

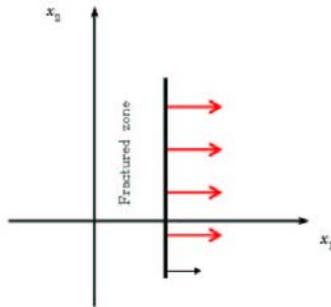


**Rupture propagation in  
a *truly* 3 – D fault model**



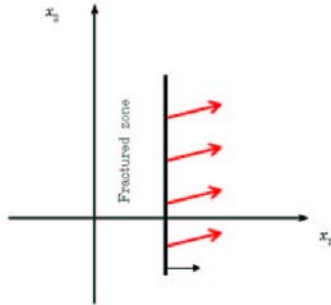
# Remembering dimensionality ...

PURE MODE II



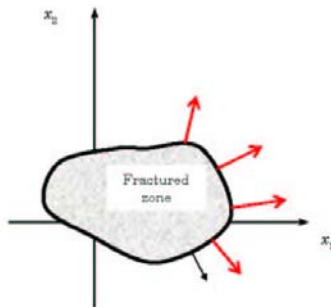
Dependence on  $x_1$   
Independence on  $x_2$   
 $\Rightarrow u_1(x_1, t)$

MIXED MODE



Dependence on  $x_1$   
Independence on  $x_2$   
 $\Rightarrow u_1(x_1, t)$   
 $u_2(x_1, t)$

TRULY 3 - D



Dependence on  $x_1$   
Dependence on  $x_2$   
 $\Rightarrow u_1(x_1, x_2, t)$   
 $u_2(x_1, x_2, t)$

— Crack tip  
→ Local crack enlargement direction

→ Local displacement

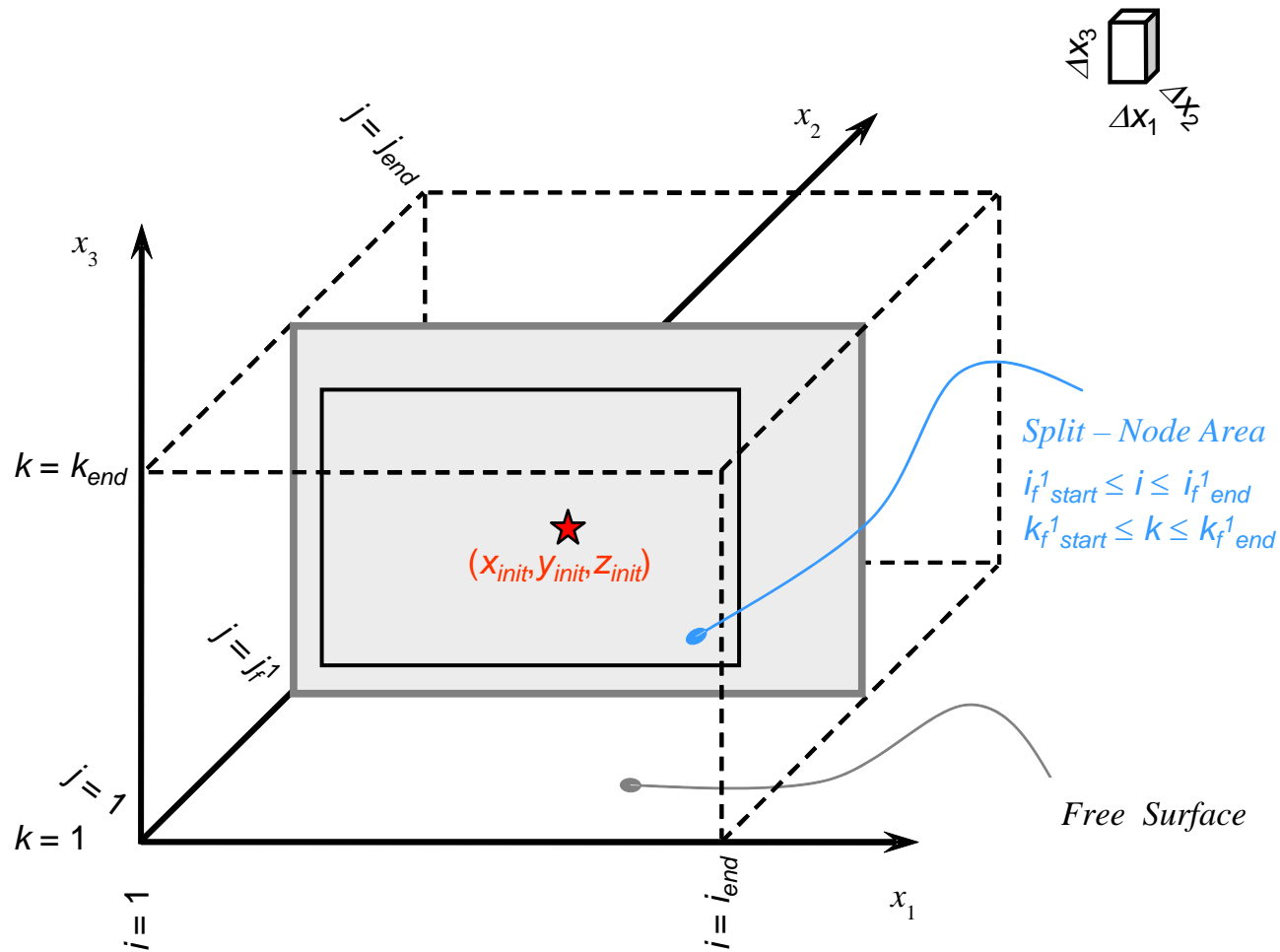
We solve a *truly* 3 - D rupture problem:

- Both two components of solutions depend on two spatial coordinates and on time;

- Shear traction is collinear with fault slip velocity (  $\mathbf{T} \parallel \mathbf{v}$  ), **but the rake** ( i. e. the fault slip velocity azimuth ) **can vary during time.**



# Numerical Method: FD 3 - D



$$\mathbf{u} = (u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t))$$



# Reference Case



**Slip**

**Traction**

Slip\_26ani\_sw\_total

Tau\_26ani\_sw\_total

$S = 0.8$

$S = 0.8$

In. rake = 0.785398 rad.

In. rake = 0.785398 rad.

Anim\_Slip\_26ani\_sw\_total.avi

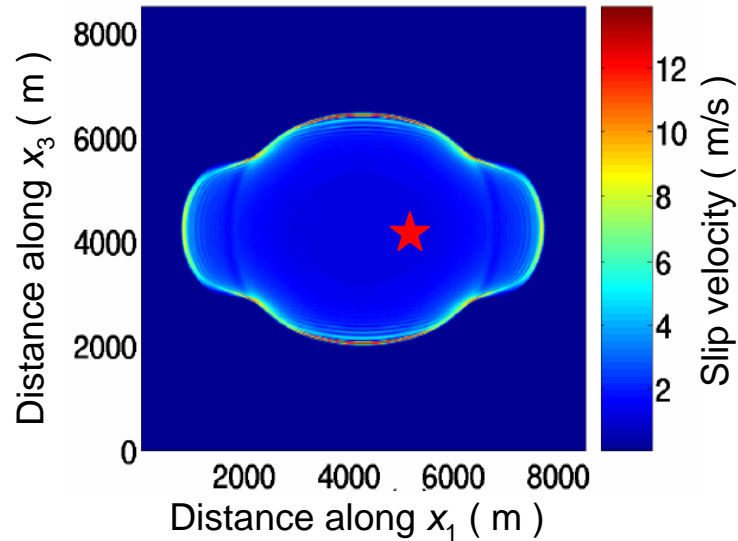
Anim\_Tau\_26ani\_sw\_total.avi





# The cohesive zone

Time snapshot  
( $t = 0.8 \text{ s}$ ) - SW law



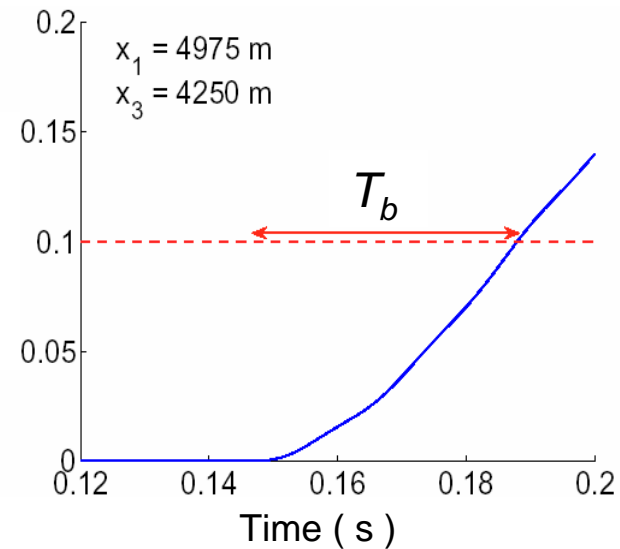
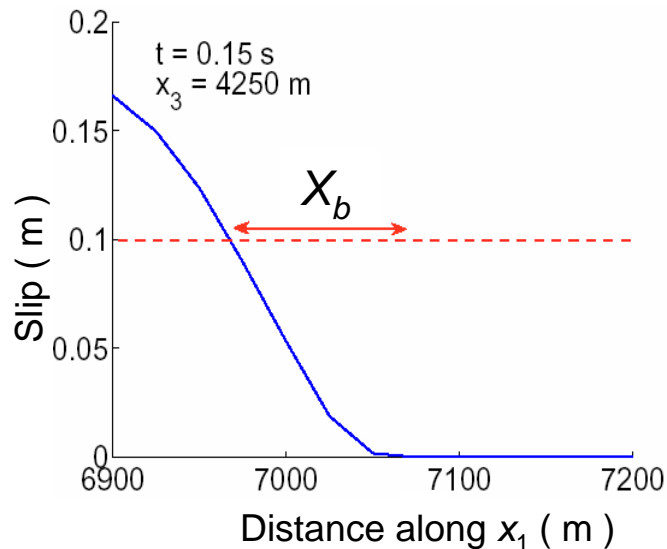
In the target location we can estimate:

$$X_b = 105 \text{ m} \quad T_b = 0.04 \text{ s}$$

From these quantities:

$$v_{rupt} = X_b / T_b = 2625 \text{ m/s}$$

Local estimate

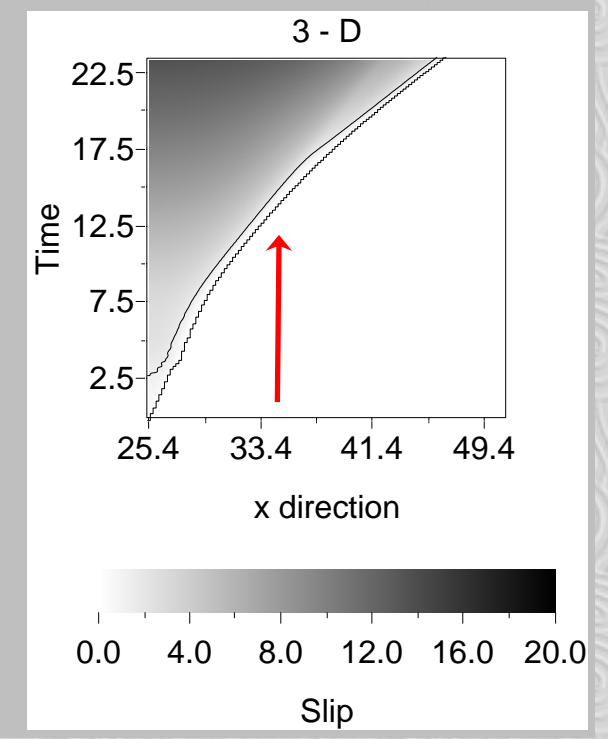
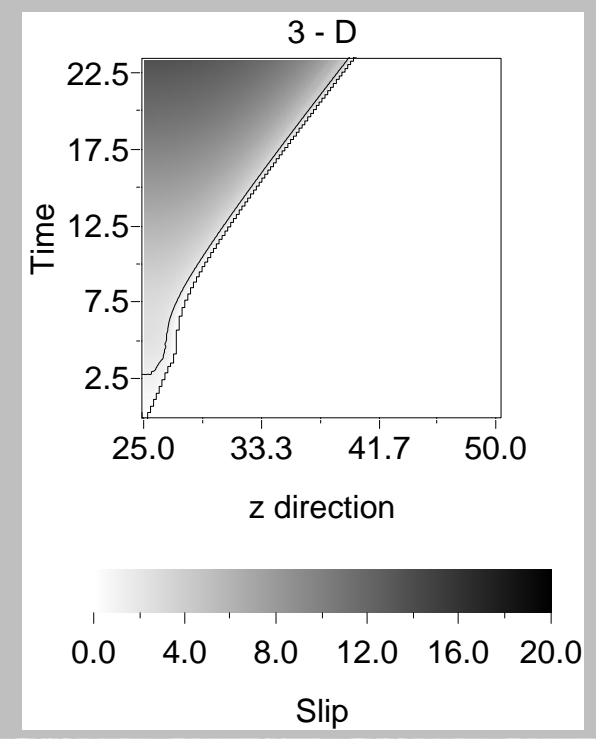
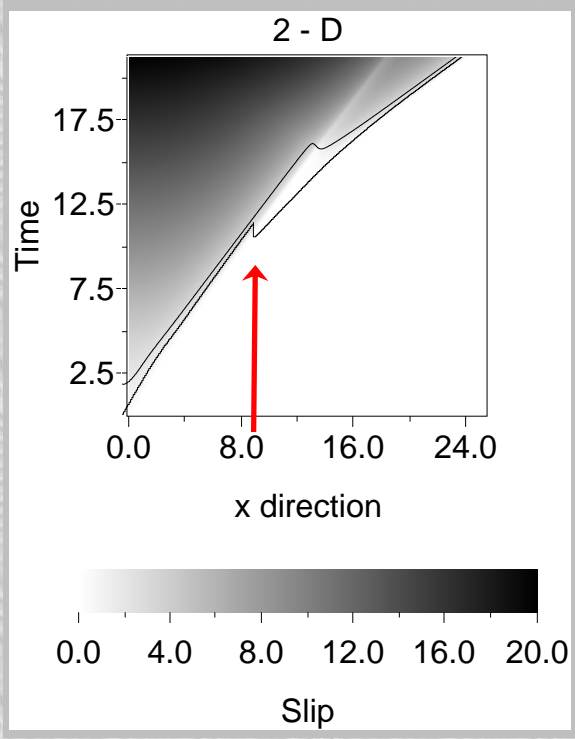




# Comparison between 2 - D and 3 - D models #1

Fixed  $x_1$  coordinate

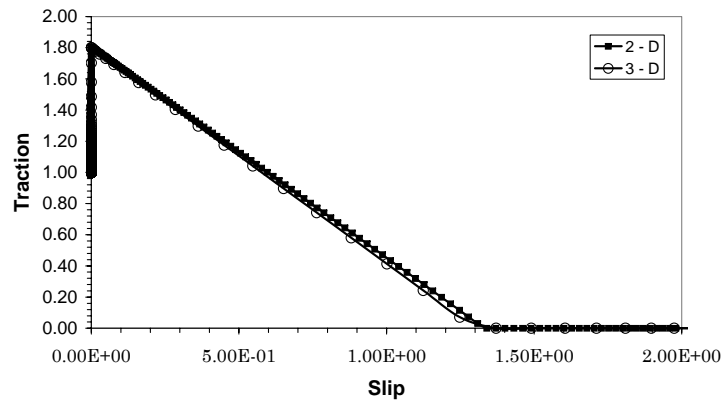
Fixed  $x_3$  coordinate



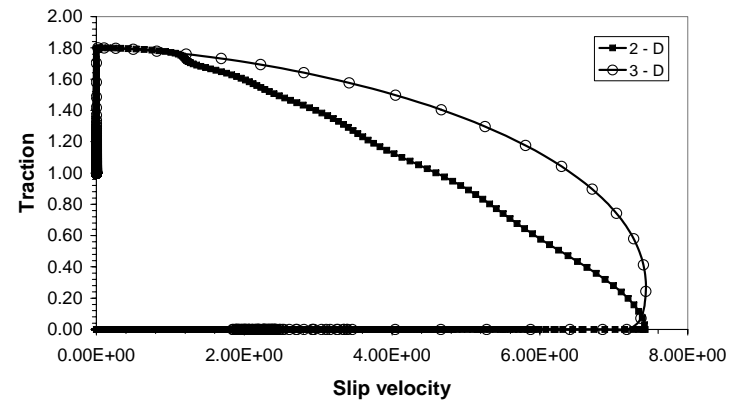


# Comparison between 2 - D and 3 - D models #2

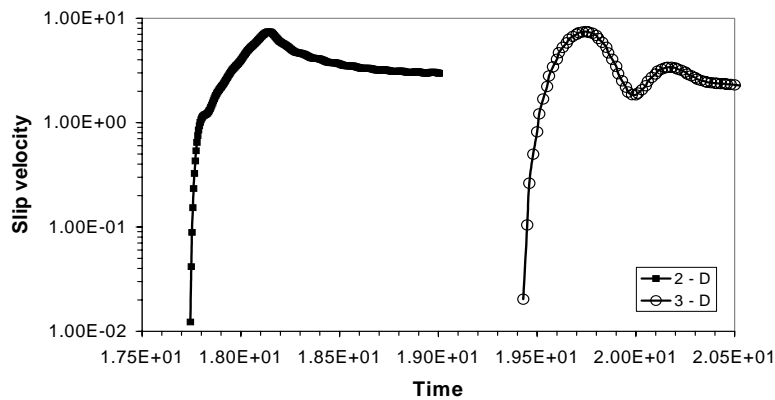
Traction vs. Slip  
at  $x_1 = x_{init} + 18.0$



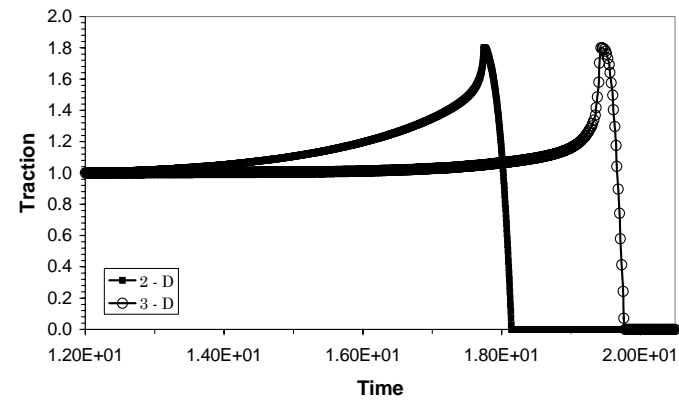
Traction vs. Slip velocity  
at  $x_1 = x_{init} + 18.0$



Slip velocity vs. Time  
at  $x_1 = x_{init} + 18.0$

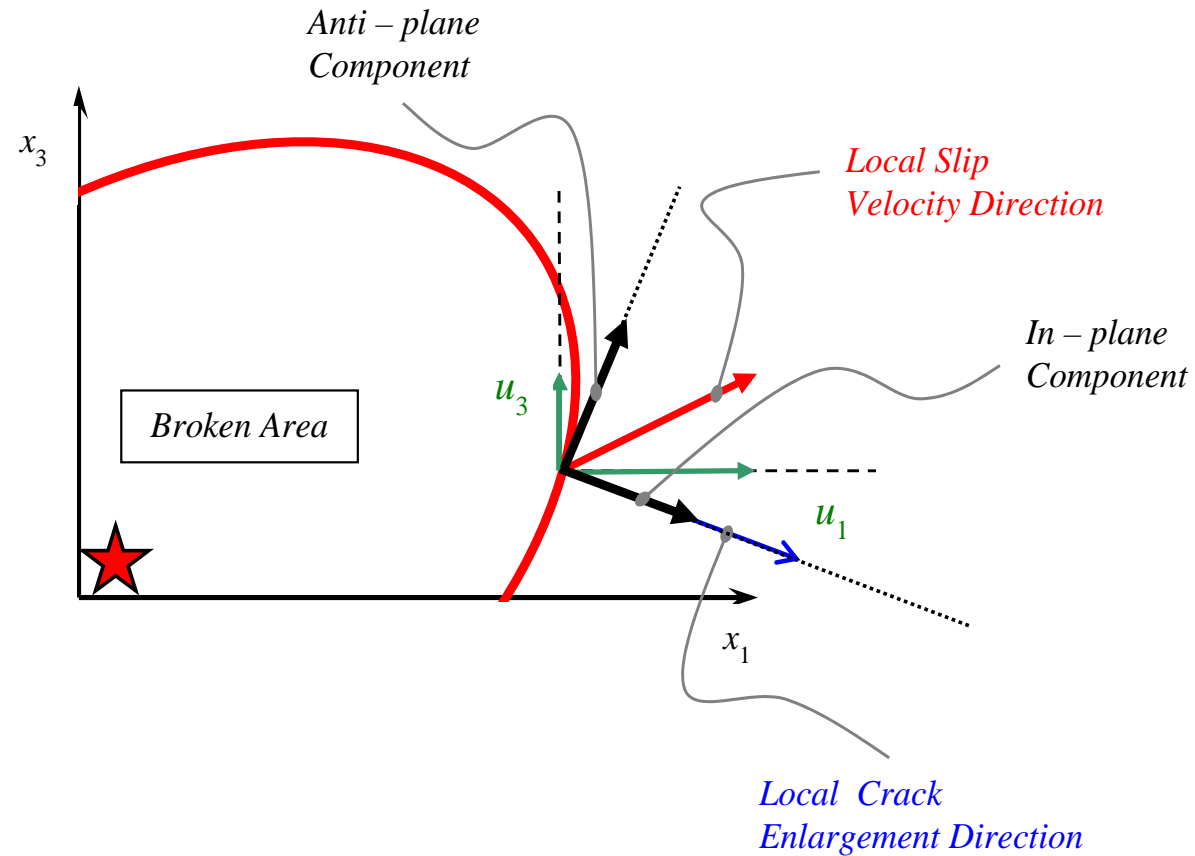


Traction vs. Time  
at  $x_1 = x_{init} + 18.0$





# The rake rotation: the coupling of the two modes of propagation





# Rake rotation #2: evidences

From Spudich et al., (1998)

## Slip paths reconstructed from striations

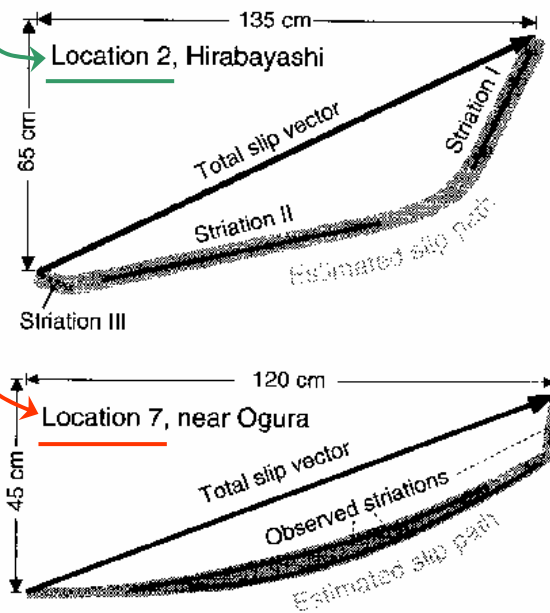


Figure 2. Black lines: striations observed at locations 2 and 7. Gray bands: slip paths inferred from striation locations 2 and 7. From Otsuki et al. (1997).

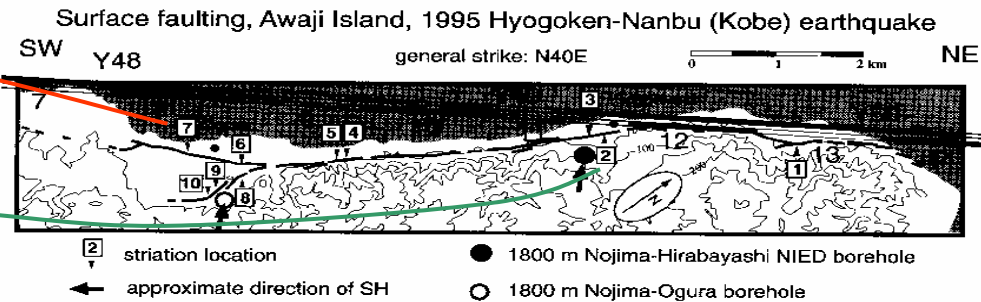


Figure 1. Map of Nojima fault on Awaji Island, showing elevation (m), surface faulting (heavy line), locations of fault striations (numbers in boxes), the Nojima-Hirabayashi NIED and the Nojima-Ogura boreholes, and the subfaults (numbered) in the original (Y48 to Y50) and interpolated (7 to 14) Yoshida slip models. Arrows show approximate direction of SH in boreholes.

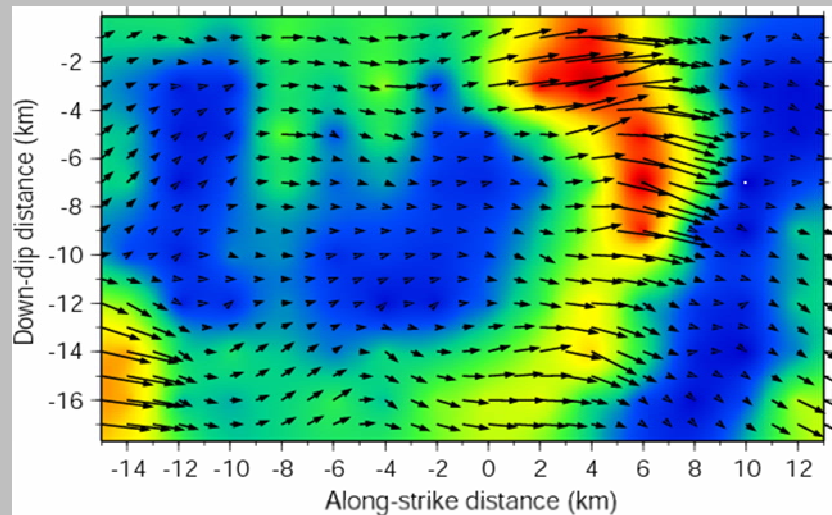
Etchecopar (1984), Florensov and Solonenko (1965), Kakimi et al. (1977), Philip and Megard (1977).

More recently curved striations (also called slickenlines) were seen in the Denali earthquake (Haeussler et al. 2004).

Curved striations were observed in the 1971 San Fernando; 1999 Hector Mine EQ; the 1992 Landers EQ; the 1980 El Asnam, Algeria EQ, and on the San Andreas in the Mecca Hills.

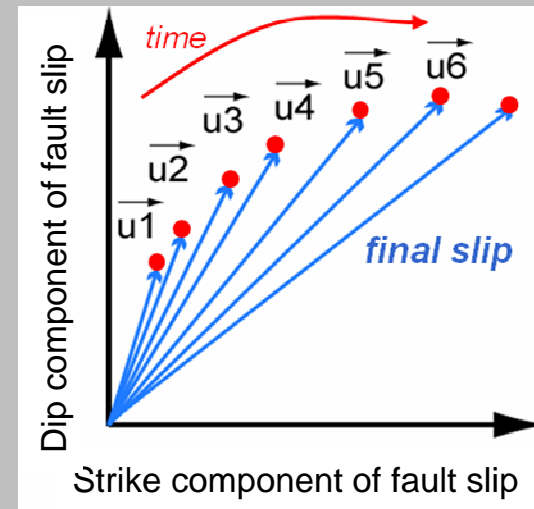
# Rake rotation: a schematic example

Spatial heterogeneous rake

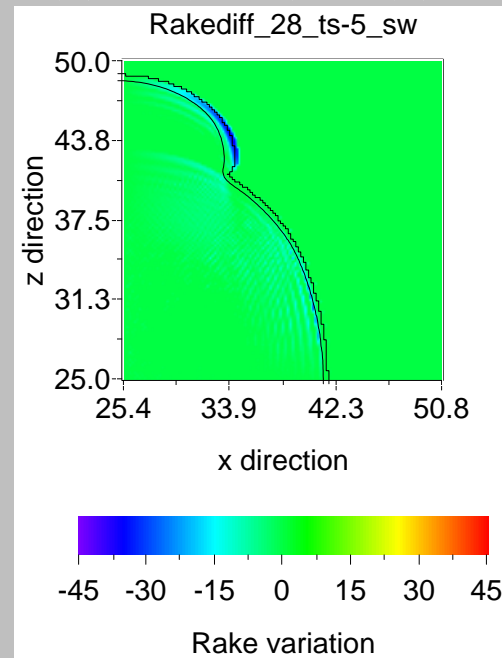
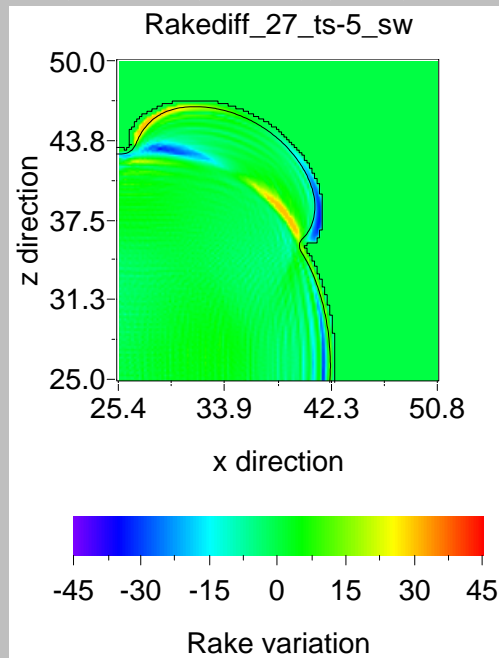
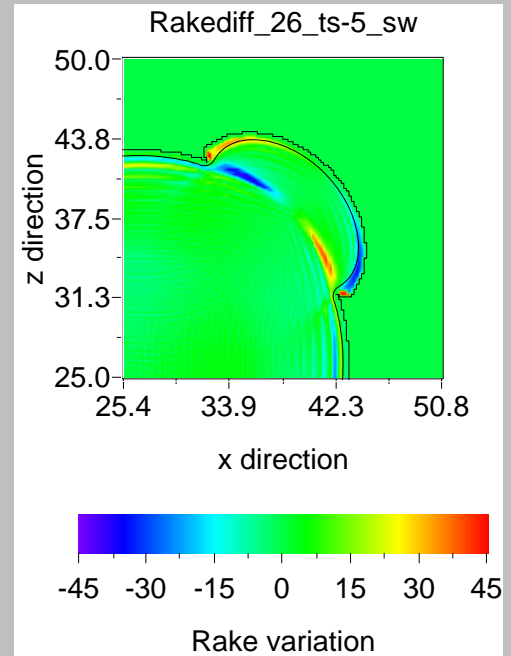
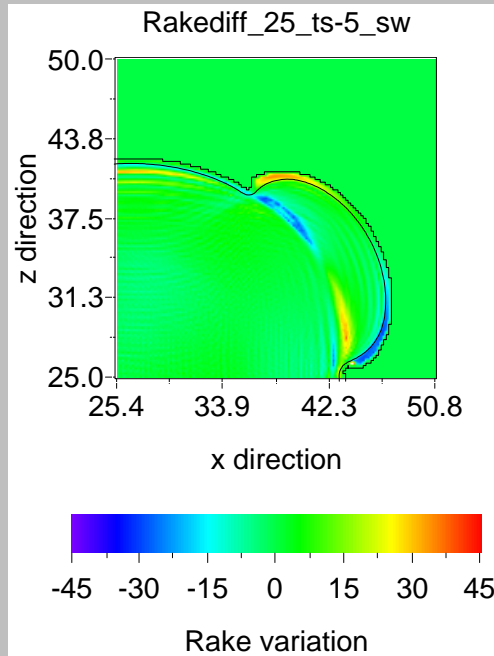
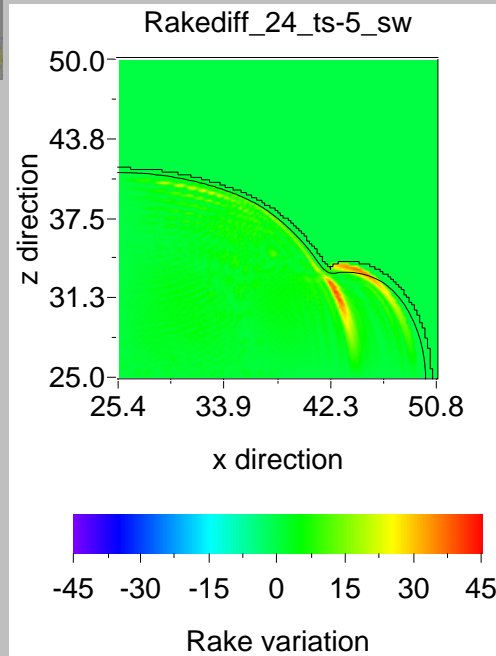


Slip distribution on the fault

Temporal heterogeneous rake

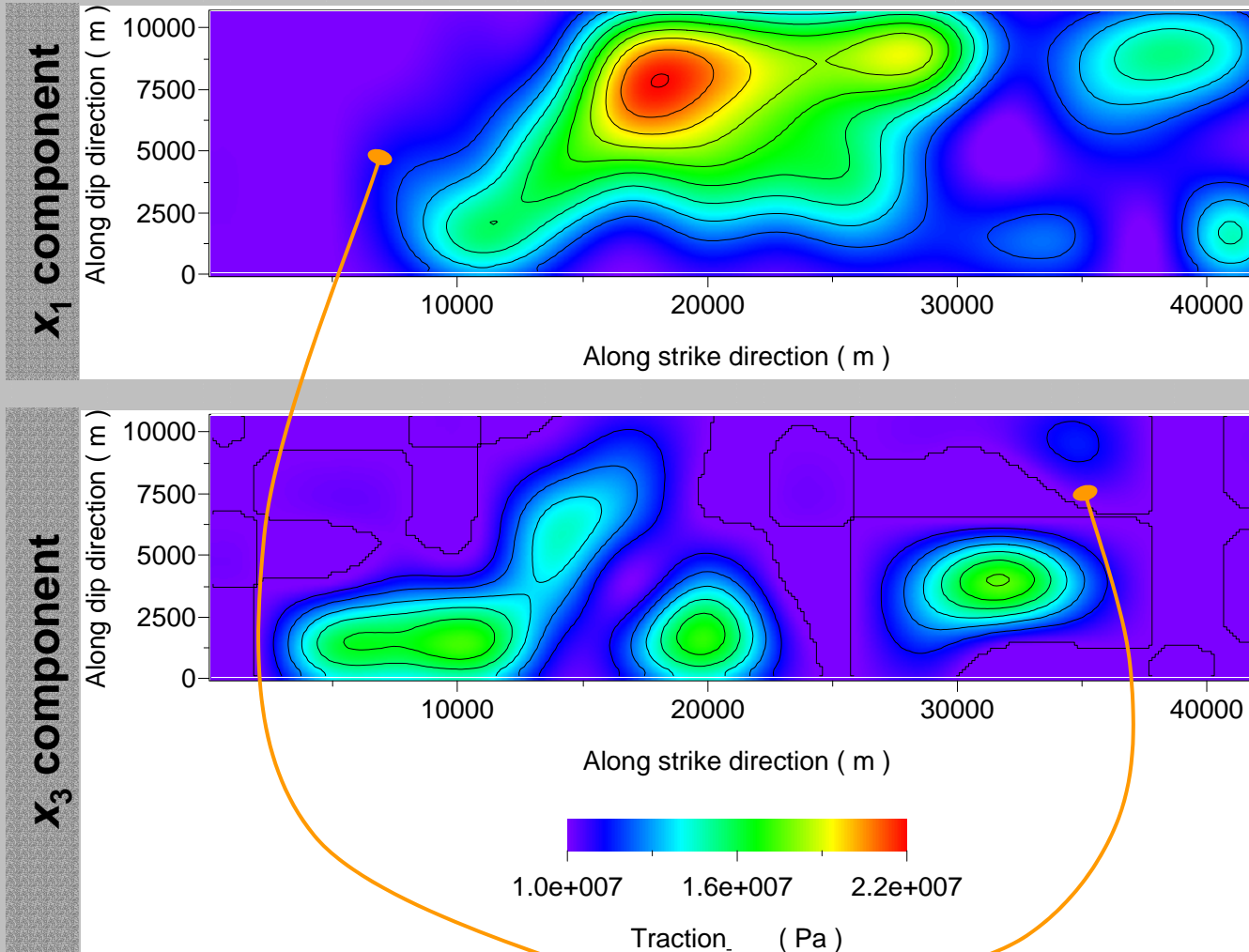


Temporal evolution of slip for a target point





# The rake rotation #5: path / modulus



$$\mathbf{T}_0(x_1, x_3) \equiv \mathbf{T}(x_1, x_3, 0) = (T_1(x_1, x_3, 0), 0, T_3(x_1, x_3, 0))$$

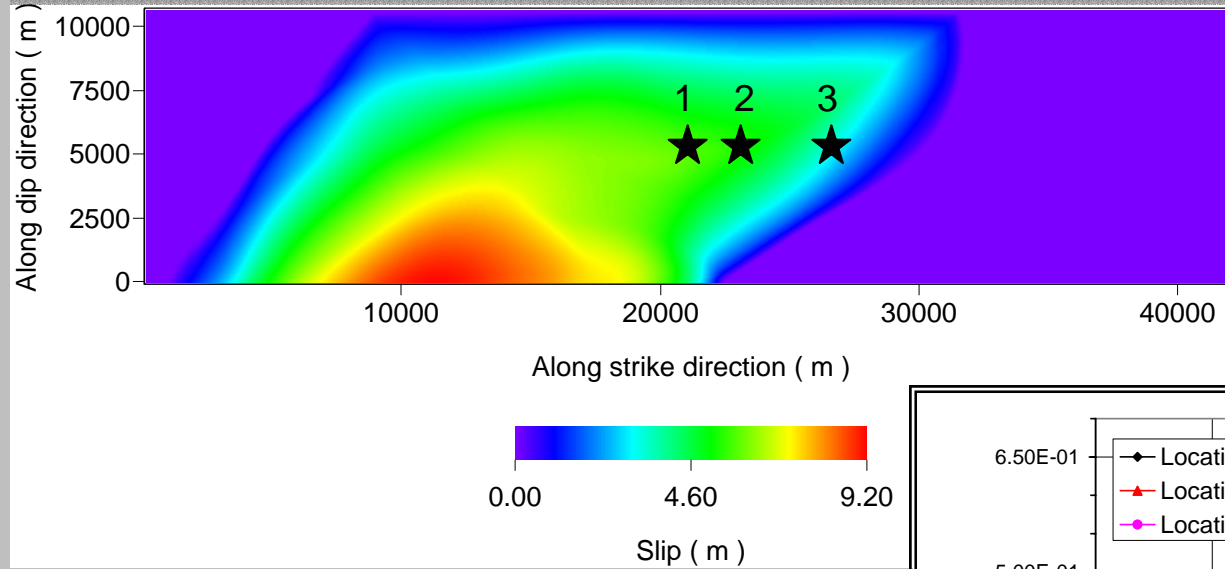
Normal Traction

$$\rightarrow \Sigma_0(x_1, x_3) \equiv \Sigma(x_1, x_3, 0) = -\sigma_n^{eff} \hat{\mathbf{n}} = (0, -30 \text{ MPa}, 0)$$

$$\mathcal{T}_0 = \mathbf{T}_0 + \Sigma_0$$

Total Traction

### Fault slip time snapshots – Linear SW assumed

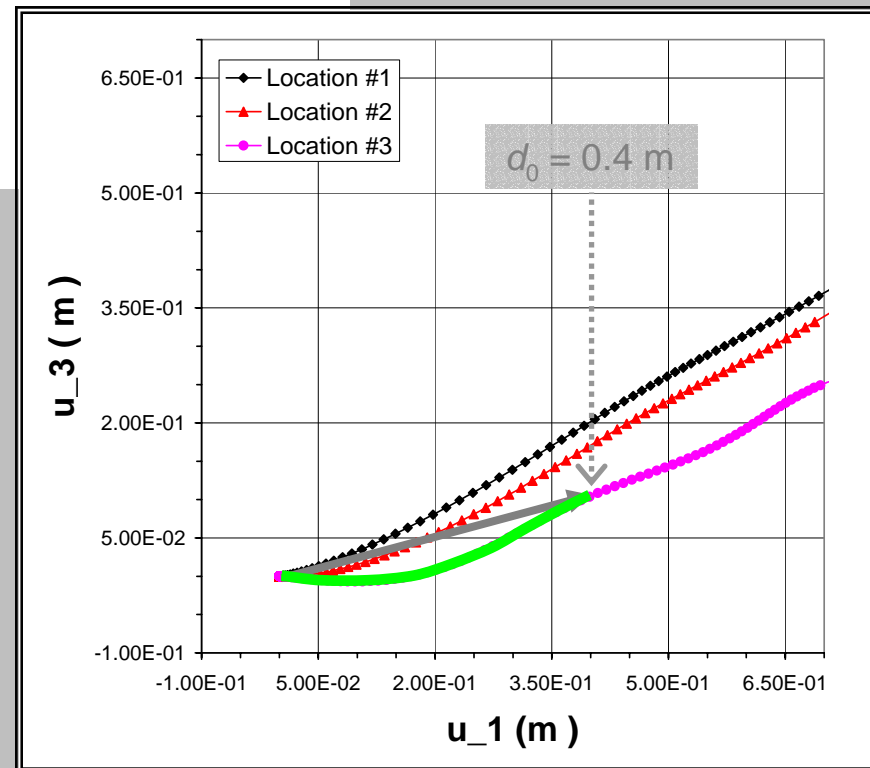


Slip modulus:

$$u = u^{(mod)}(x_1, x_3, t) \equiv \|\mathbf{u}(x_1, x_3, t)\|$$

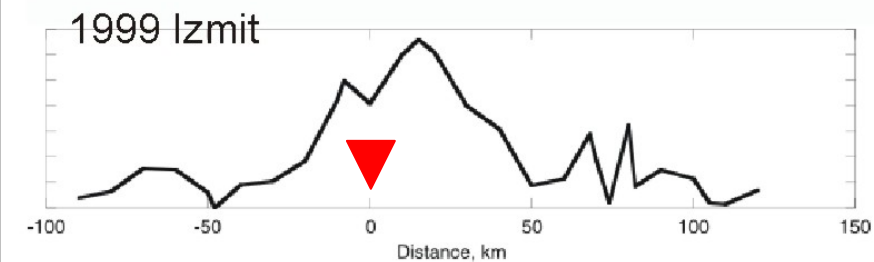
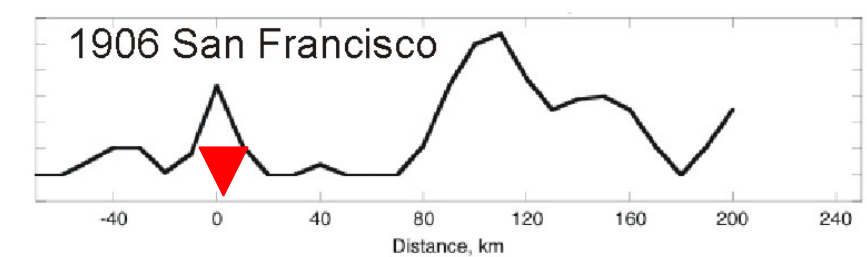
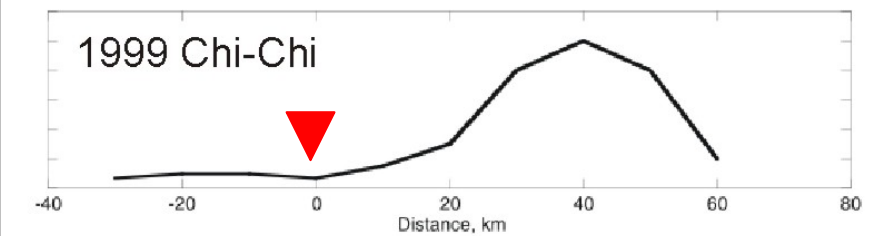
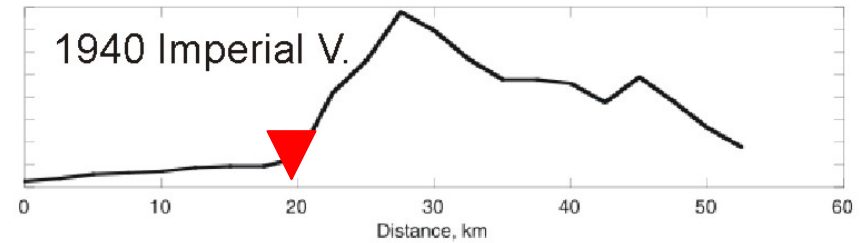
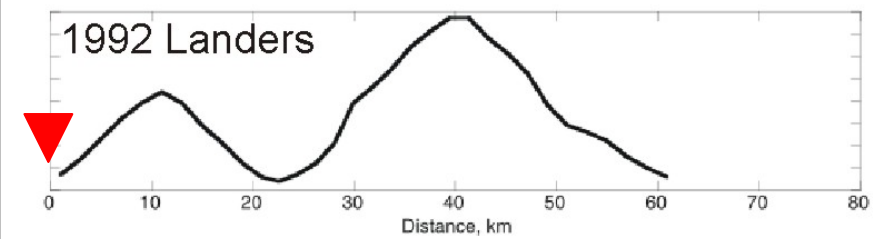
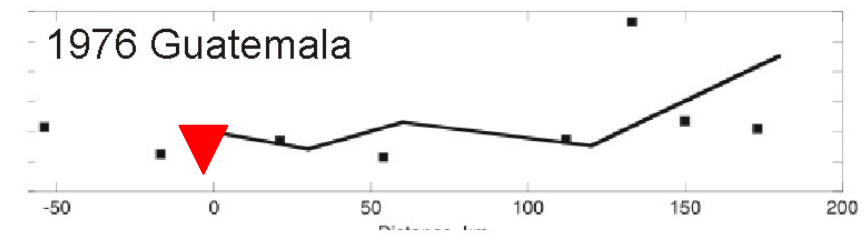
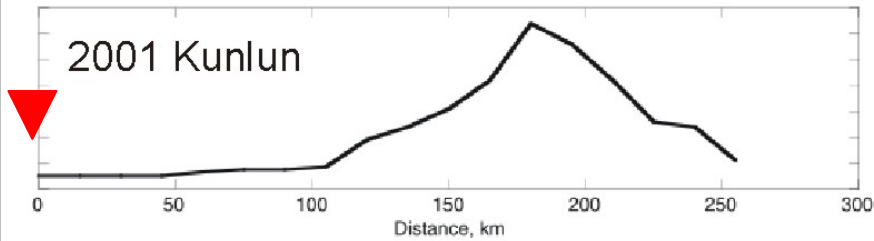
Slip path:

$$u = u^{(path)}(x_1, x_3, t) \equiv \int_0^t \|\mathbf{v}(x_1, x_3, t')\| dt'$$





# Slip distribution of large earthquakes

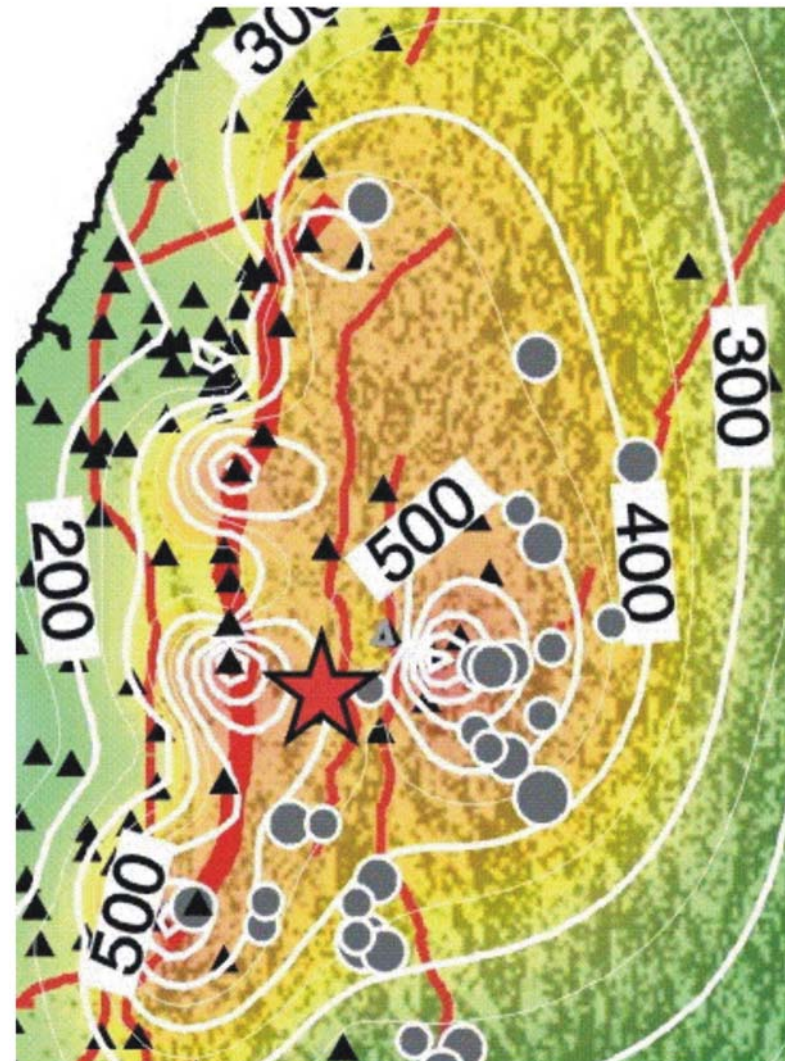
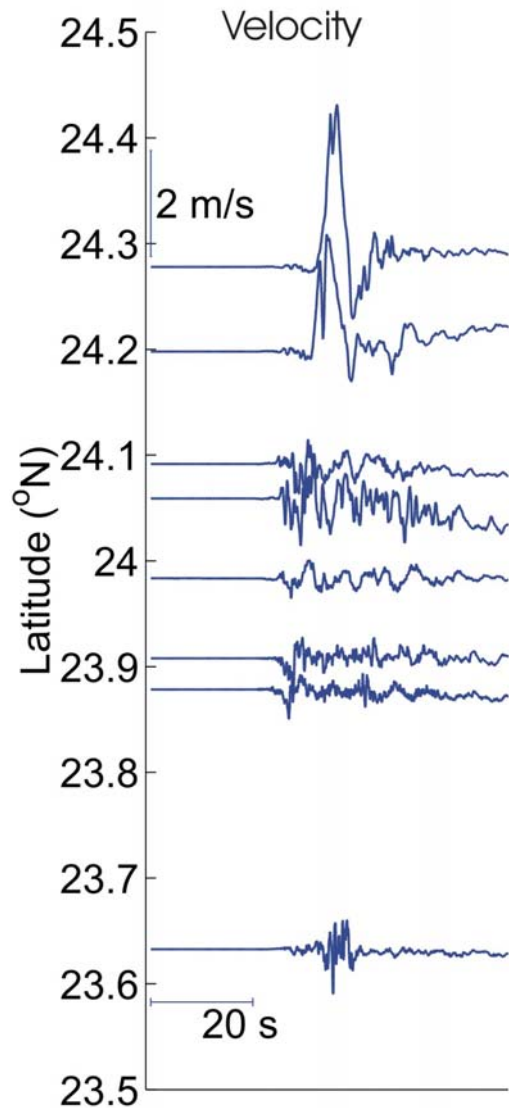




# Ground motion from Chi – Chi, Taiwan, EQ

*Brodsky and Kanamori ( 2001 )*

*Ma et al. ( 1993 )*





# Effects of Strength Heterogeneity #1

Slip\_var10ani\_sw\_total

$$S_3 = 0.8$$

$$S_2 = S_1 = 3.0$$

In. rake = 0.785398 rad.

Anim\_Slip\_var10ani\_sw\_total.avi

## Homogeneous

Rakediff\_26ani\_sw

$$S = 0.8$$

In. rake = 0.785398 rad.

Anim\_Rakediff\_26ani\_sw\_total.avi

## Heterogeneous

Rakediff\_var10ani\_sw

$$S_3 = 0.8$$

$$S_2 = S_1 = 3.0$$

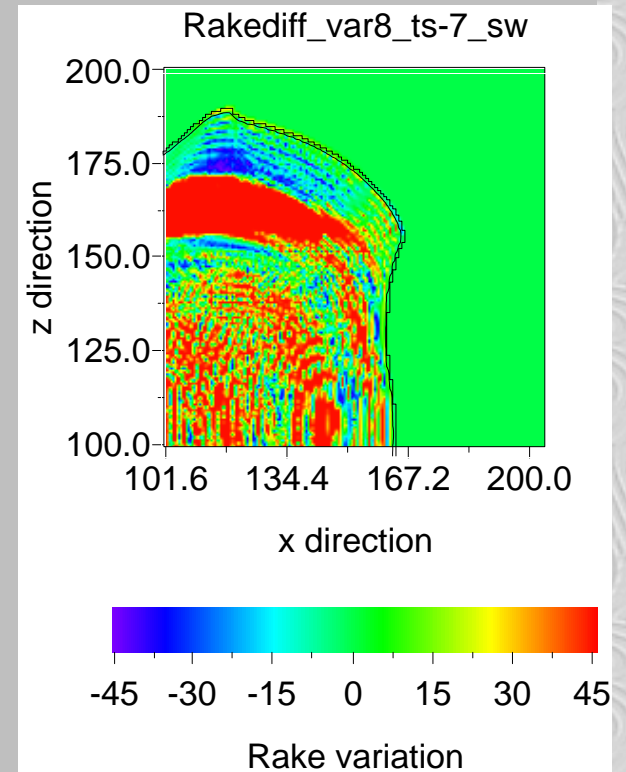
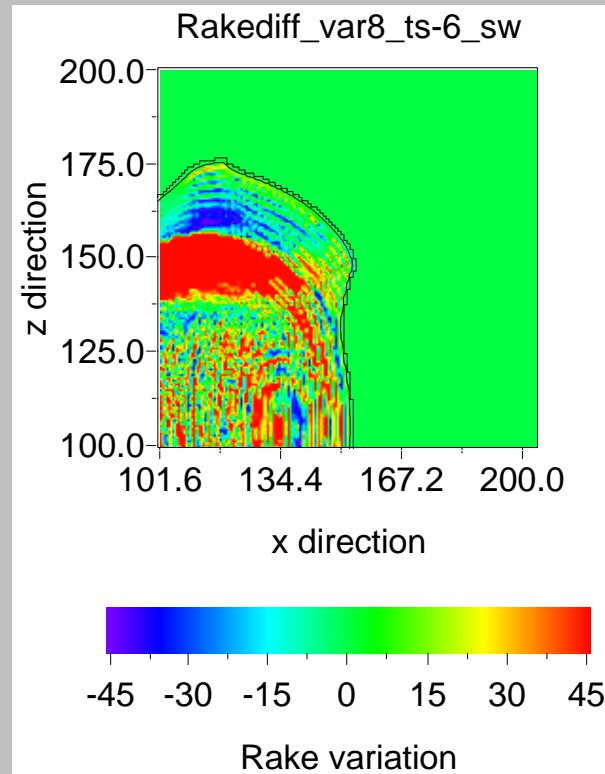
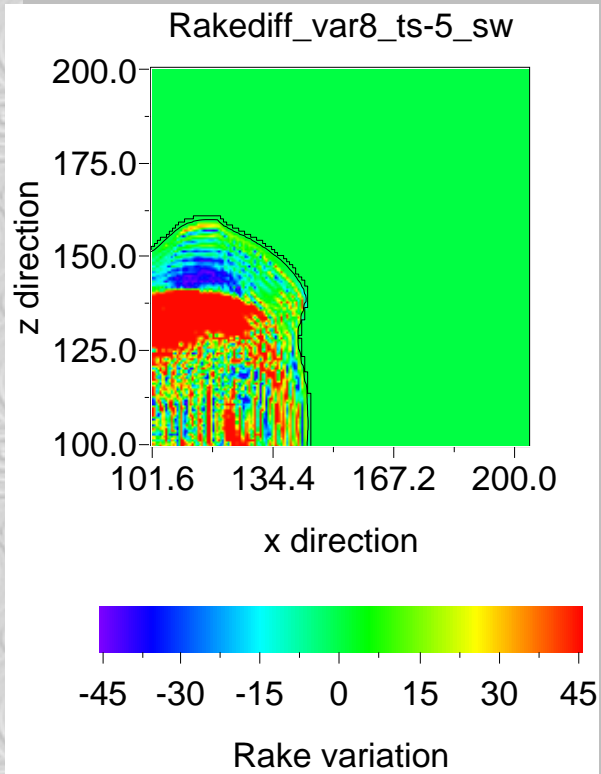
In. rake = 0.785398 rad.

Anim\_Rakediff\_var10ani\_sw\_total.avi





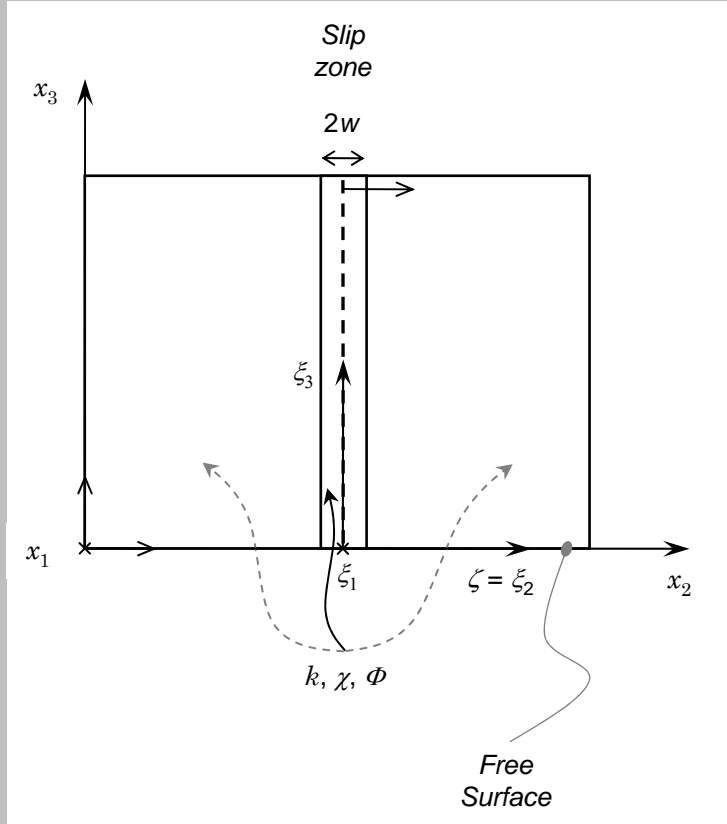
# Effects of Strength Heterogeneity #2



An aerial photograph of a coastal wetland or marsh area. The landscape is a mix of light-colored, sandy or silty ground and darker, water-saturated mudflats. There are several small, irregular pools of water scattered throughout. The overall appearance is that of a flat, low-lying terrain with varying moisture levels. A semi-transparent grey rectangular box is centered over the image, containing the text "Thermal pressurization of pore fluids" in a bold, red, sans-serif font.

**Thermal pressurization of  
pore fluids**

# Mathematical background



1 - D Fourier's heat conduction equation:

$$\frac{\partial}{\partial t} T = \chi \frac{\partial^2}{\partial \zeta^2} T + \frac{1}{c} q$$

Coupling of temperature  $T$  with pore fluid pressure  $p_{fluid}$ :

$$\frac{\partial}{\partial t} p_{fluid} = \frac{\alpha_{fluid}}{\beta_{fluid}} \frac{\partial}{\partial t} T - \frac{1}{\beta_{fluid} \Phi} \frac{\partial}{\partial t} \Phi + \omega \frac{\partial^2}{\partial \zeta^2} p_{fluid}$$

where  $\chi$  is the thermal diffusivity,  $c$  the heat capacity for unit volume,  $\alpha_{fluid}$  the coefficient of thermal expansion,  $\beta_{fluid}$  the compressibility coefficient,  $\Phi$  the porosity and  $\omega = k/\eta_{fluid}\beta_{fluid}\Phi$  the hydraulic diffusivity (being  $k$  the permeability of the medium and  $\eta_{fluid}$  the dynamic fluid viscosity). Analytical solutions at  $\zeta = 0$  are:

$$T^{wf}(\xi_1, \xi_3, t) = T_0^f + \frac{1}{2cw(\xi_1, \xi_3)} \int_0^{t-\varepsilon} dt' \operatorname{erf}\left(\frac{w(\xi_1, \xi_3)}{2\sqrt{\chi(t-t')}}\right) \tau(\xi_1, \xi_3, t') v(\xi_1, \xi_3, t')$$

$$\begin{aligned} \tilde{p}_{fluid}^{wf}(\xi_1, \xi_3, t) = & p_{fluid_0}^f + \frac{\gamma}{2w(\xi_1, \xi_3)} \int_0^{t-\varepsilon} dt' \left\{ -\frac{\chi}{\omega - \chi} \operatorname{erf}\left(\frac{w(\xi_1, \xi_3)}{2\sqrt{\chi(t-t')}}\right) + \frac{\omega}{\omega - \chi} \operatorname{erf}\left(\frac{w(\xi_1, \xi_3)}{2\sqrt{\omega(t-t')}}\right) \right\} \\ & \left\{ \tau(\xi_1, \xi_3, t') v(\xi_1, \xi_3, t') - \frac{2w(\xi_1, \xi_3)}{\gamma} \frac{1}{\beta_{fluid} \Phi(t')} \frac{\partial}{\partial t'} \Phi(\xi_1, 0, \xi_3, t') \right\} \end{aligned}$$

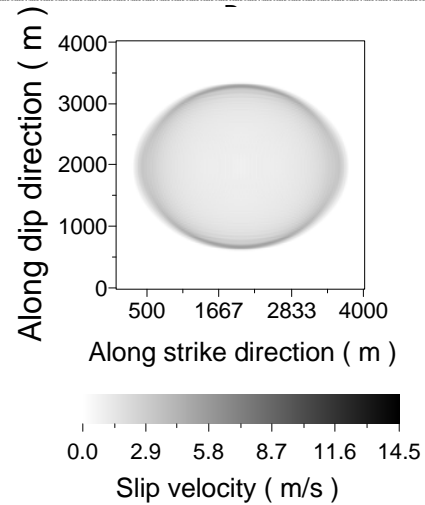




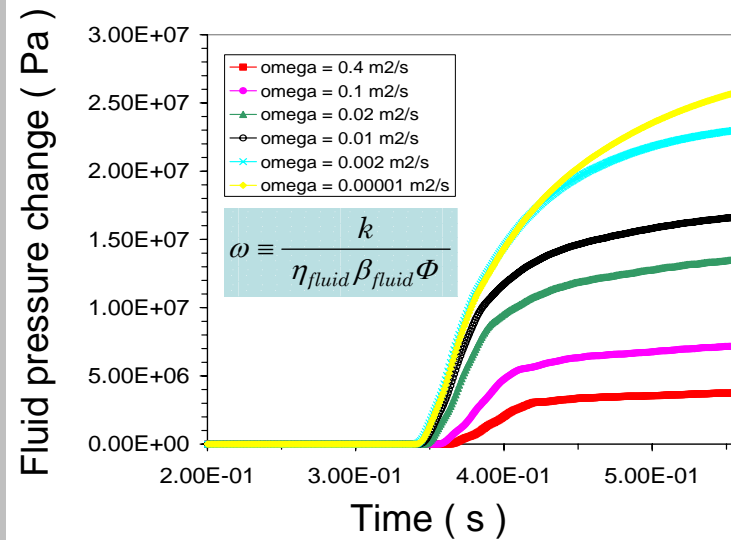
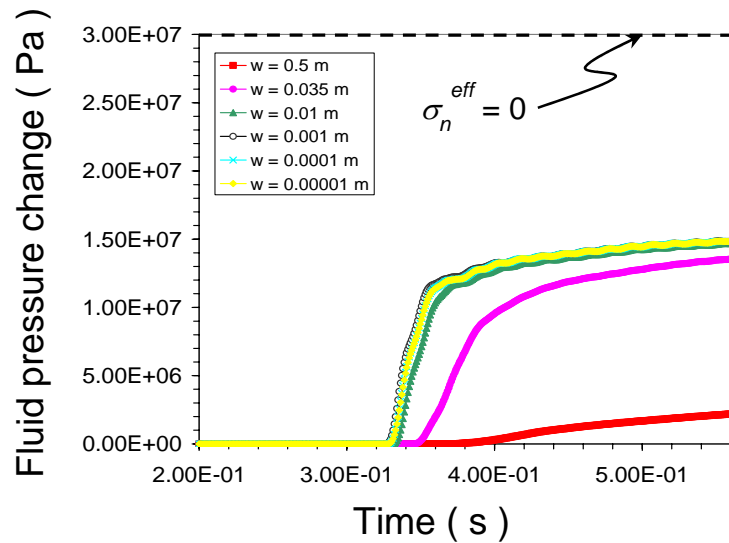
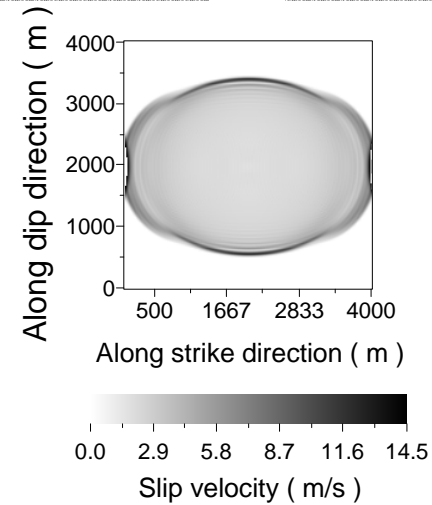
# Results with SW law



## Dry fault ( $\sigma_n^{eff} = \text{const}$ )

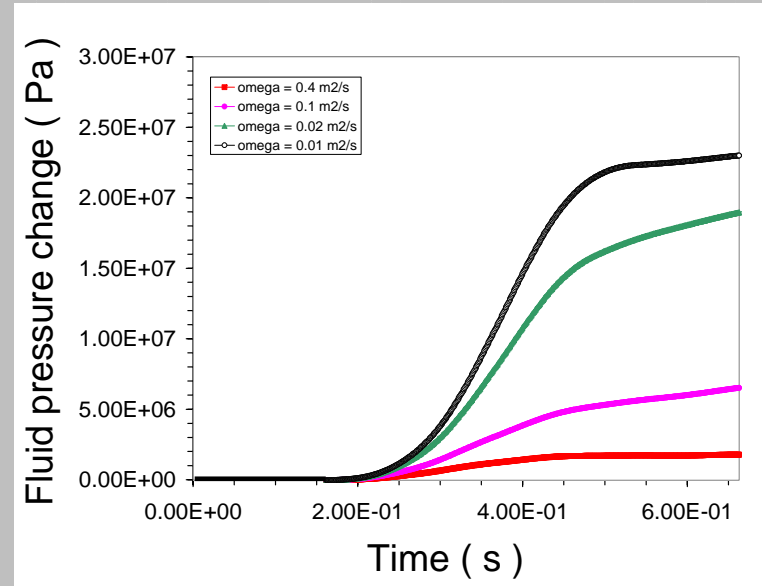
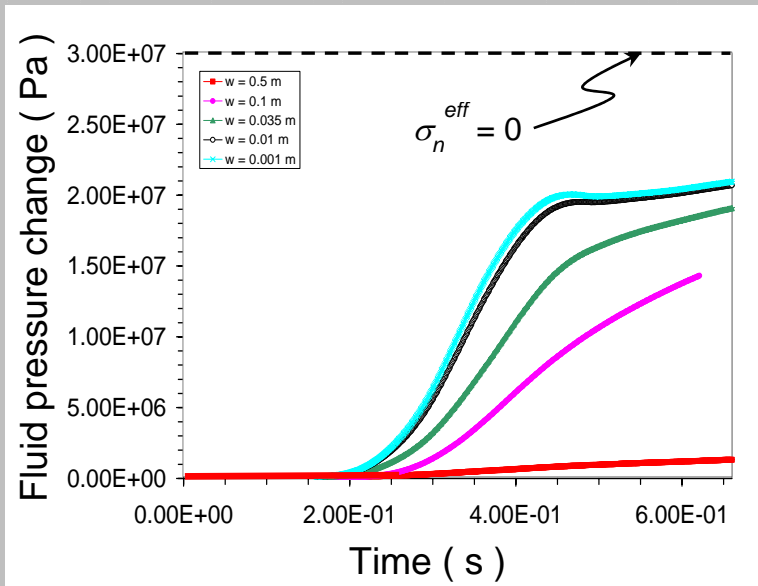


## Wet fault ( $\sigma_n^{eff}$ varies)



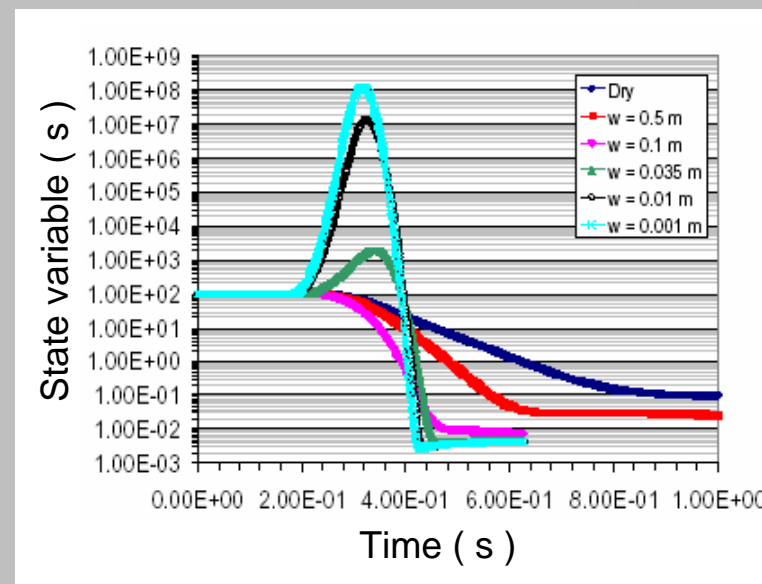
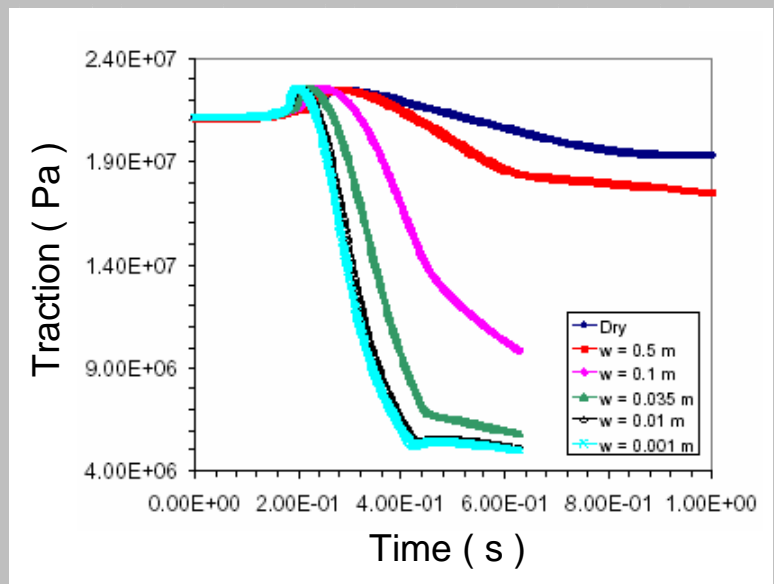
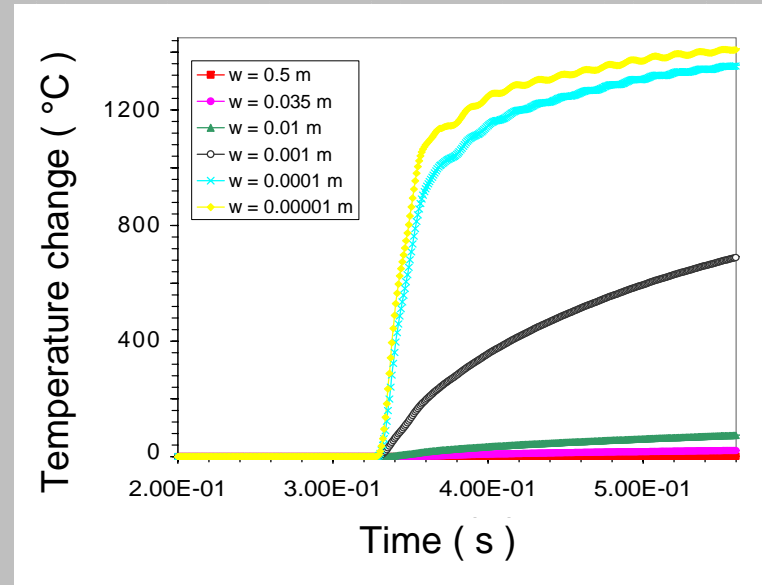
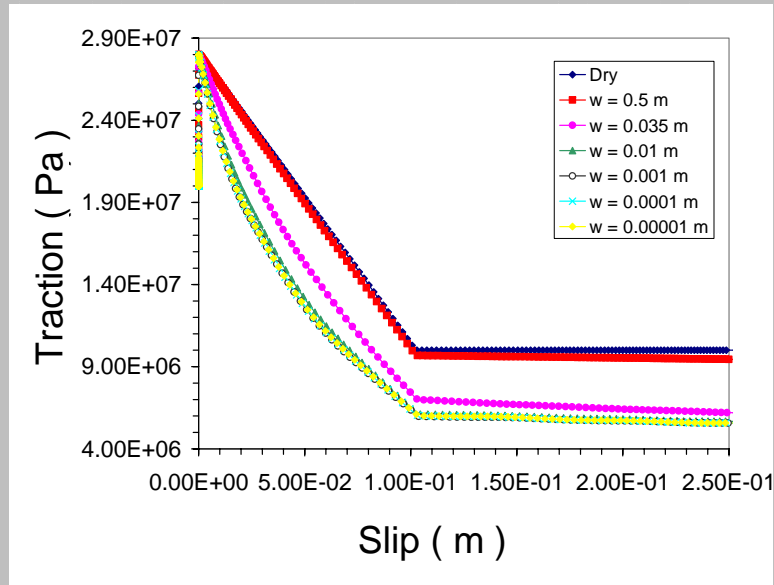


# Results with DR law



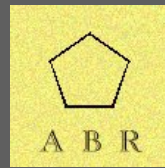


# The breakdown zone



**Thank you!**

**This slide is empty intentionally.**



# **Support Slides: Parameters, Notes, etc.**

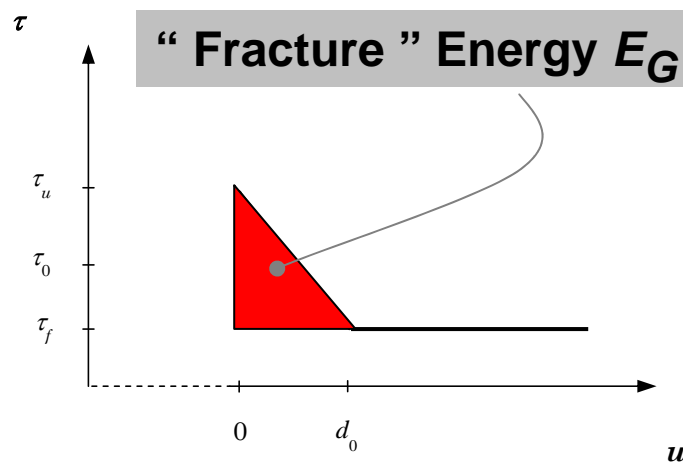
*To not be displayed directly. Referenced above.*



# Slip - Weakening Friction Law



$$\tau = \begin{cases} \left[ \mu_u - (\mu_u - \mu_f) \frac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$



*Barenblatt ( 1959a, 1959b ), Ida ( 1972 ), Andrews ( 1976a, 1976b ), and many authors thereafter*

$d_0$  is the characteristic slip –  
weakening distance

# Rate - and State - Dependent



## DIETERICH – RUINA WITH VARYING NORMAL STRESS

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - \alpha \ln \left( \frac{v_*}{v} \right) + b \ln \left( \frac{\Psi v_*}{L} \right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = 1 - \frac{\Psi v}{L} - \left( \frac{\alpha_{LD} \Psi}{b \sigma_n^{eff}} \right) \frac{d}{dt} \sigma_n^{eff} \end{array} \right.$$

*Linker and Dieterich ( 1992 ), Dieterich and Linker ( 1992), Bizzarri and Cocco ( 2006a, 2006b )*

### Response to an abrupt jump in load

