linker and Dieterich (1992), Dieterich and Linker (1992), Bizzarri and Cocco (2005b, 2005c)

4. RUINA – DIETERICH WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{\,eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) - \left(\dfrac{\alpha_{LD} \Psi}{b\, \sigma_n^{\,eff}} \right) \dfrac{\mathrm{d}}{\mathrm{d}t} \sigma_n^{\,eff} \end{cases}$$

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ilaw = 32

decis10=T
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RD

Linker and Dieterich (1992), Bizzarri and Cocco (2006b, 2006c)

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Support Slides: Parameters, Notes, etc.

To not be displayed directly. Referenced above.

Thermal pressurization:

Sibson (1973), Lachenbruch (1980), Mase and Smith (1985, 1987), Andrews (2002), Bizzarri and Cocco (2006b, 2006c) .

Morrow et al. (1984) show that gouge contain water

Gouge behaviour:

Marone et al. (1990), Marone and Kilgore (1993), Mair and Marone (1999), Mair et al. (2002)

Frictional melting:

Jeffreys (1942), McKenzie and Brune (1972), Richards (1977), Sibson (1977), Cardwell et al. (1978), Allen (1979)

Mechanical lubrication:

Spray (1993), Brodsky and Kanamori (2001), Kanamori and Brodsky (2001)

Acustic fluidization:

Melosh (1979, 1996)

Gouge gelation:

Goldbsy and Tullis (2002), Di Toro et al. (2004)

Bi – material interface:

Andrews and Ben – Zion (1997), Harris and Day (1997)

Humidity effects:

Dieterich and Conrad (1984)

Characteristic length of surface effects:

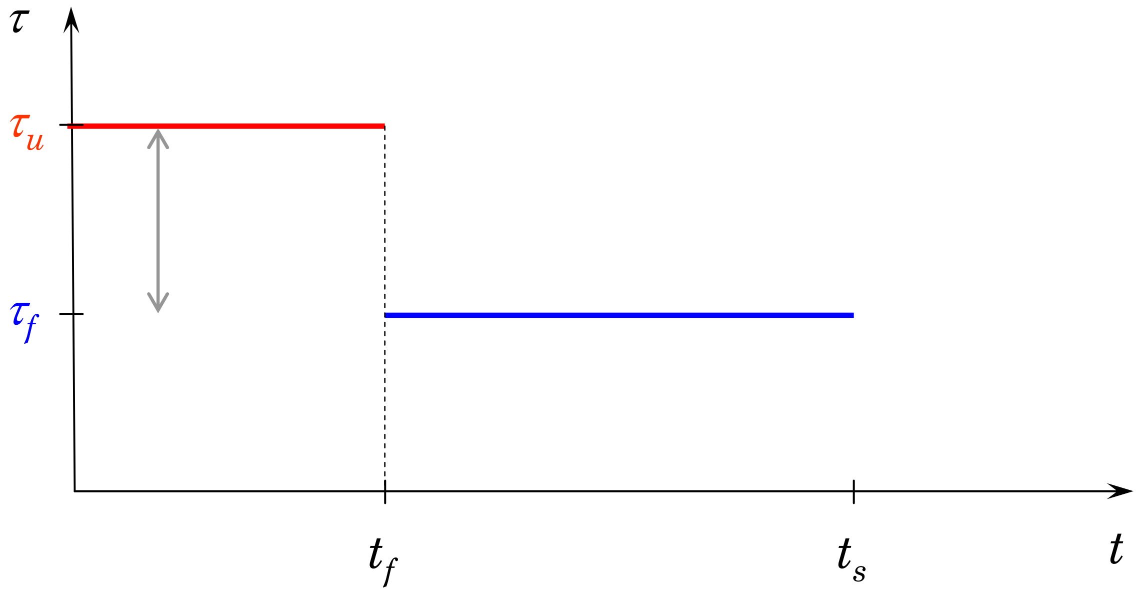
Ohnaka and Shen (1999), Ohnaka (2003)

Simplest friction models

At a particular fault point ξ (following *Savage and Wood, 1971*; *Scholz, 1990*)

Maximum (or upper, or yield) stress

Kinetic (or frictional) stress



Failure time (or rupture onset)

Rupture arrest

Strength excess: $\tau_u - \tau_0 = 0$

Dynamic stress drop: $\Delta\tau_d = \tau_0 - \tau_f$

Simplest friction models

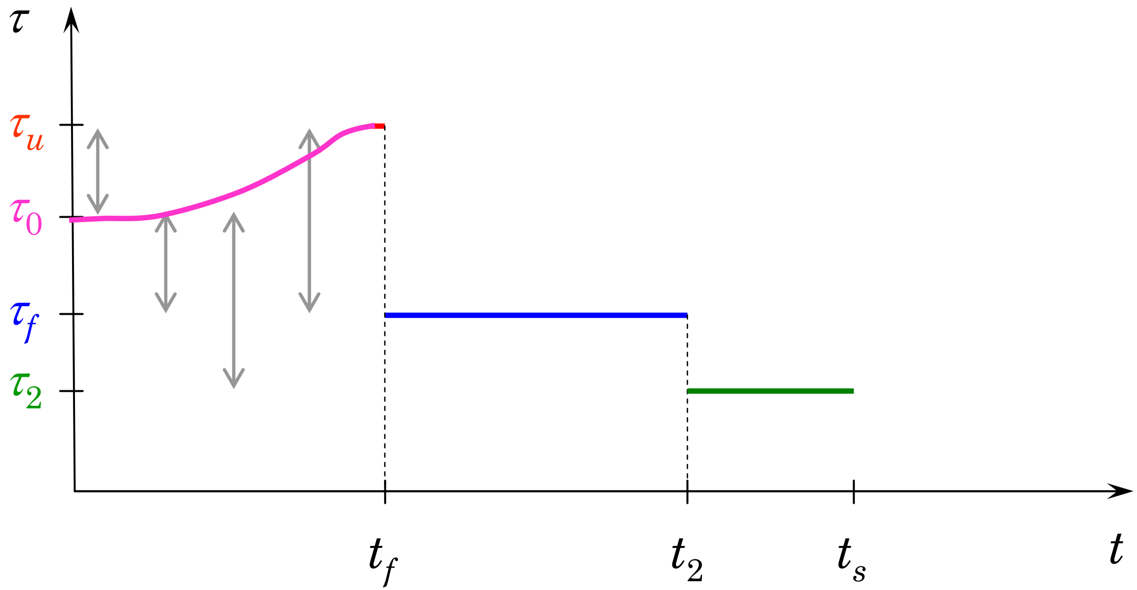
At a particular fault point ξ (following *Savage and Wood, 1971*; *Scholz, 1990*)

Maximum (or upper, or yield) stress

Initial stress

Kinetic (or frictional) stress

Residual stress



Strength excess: $\tau_u - \tau_0$

Dynamic stress drop: $\Delta\tau_d = \tau_0 - \tau_f$

Static stress drop: $\Delta\tau_s = \tau_0 - \tau_2$

Breakdown str. drop: $\Delta\tau_b = \tau_u - \tau_f$

Failure time (or rupture onset)

Dynamic overshoot

Rupture arrest

- *Savage and Wood (1971)* also define:

Mean stress: $<\tau> = ½ (\tau_u + \tau_2)$

Seismic efficiency: $\eta = E_s/E$, where: E_s is the seismic energy
E is the total available energy

Apparent stress: $\tau_a = \eta <\tau>$

- Direct observation of the absolute stress near an earthquake is not feasible, but it is possible (*Wyss and Brune, 1968*) calculate τ_a and stress drop from physical observables.

Slip - hardening effect

- **The slip – hardening (SH) phenomenon has been also found in seismological inversion studies (e. g. *Quin, 1990*; *Miyatake, 1992*; *Mikumo and Miyatake, 1993*; *Beroza and Mikumo, 1996*; *Ide, 1997*; *Bouchon, 1997*).**