

CONVERGENZA

Consistency requirement

Quando la dimensione dell' elemento (i.e. la discretizzazione) tende a zero le equazioni approssimate ben rappresentano le equazioni (differenziali) esatte e le condizioni al contorno

Stability condition

- i) La soluzione del sistema di equazioni discretizzate è unica ;
- ii) sono rimosse tutte le cause che contaminano le soluzioni per tutte le dimensioni degli elementi (i.e. della discretizzazione).

Consistency requirement

Stability condition



CONVERGENZA

- quanto buona è l' approssimazione computata
- come essa può essere sistematicamente migliorata per avvicinare la soluzione "esatta" del problema

Errore : differenza tra la soluzione "esatta" e quella approssimata.

3) BIE 2-D CON SLIP-WEAKENING



1) Unicità della soluzione nell'integrazione numerica del sistema linearizzato (Andrews, 1985):

$$\Delta x < - \frac{\mu v_p}{v_s \frac{d\tau}{du}}$$

o:

$$\Delta x < l_c^{(II)} \cdot \frac{\pi}{2} \frac{a \sqrt{a}}{a-1} \frac{1}{(1+S)^2}$$

$$a \equiv \sqrt{\frac{v_p}{v_s}}$$

$$\tau \equiv \frac{\tau_u - \tau_o}{\tau_o - \tau_f}$$

2) Risoluzione del breakdown process nella zona coesiva:

$$\Delta t \ll \tau_b$$

o

$$\Delta x \ll X_b$$

ove:

$$\tau_b = \frac{d_o}{\langle v \rangle \tau_b} = w_{CFL} \frac{v_{crack}}{v_s} \frac{d_o}{\langle v \rangle \tau_b}$$

$$w_{CFL} \equiv v_s \frac{\Delta t}{\Delta x}$$

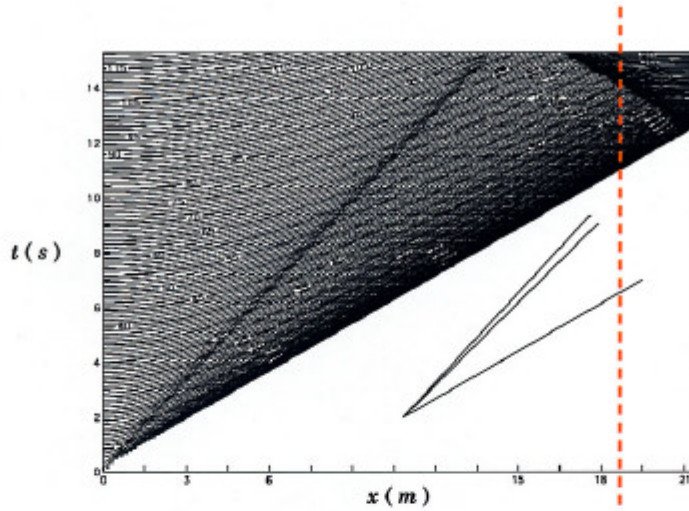
$$X_b = \frac{1}{w_{CFL}} \frac{v_s}{\langle v \rangle \tau_b} d_o = \frac{v_{crack}}{\langle v \rangle \tau_b} d_o$$

3) Disaccoppiamento dei primi vicini :

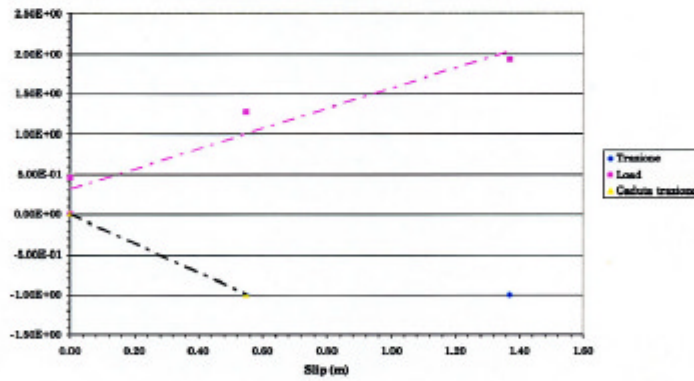
$$\Delta t \leq \frac{\Delta x}{v_p}$$

Esempio # 1

2) Violazione della condizione di unicità

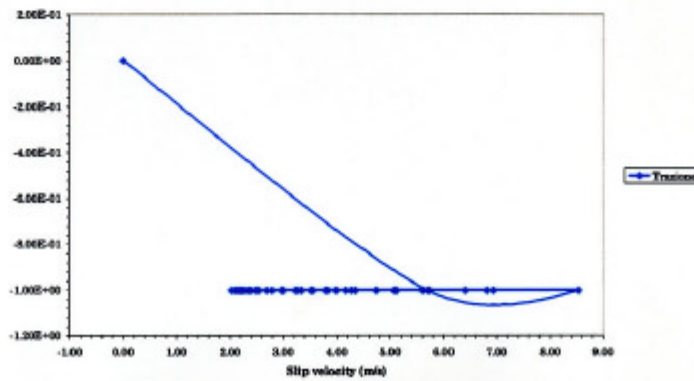


Particolare : Slip - Trazione / Slip - Load



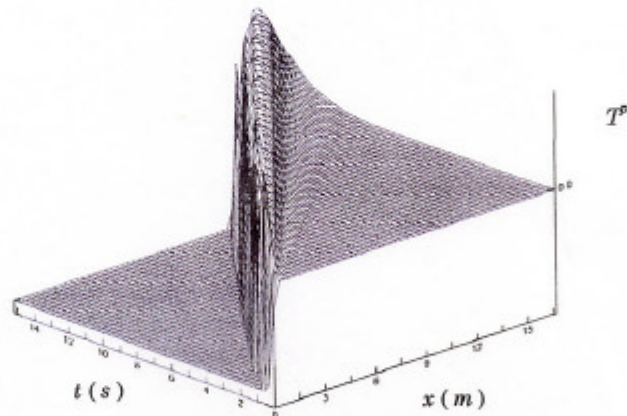
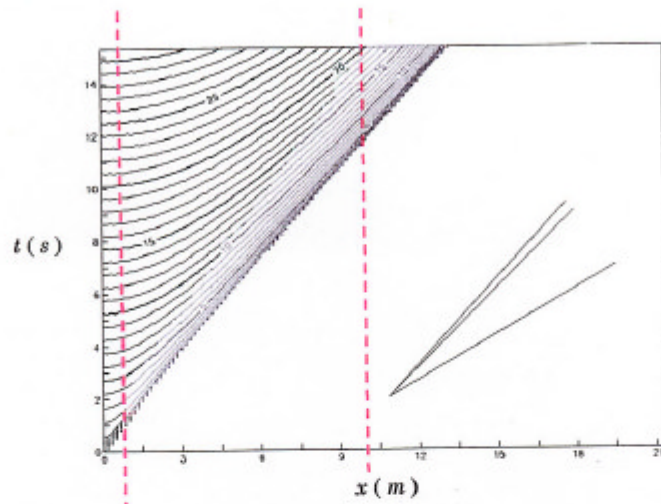
x = 18.84 m

Slip velocity - Trazione



Esempio #2

3) Violazione della condizione $dx < d_0$

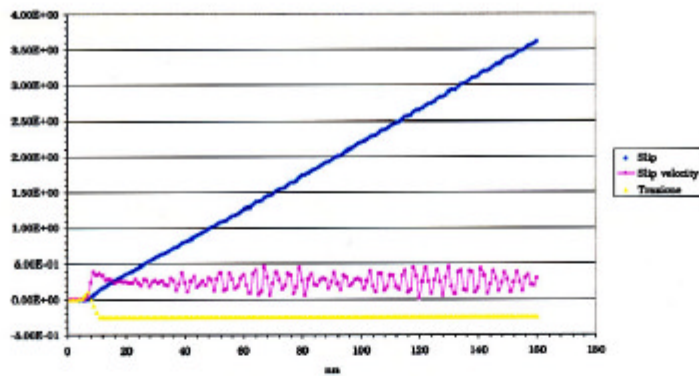


$$dx = 0.1667 \text{ m}$$

$$d_0 = 0.7 dx$$

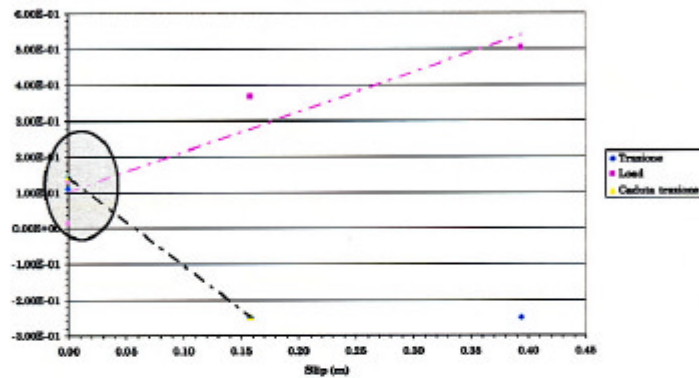
$$S = 3.2$$

Grandezze - Tempo



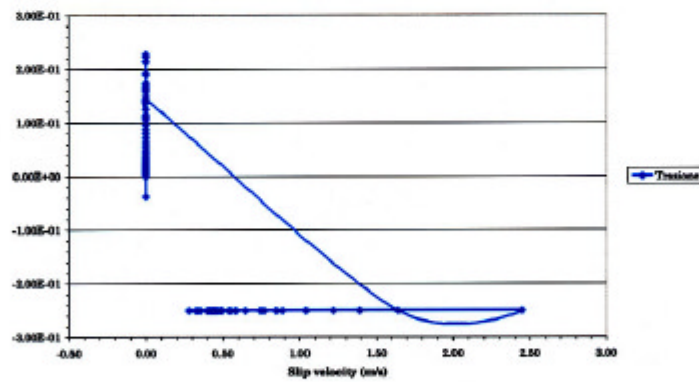
$x = 0.50 \text{ m}$

Particolare : Slip - Trazione / Slip - Load

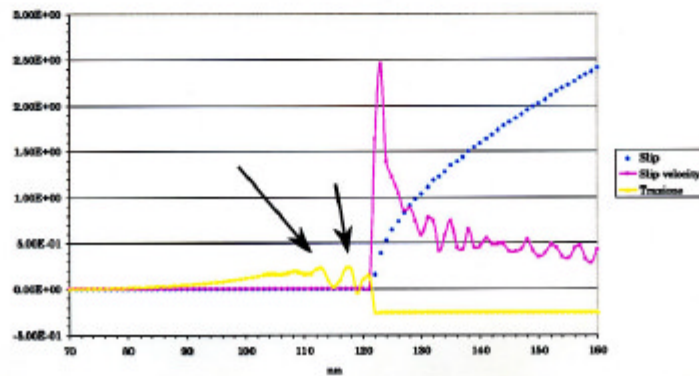


$x = 9.83 \text{ m}$

Slip velocity - Trazione



Particolare : Grandezze - Tempo



FD 2-D CON RATE- AND STATE-DEPENDENT FRICTION LAWS

1) Condizioni di Rice (1993) per l'approssimazione al continuo:

$$k_{diag} \gg k_{cr}$$

$$k_{diag} = \frac{1}{\epsilon} = \frac{v_s \rho}{8 w_{CFL}} \frac{\sqrt{3}}{\Delta t}$$

$$k_{cr} = \frac{(b-a) \sigma_n^{eff}}{L}$$

$$\Delta t \ll \Delta t_*$$

σ

$$\Delta x \ll \Delta x_*$$

ove:

$$\Delta t_* = \frac{v_s \rho L}{(b-a) \sigma_n^{eff}} \frac{8}{\sqrt{3}} w_{CFL}$$

$$w_{CFL} \equiv v_s \frac{\Delta t}{\Delta x}$$

$$\Delta x_* = \frac{v_s^2 \rho L}{(b-a) \sigma_n^{eff}} \frac{8}{\sqrt{3}}$$

2) Risoluzione della zona coesiva equivalente:

$$\Delta t \ll \tau_b^{eq}$$

o

$$\Delta x \ll x_b^{eq}$$

ove:

$$\tau_b^{eq} = \frac{d_o^{eq}}{\langle v \rangle_{\tau_b^{eq}}} \approx \frac{L \ln \left(\frac{v_o}{v_{init}} \right)}{\langle v \rangle_{\tau_b^{eq}}} = \frac{\omega_{CFL}}{v_s} \frac{v_{crack}}{\langle v \rangle_{\tau_b^{eq}}} d_o^{eq}$$

$$x_b^{eq} = \frac{1}{\omega_{CFL}} \frac{v_s}{\langle v \rangle_{\tau_b^{eq}}} d_o^{eq} \approx \frac{1}{\omega_{CFL}} \frac{v_s}{\langle v \rangle_{\tau_b^{eq}}} L \ln \left(\frac{v_o}{v_{init}} \right) =$$

$$= \frac{v_{crack}}{\langle v \rangle_{\tau_b^{eq}}} d_o^{eq}$$

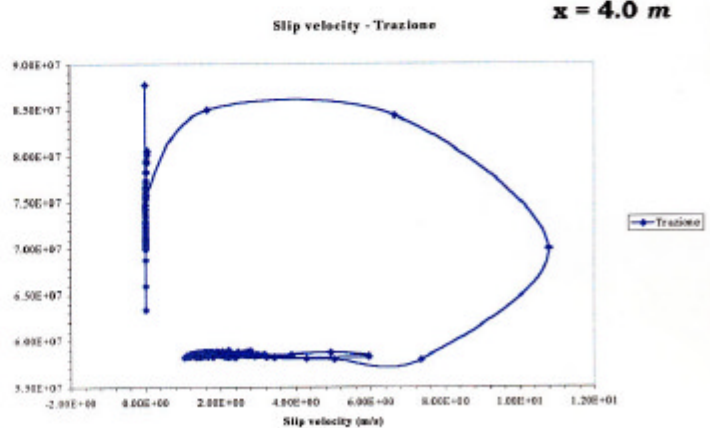
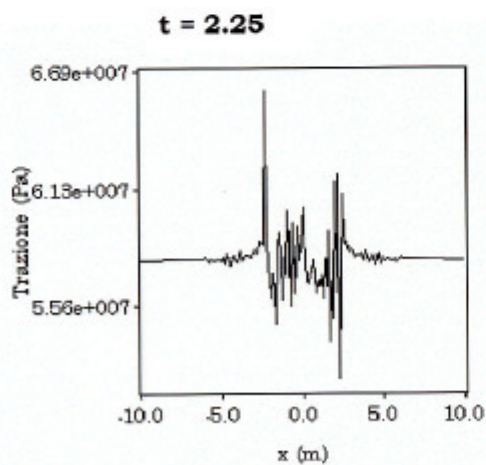
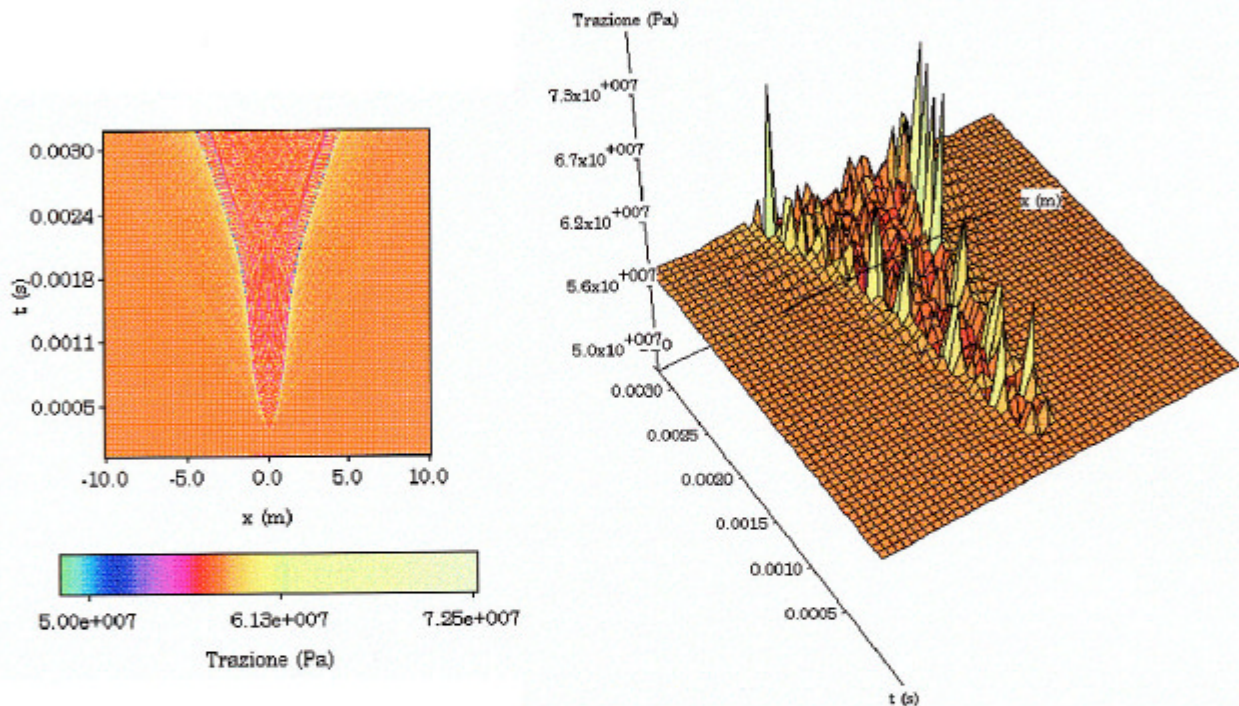
3) Disaccoppiamento dei primi vicini:

$$\Delta t \leq \frac{\Delta x}{v_p}$$

Esempio #1

Simulazioni di modelli 2 - D : legge costitutiva di Dieterich in forma ridotta

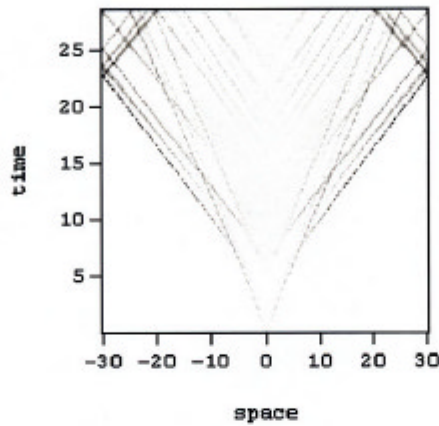
Esempio di regime *weak seismic* ($B - A \geq 0$)



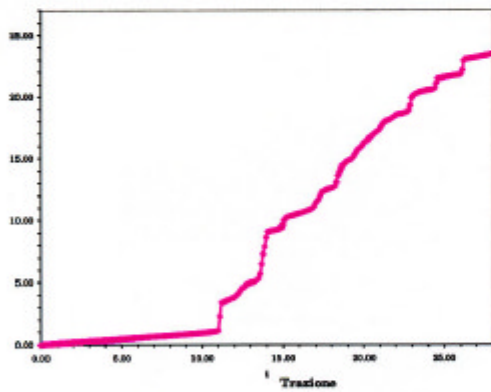
Esempio #2

Violation of the Rice (1993) condition

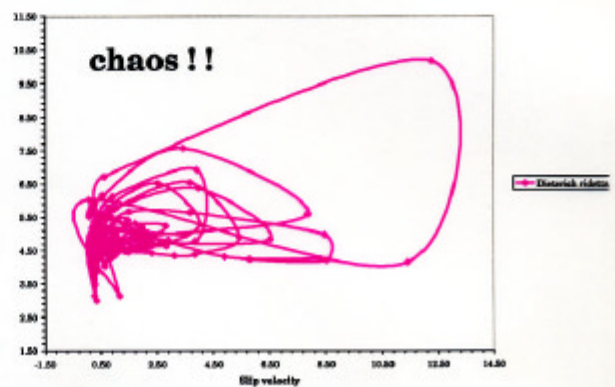
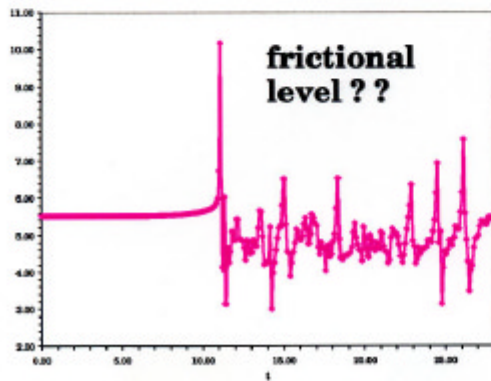
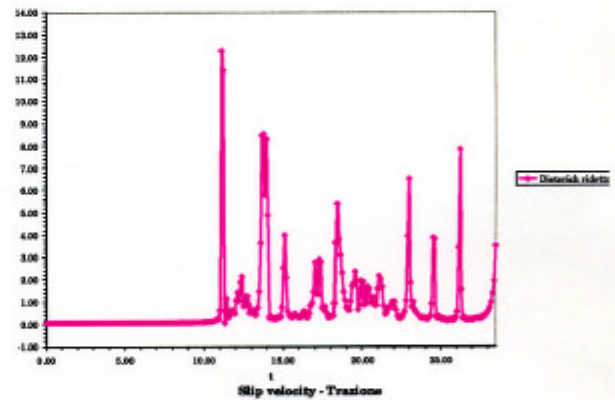
Slip velocity



Slip

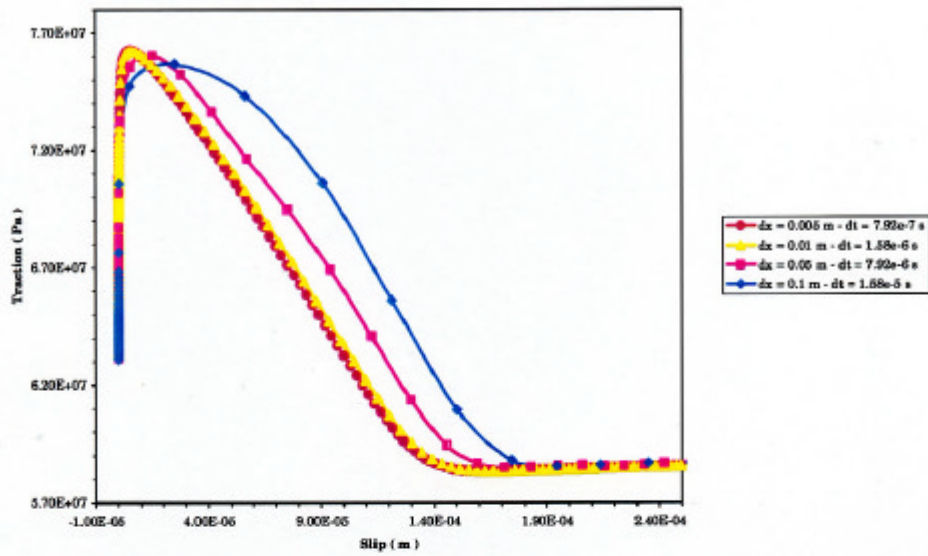


Slip velocity

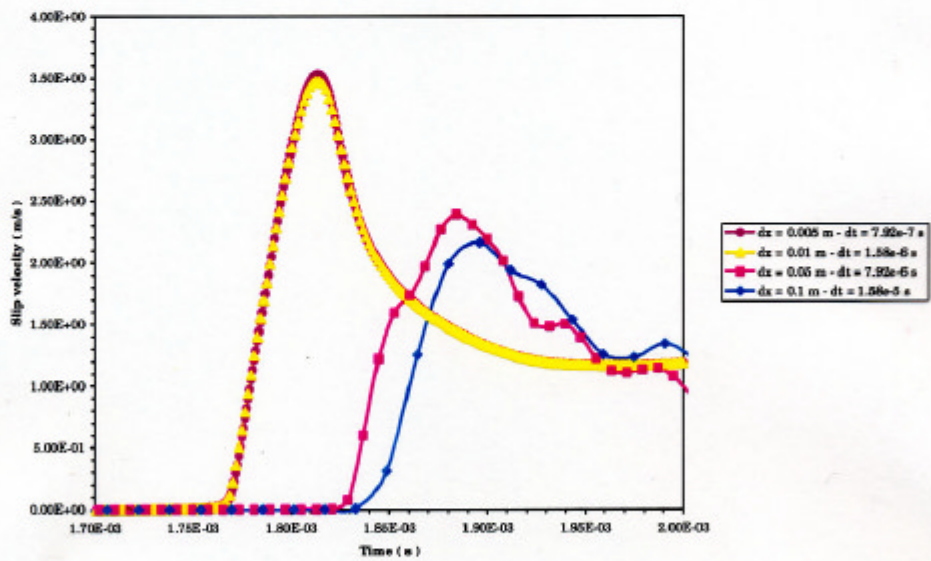


Esempio # 3

Condizioni di Convergenza e Stabilità #1

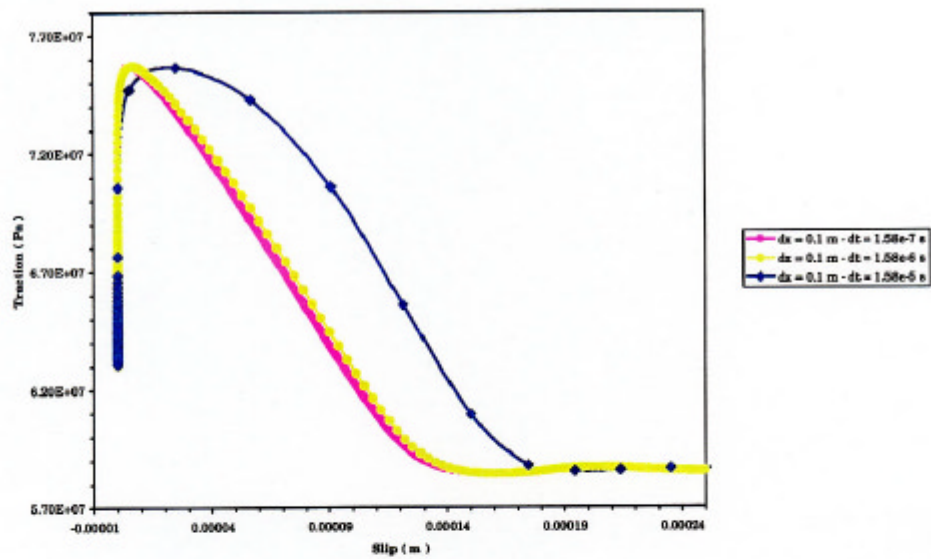


(a)

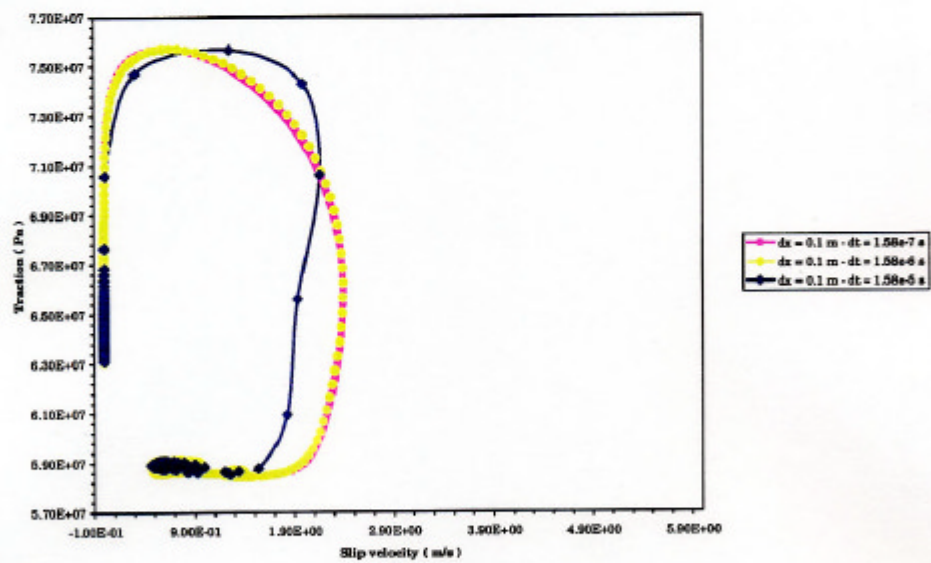


(b)

Condizioni di Convergenza e di Stabilità #2

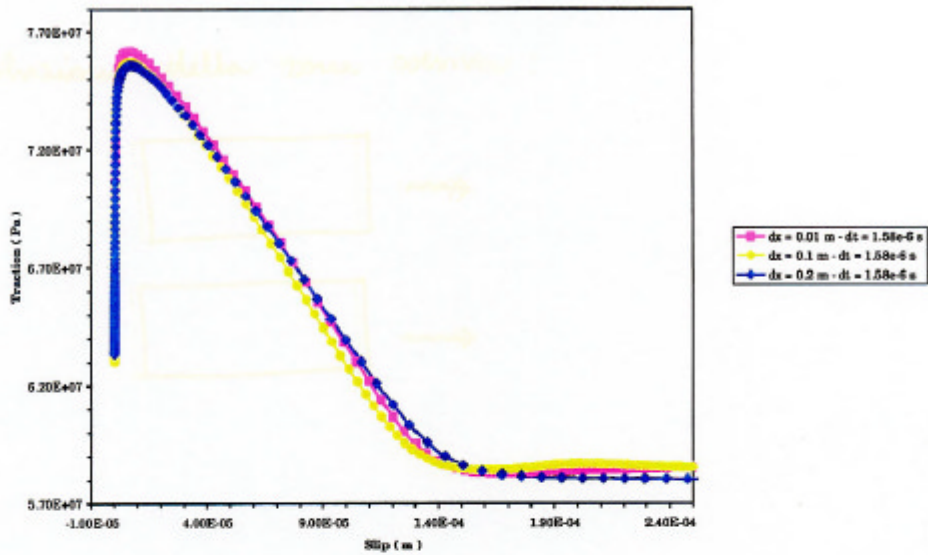


(a)

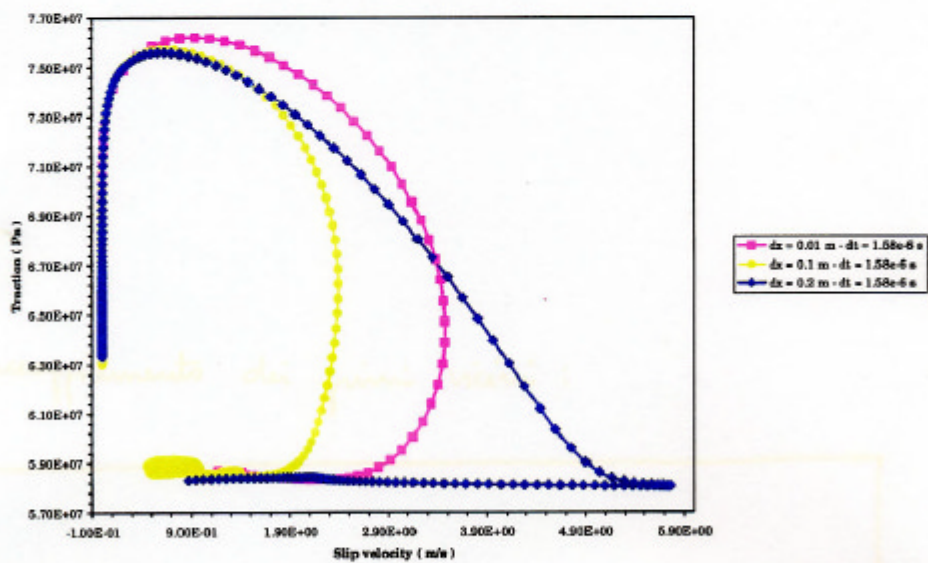


(b)

Condizioni di Convergenza e di Stabilità #3



(c)



(d)

FD 3-D CON SLIP-WEAKENING

1) Risoluzione della zona coesiva:

$$\Delta t \ll \tau_b \implies \Delta t \ll \text{Min} \{ \tau_b(\tau_1), \tau_b(\tau_2) \}$$

o

$$\Delta x \ll X_b \implies \Delta x \ll \text{Min} \{ X_b(\tau_1), X_b(\tau_2) \}$$

ove:

$$\tau_b(\tau_1) = \frac{d_0 \cos \varphi}{\langle v \rangle_{\tau_b(\tau_1)}}$$

breakdown zone time relative
alla componente 1

$$\tau_b(\tau_2) = \frac{d_0 \sin \varphi}{\langle v \rangle_{\tau_b(\tau_2)}}$$

2) Disaccoppiamento dei primi vicini:

$$\Delta t \leq \frac{1}{\sqrt{\left(\frac{v_p}{\Delta x}\right)^2 + \left(\frac{v_p}{\Delta y}\right)^2 + \left(\frac{v_p}{\Delta z}\right)^2}}$$