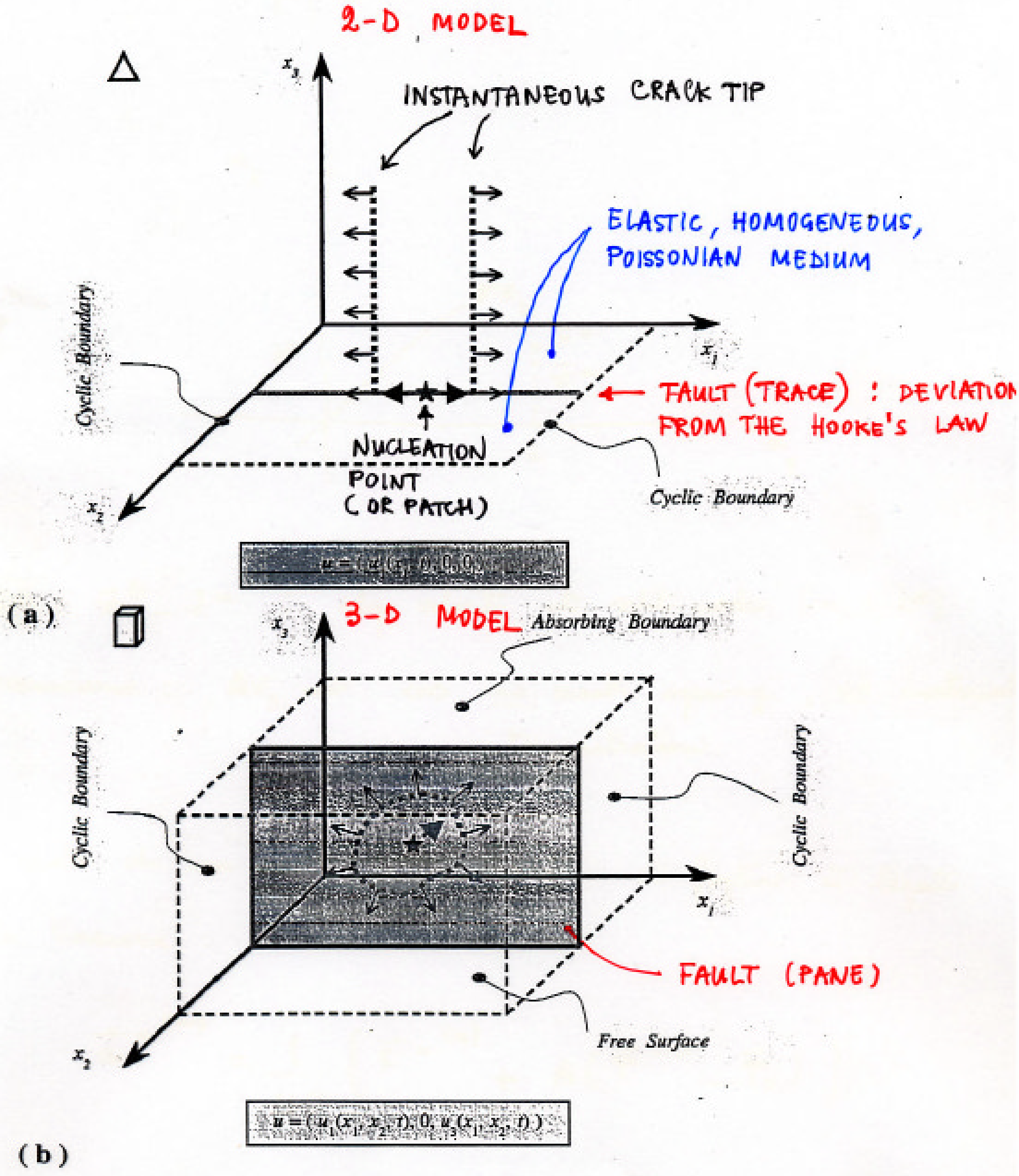
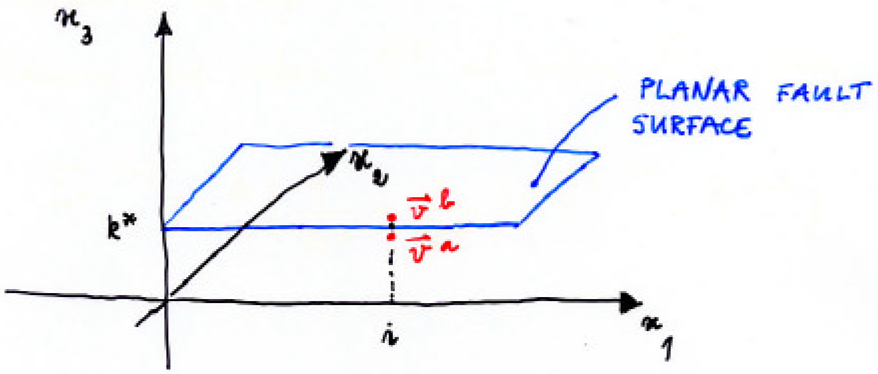
THE FAULT BOUNDARY CONDITION
(FBC)



From Bizzarri (2003)

- **Traction at Split Nodes (TSN) FBC**

La faglia è rappresentata da una superficie nella quale ogni nodo (grid point) è splittato ed ogni parte appartiene ad un lato della superficie.



$\Delta\vec{v} = \vec{v}^{\,b} - \vec{v}^{\,a}$: slip velocity at split node $(i, 1, k^*)$

Assumiamo : $\Delta v_3 = 0 \iff$ no crack opening, no material interpenetration

Il moto di un nodo della superficie di faglia è legato alla trazione :

$$\vec{a}^{\,a\,(n)} = \frac{1}{M^a}\left(\vec{F}^{\,a\,(n)} + A\left(\vec{T}^{\,(n)} - \vec{T}_0\right)\right)$$

$$\vec{a}^{\,b\,(n)} = \frac{1}{M^b}\left(\vec{F}^{\,b\,(n)} - A\left(\vec{T}^{\,(n)} - \vec{T}_0\right)\right)$$

ove: $\vec{T}_0$ è il valore iniziale (di riferimento) della trazione sulla superficie di faglia

A è l'area del nodo

$\vec{F}^a$, $\vec{F}^b$ sono le forze che agiscono sul nodo

$\vec{a}^a$, $\vec{a}^b$ sono le accelerazioni del nodo, la cui variazione in seguito ad un cambiamento della trazione dal valore $\vec{T}^t$ al valore $\vec{T}$ è data da:

$$\vec{\Delta a}^{(n)} = A\left(\frac{1}{M^a}+\frac{1}{M^b}\right)\left(\vec{T}^{t\,(n)} - \vec{T}^{(n)}\right)$$

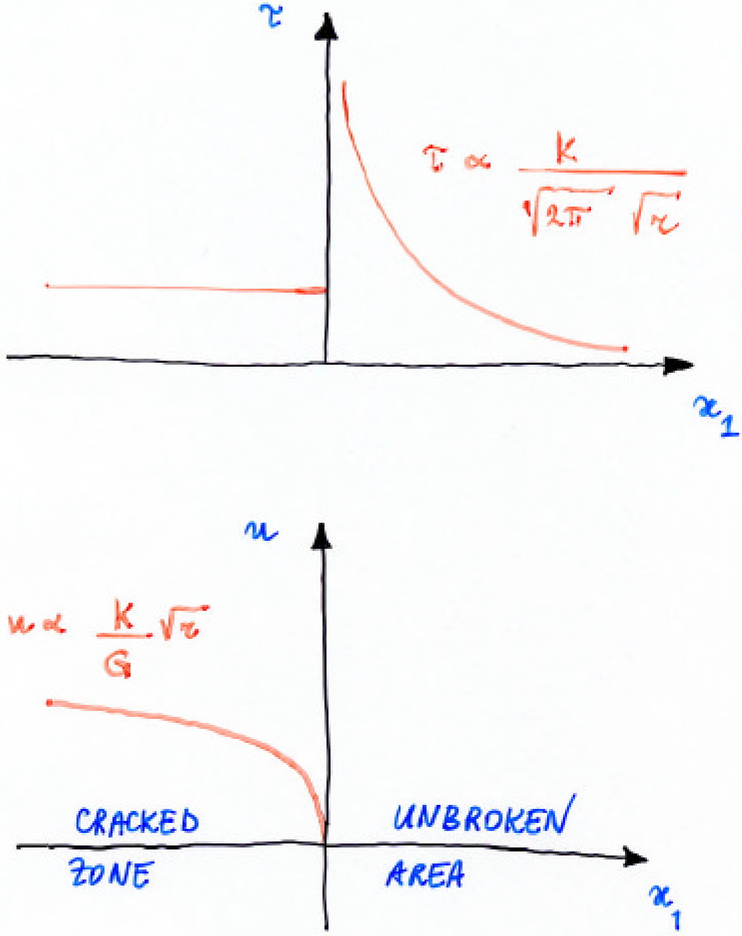
Spostamenti, forze e trazioni sono note al time step n.
Velocità sono note al time step $n-\frac{1}{2}$. Esse sono esplicitamente aggiornate mediante lo schema:

$$\Delta v_i^{(n+\frac{1}{2})} = \Delta v_i^{(n-\frac{1}{2})} + \Delta t\, A\left(\frac{1}{M^a}+\frac{1}{M^b}\right)\left(T_i^{t\,(n)} - T_i^{(n)}\right)$$

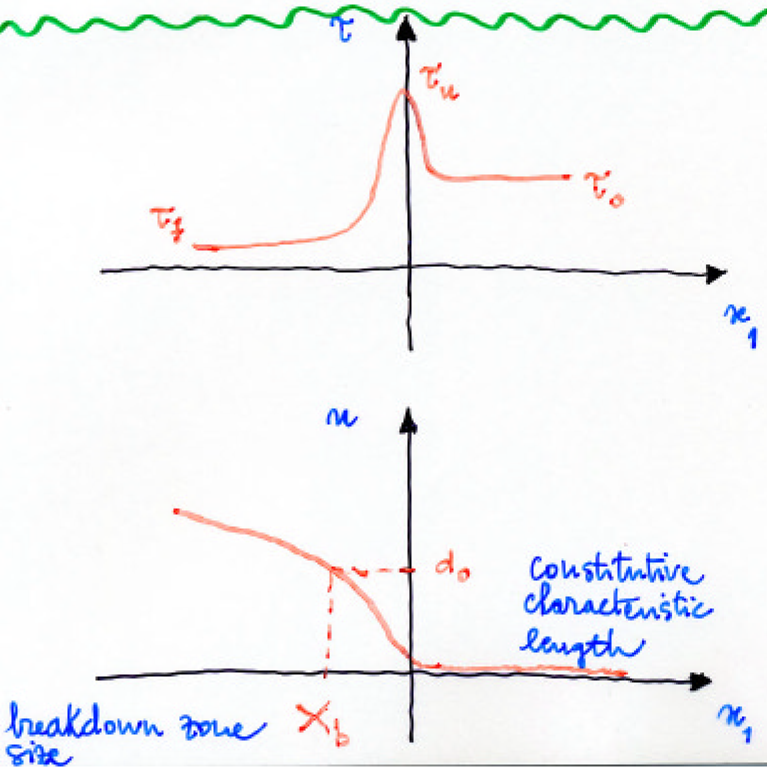
$i = 1,2$

Classical elastic crack model

[$x_1 = 0$: crack tip]



Crack model with constitutive equation



Dolt 2003 _ V.4
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Governing equations

- **Slip – weakening**
  - * Classical form
    Andrews (1976a, 1976b)
  - * Modified forms
    (with initial slip – hardening)
    Ohnaka & Yamashita (1989)
    Matsu'ura et al. (1992)

- **Rate and state dependent friction laws**
  - * Original forms :
    - – Dieterich
    - – Dieterich in reduced form
    - – Ruina

    Dieterich (1978, 1979a, 1979b, 1981) ;
    Ruina (1980, 1983)
  - * Regularized forms :
    - – Dieterich in reduced form regularized
    - – Ruina regularized

    Perrin et al. (1995)

- **Rate dependent friction law**
  Madariaga & Cochard (1994)

- **Slip and state dependent friction law**
  Cochard & Madariaga (1996)

Constitutive Equations

- The solution of the dynamic rupture problem requires the use of a constitutive law that relates the total dynamic traction to fault friction.

$$\tau = \Im(u, \dot{u}, \sigma_n^{eff}, c_e, \lambda_c, T, \phi)$$

u is the slip,

$\dot{u}$ the slip velocity,

σ_n^{eff} the effective normal stress (which include the pore pressure)

c_e the chemical effect of the fluid pressure,

λ_c a parameter describing the geometric characteristics of the fault surface (roughness, fault gouge, etc.),

T the temperature

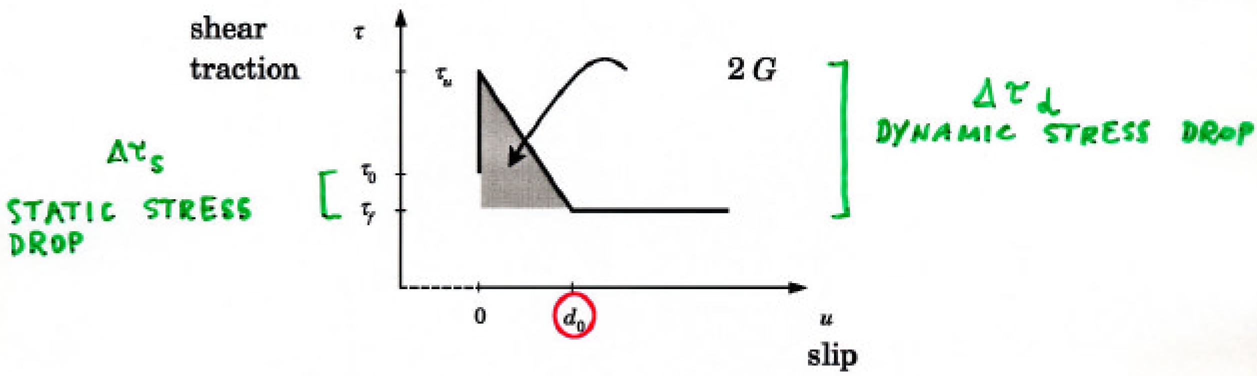
$\Phi \equiv (\Phi_1, \ldots, \Phi_N)$ is the state variable

$$\boxed{\frac{d}{dt}\phi_i = g_i(u, \dot{u}, \sigma_n^{eff}, c_e, \lambda_c, T, \phi) \quad i = 1, \ldots, N}$$

The slip – weakening model

Ida (1972)
Andrews (1976a,b)

$$\boxed{\tau(u)=\begin{cases}\tau_u-(\tau_u-\tau_f)\dfrac{u}{d_0} & ,u<d_0\\[2ex] \tau_f & ,u\geq d_0\end{cases}}$$



Critical half – length L_c :

$$L_c^{(\mathrm{II})}=8\,\frac{\mu}{\pi}\,\frac{\lambda+\mu}{\lambda+2\,\mu}\,\frac{G}{(\tau_0-\tau_f)^2}$$

$$L_c^{(\mathrm{III})}=4\,\frac{\mu}{\pi}\,\frac{G}{(\tau_0-\tau_f)^2}$$

- cracked < L_c : quasi-static nuclea-tion and propagation [STOP IF G LOW]
- cracked > L_c : dynamic propagation [SPONTANEOUS CRACK ACCELERATION]

Fracture energy :

$$G=(\tau_u-\tau_f)\,d_0/4$$

The rate and state dependent friction laws

$$\tau(t) = F\left(\sigma_n, v(t)\, , state(t) \right)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\, state(t) = G\left(\sigma_n, v(t)\, , state(t), L \right)$$

$$state \;\longleftrightarrow\; \Psi \equiv (\, \Psi_1, \ldots, \Psi_N \,)$$

$$\Psi = \Psi\,(\, T,\, \sigma_n \,, \text{chemical environment} \,)$$

Assumption : the functions F e G are effective – normal – stress – independents .

Dieterich in reduced form (DRF) model :

if $(A \,/\, \tau_*) \ln(v_* / v + 1) << 1$

Dieterich (1986)

$$\boxed{\begin{cases} \tau = \tau_* - A \ln\left(\dfrac{v_*}{v} + 1 \right) + B \ln\left(\dfrac{\Phi\, v_*}{L} + 1 \right) \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Phi = 1 - \dfrac{\Phi\, v}{L} \end{cases}}$$

v_* : reference slip velocity

τ_* : friction at velocity v_*

Steady state value ($(\mathrm{d}/\mathrm{d}t)\ \Phi = 0$) :

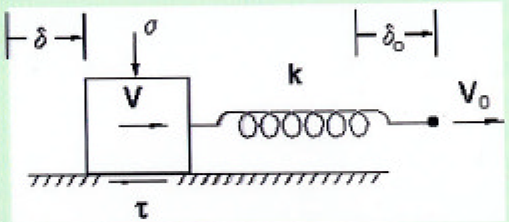
$$\Phi^{ss} = \frac{L}{v}$$

Rate & State dependent parameters

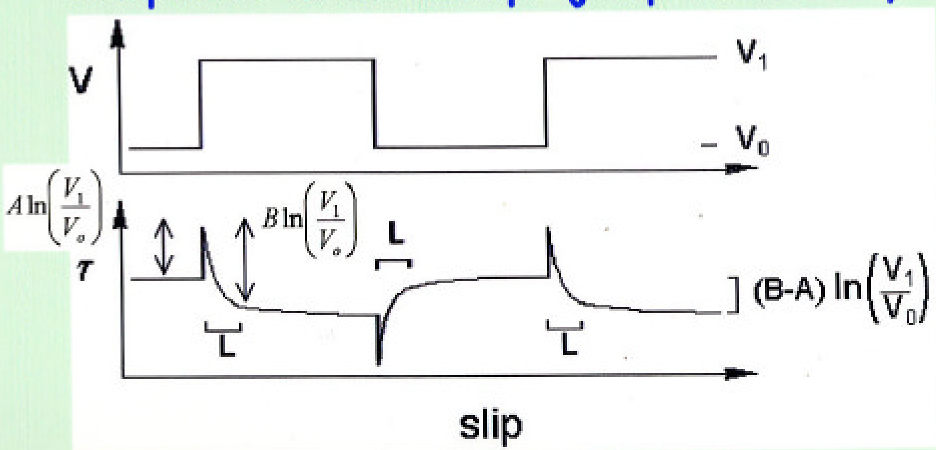
$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} + 1 \right) + b \ln\left(\dfrac{\Phi v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Phi = 1 - \dfrac{\Phi v}{L} \end{cases}$$

- Although a propagating crack shows a quite different sliding velocity behavior, a spring slider model represents a useful analog system to study analytically complex constitutive laws

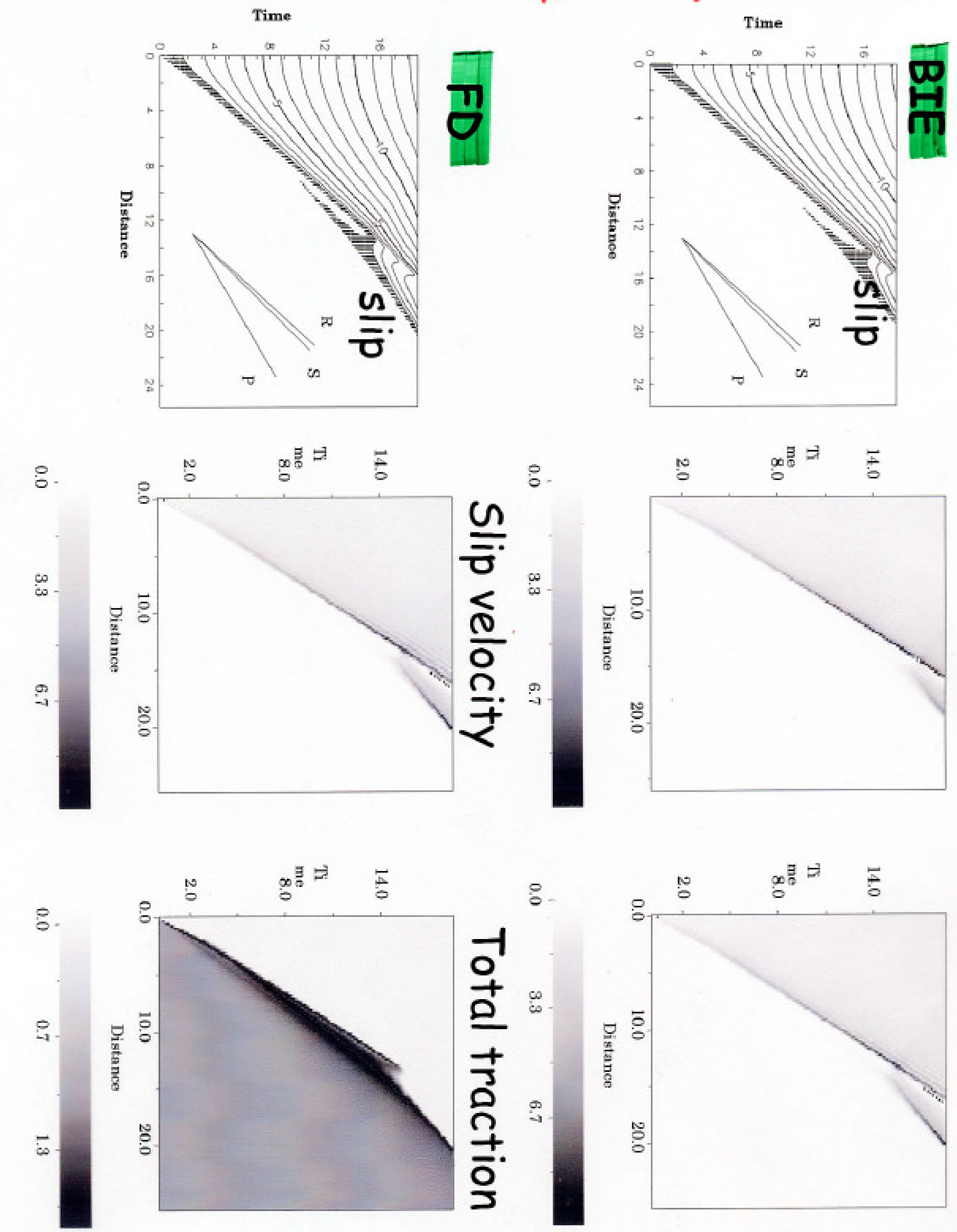
Spring slider models



Response to an abrupt jump in velocity

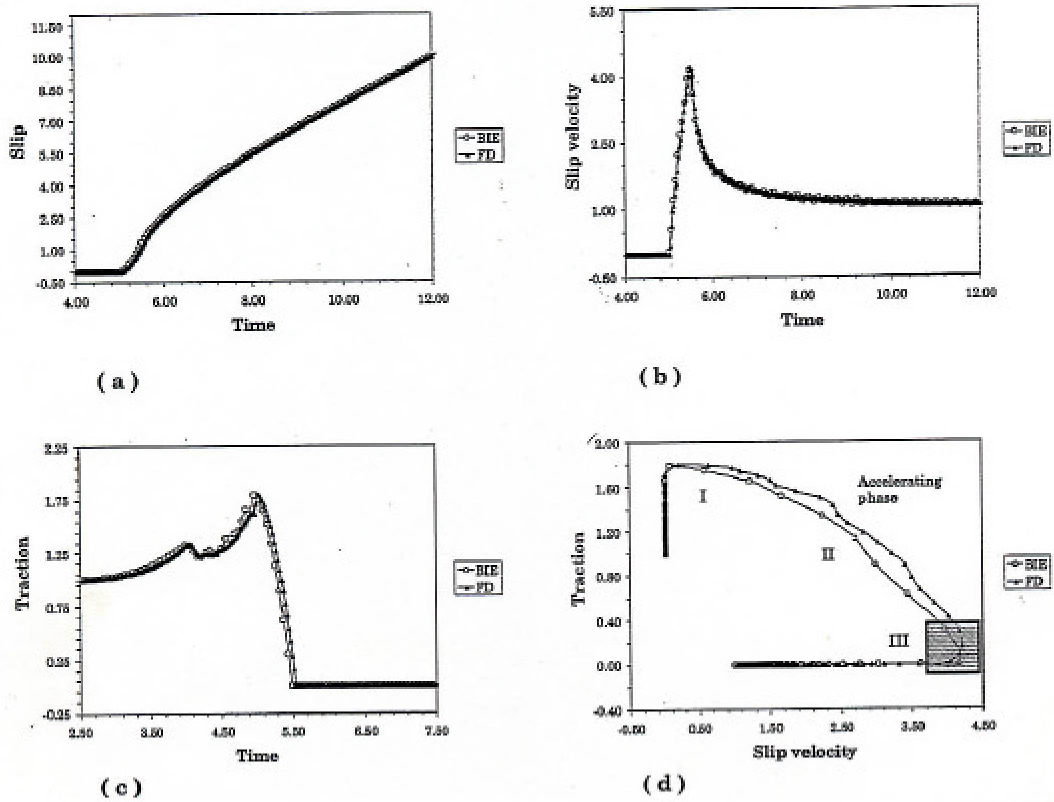


BIE vs. FD for a 2-D fault governed by SW law #1



From Bizzarri et al. (2001)

BIE vs. FD for a 2-D fault governed by SW law #2



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