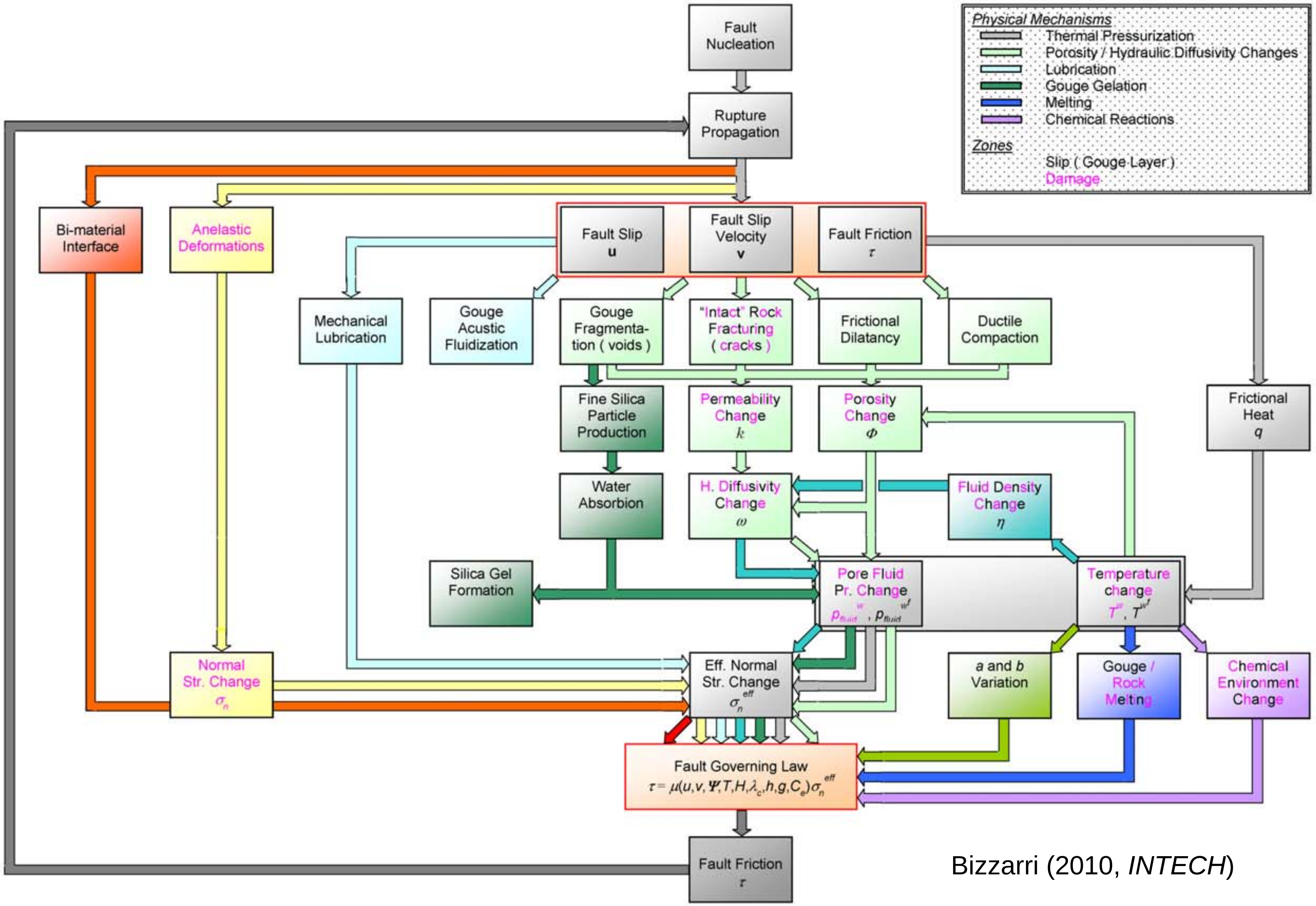
Fault governing laws
(constitutive equations)



Bizzarri (2010, *INTECH*)

Fracture Criteria & Constitutive Laws

- In full of generality we can express the constitutive (or governing) as:

$$\tau = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\ \sigma_n^{eff}(\sigma_n, p_f)$$

where:

1st – order dependencies

u is the slip (i. e. displ. disc.) modulus,

v is the slip velocity modulus (its time der.),

$\Psi = (\Psi_1, \ldots, \Psi_N)$ is the state variable vector,

T is the temperature (related to ductility, plastic flow, melting and vaporization),

H is the humidity,

λ_c is the characteristic length of surface (accounting for roughness and topography of asperity contacts),

h is the hardness,

g is the gouge (accounting for surface consumption and gouge formation),

C_e is the chemical environment

Towards real – world conditions

u_{tot} ~ several m

v ~ several m/s

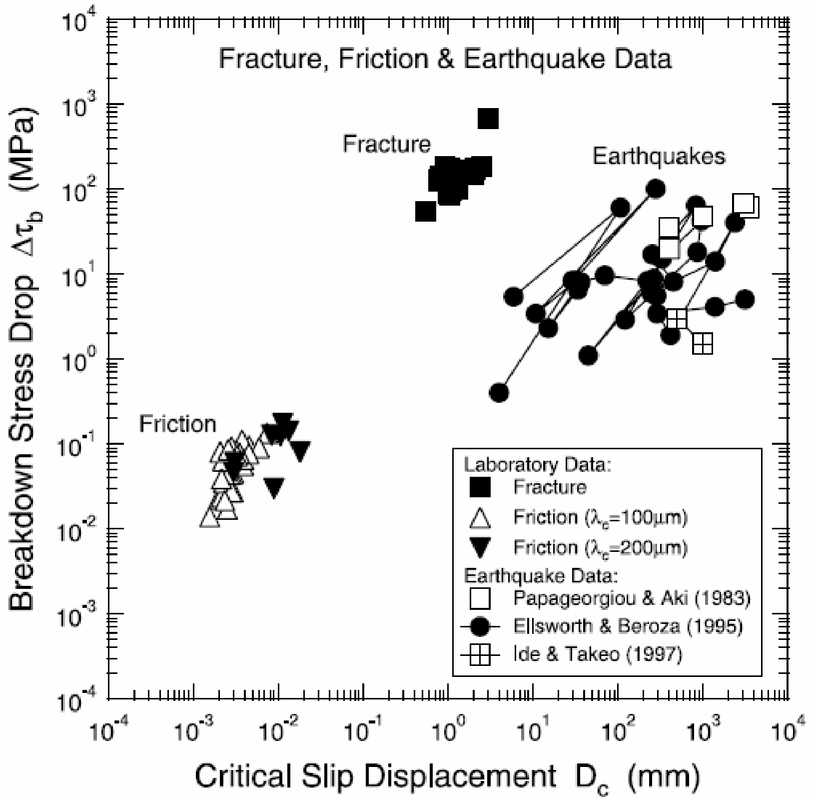
σ_n^{eff} = 100 – 200 MPa

Classical laboratory stick – slip experiments (Dieterich, 1981)

u_{tot} up to 1.4 mm

v up to 25 μm/s

σ_n^{eff} = 10 MPa



From Ohnaka (2003)

High velocity rotary friction apparatus

u_{tot} = infinite

v = 0.1 μm/s – 10 m/s

σ_n^{eff} < 20 MPa

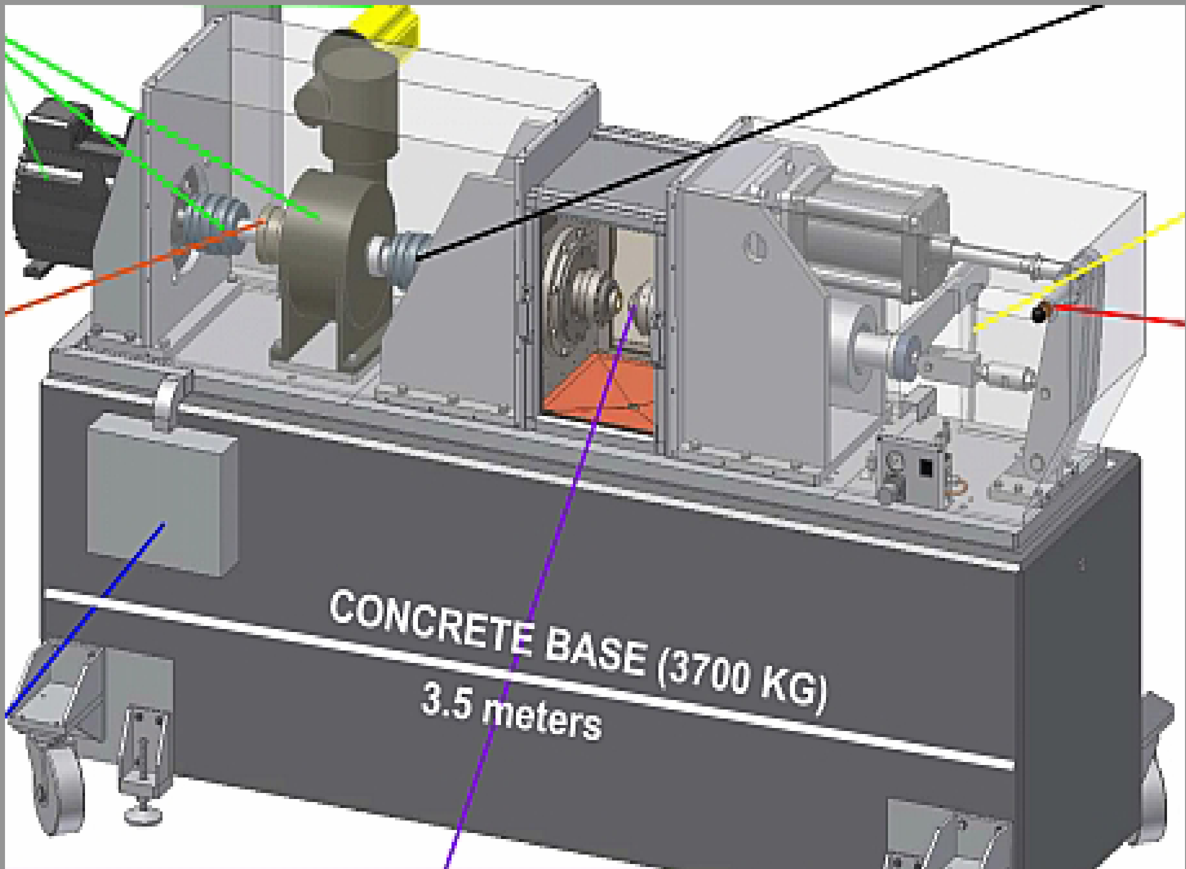
Shimamoto and Tsutumi (2004, *Str. Geol.*)

High velocity rotary friction apparatus @ INGV

u_{tot} = infinite

v = 1 μm/s – 9 m/s

σ_n^{eff} < 70 MPa

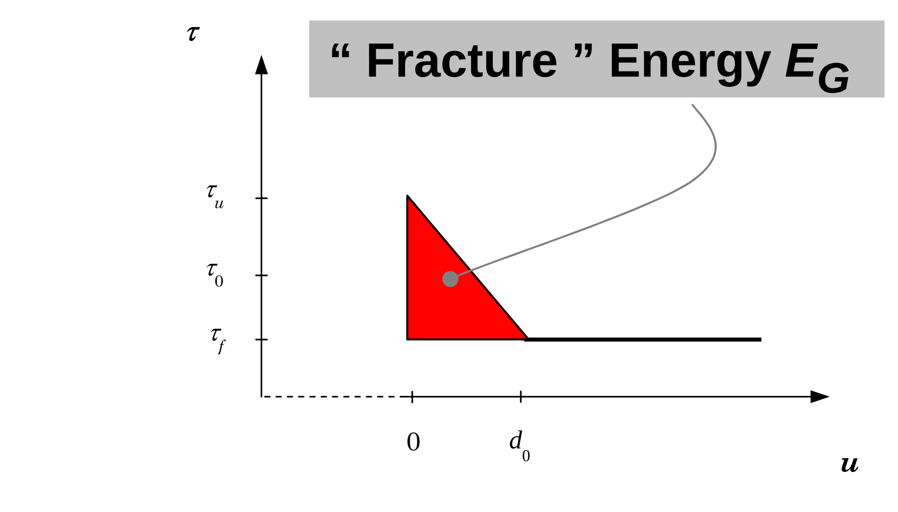


Niemeijer et al. (2009, *AGU Fall Meeting*)

Slip - Dependent Friction Laws

1. LINEAR SLIP – WEAKENING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$



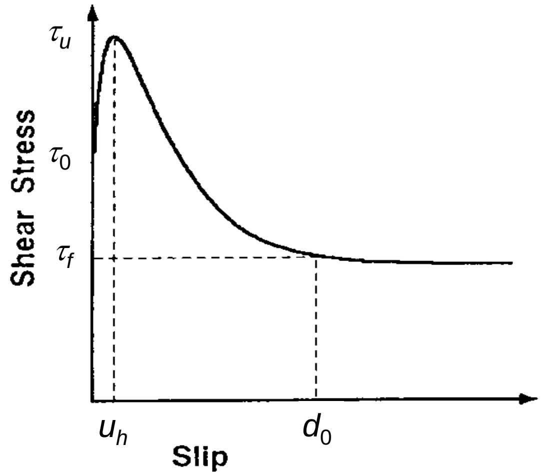
Barenblatt (1959a, 1959b), Ida (1972), Andrews (1976a, 1976b), and many authors thereinafter

d_0 is the characteristic slip – weakening distance

2. NON LINEAR SLIP – WEAKEING LAW WITH SLIP – HARDENING

$$\tau = \left\{ \left[\left(\frac{\tau_0}{\sigma_n^{\ eff}} - \mu_f \right) \left(1 + \alpha_{OY} \ln \left(1 + \frac{u}{\beta_{OY}} \right) \right) \right] \mathrm{e}^{-\frac{u}{d_0}} + \mu_f \right\} \sigma_n^{\ eff}$$

$$u_h : \left. \frac{\mathrm{d}\tau}{\mathrm{d}u} \right|_{u_h} = 0 ; \qquad \begin{cases} u_h = r d_0 \quad (\mathrm{e.g.}\ r = 0.1) \\ \tau(u_h) = \tau_u \end{cases}$$



Ohnaka and Yamashita (1989) and the following papers by Ohnaka and coworkers

u_h is associated with the preparatory phase of the imminent macroscopic failure in the cohesive zone. It accounts for micro–cracking

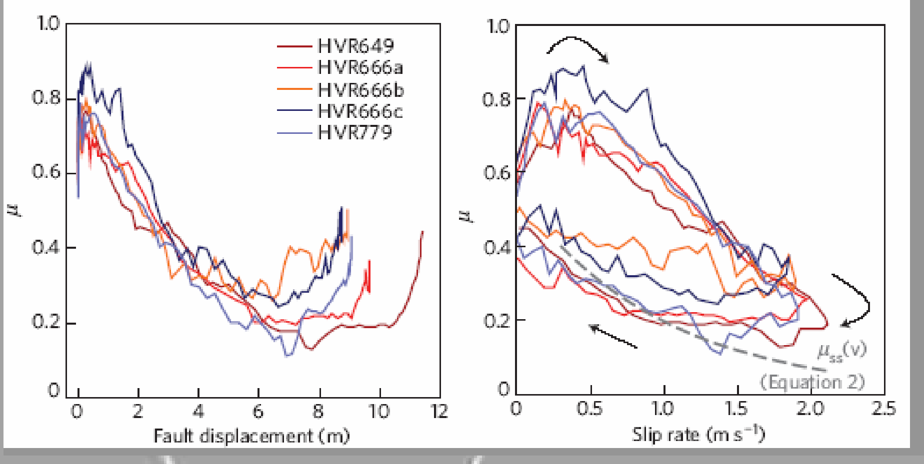
Slip - and Rate - Dependent Friction Laws

$$\tau = \left\{ \mu^{ss}(v) + \left[F(u)\mu_i - \mu^{ss}(v) \right] e^{\frac{\ln(0.05)u}{d_0}} \right\} \sigma_n^{eff}$$

$$\mu^{ss}(v) = \mu^{ss}(0)\, e^{-\frac{v}{v_{SS}}}$$

$$F(u) = \alpha_{SS} + (1 - \alpha_{SS})\, e^{\frac{\ln(0.05)u}{u_h}}$$

Sone and Shimamoto (2009)



u_h controls the duration in slip of the slip – hardening phase, described by the function $F(u)$.

$\mu^{ss}(0) = 0.55 \pm 0.09$ $\mu_i = 0.6$

$v_{SS} = 0.99 \pm 0.23$ m/s

$\alpha_{SS} = 1.26 \div 1.54$

$u_h = 23 \div 160$ mm

Rate - and State - Dependent Friction Laws

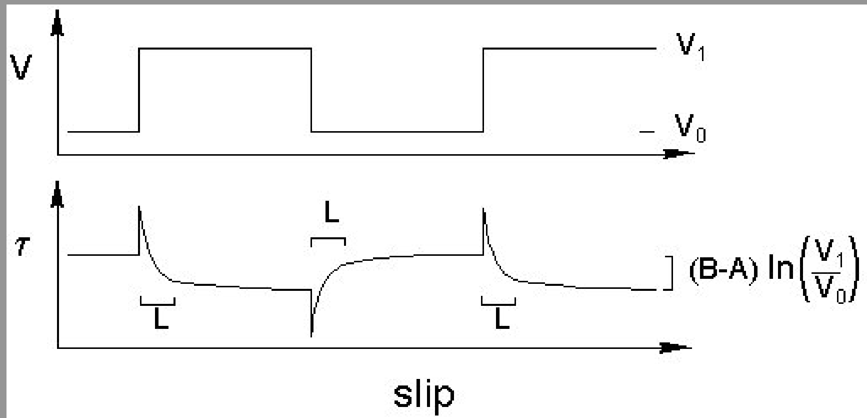
1. DIETERICH IN REDUCED FORMULATION

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\frac{v_*}{v} \right) + b \ln\left(\frac{\Psi v_*}{L} \right) \right] \sigma_n^{eff} \\ \frac{d}{dt}\Psi = 1 - \frac{\Psi v}{L} \end{cases}$$

However, while in velocity stepping experiments the traction response following the velocity variation is directly controlled by the parameter *L*, its effects are much less evident during the dynamic rupture propagation.

Bizzarri and Cocco (2005)

Response to an abrupt jump in load



2. RUINA – DIETERICH (RUINA MODERN FORM.)

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{\,eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = -\dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) \end{cases}$$

Beeler et al. (1994), Roy and Marone (1996)

How to relate relevant quantities to contitutive parameters

Dynamic Parameters

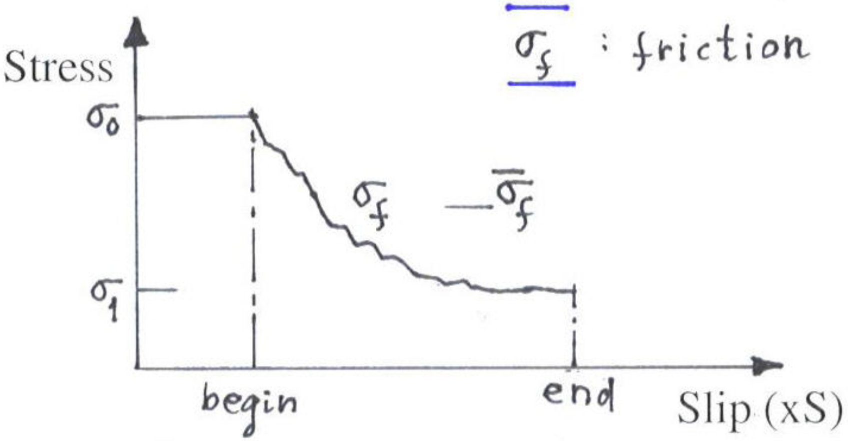
E_R : radiated energy

ruputre speed, $V \approx 0.8$ to $0.9\ \beta$

$E_{NR} = E_F + E_G + \dots$: non-radiated energy

E_F : friction (heat), E_G : fracture energy

σ_f : friction



$\Delta\sigma_s = \sigma_0 - \sigma_1$: static stress drop

$\Delta\sigma_d = \sigma_0 - \bar{\sigma}_f$: dynamic stress drop

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