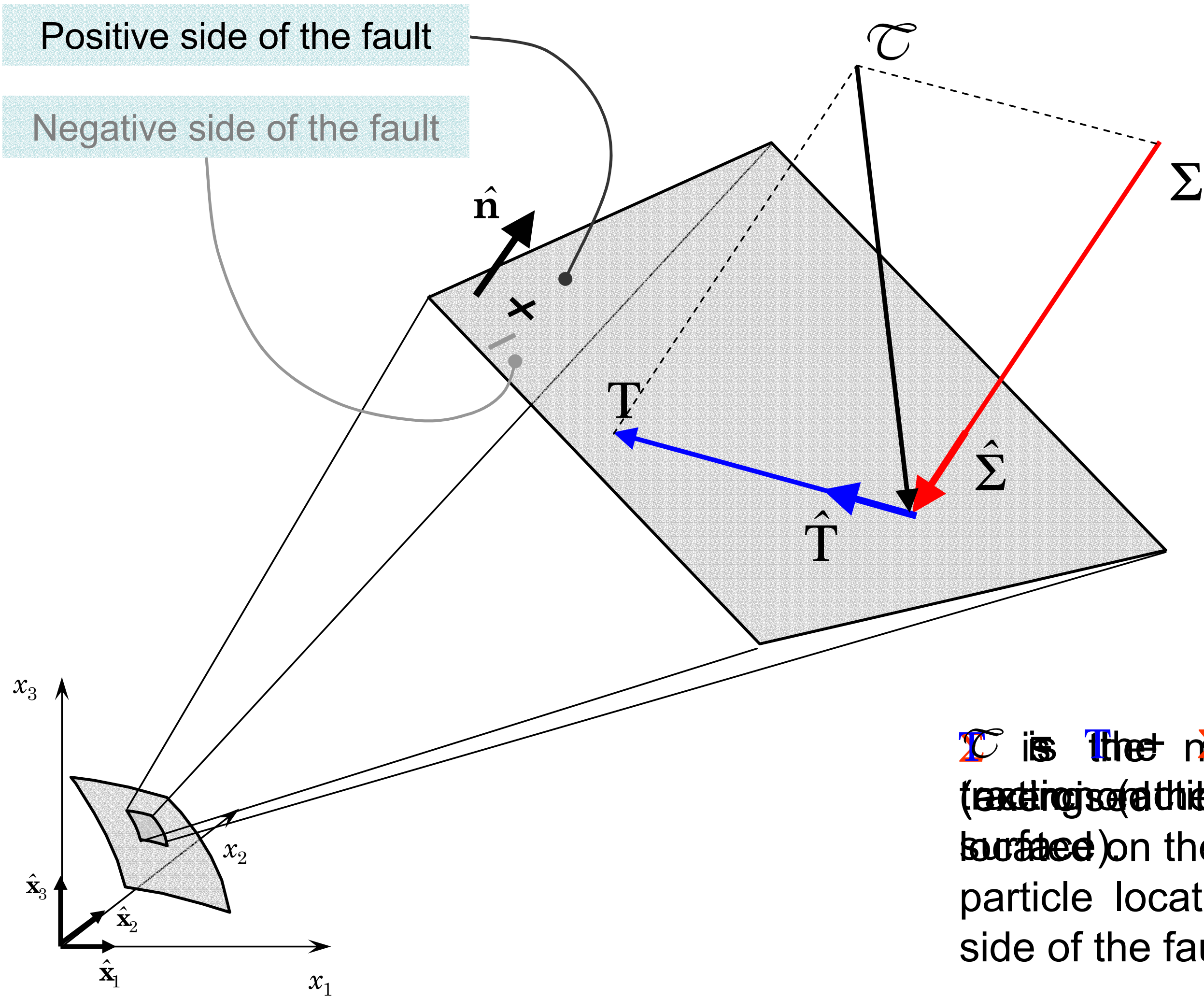
Fault governing laws
(constitutive equations)

Notations and symbols



located on the +ve side on a particle located on the –ve side of the fault surface)

$\mathcal{T} = \mathrm{T} + \Sigma$ <u>total</u> traction (acting on the fault surface).

$\mathcal{T}_j = n_i \sigma_{ij}^{eff}$ *Cauchy*' s formula, where $\mathcal{T} = (\mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3)$, $\mathbf{n} = (n_1, n_2, n_3)$ and

$$\sigma_{ij}^{\ eff} = \sigma_{ij} - p_{fluid}\,\delta_{ij} = \begin{bmatrix} \sigma_{11} - p_{fluid} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} - p_{fluid} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} - p_{fluid} \end{bmatrix}$$

$T_j = n_i \sigma_{ij}^{eff} - n_j (n_i \sigma_{ij}^{eff} n_j^{\mathrm{T}})$ <u>shear</u> traction

$\Sigma_j = n_j (n_i \sigma_{ij}^{eff} n_j^{\mathrm{T}})$ <u>normal</u> traction

Fracture Criteria & Constitutive Laws

1. FRACTURE CRITERION

- Condition that specify, at a given fault point and at a given time, if there is a rupture or not.
- It can be expressed in terms of **energy**, in terms of **maximum frictional resistence**, and so on.
- It is based on (*i*) the *Benioff (1951)* hypothesis: The fracture occours when the stress in a volume reaches the rock strength
  or, analogoulsy,
  (*ii*) the *Reid (1910)* statement: The fracture takes place when the stress attains a value greater than the rock can endure.

2. CONSTITUTIVE LAW

- Analytical relation existing between the components of the stress tensor and physical observable(s), like the slip, the slip velocity, the state variable, etc..
- From a mathematical point of view it is a **Fault Boundary Condition** (**FBC**) that controls earthquake dynamics and its complexity in space and in time.
- Its simplest form consider only **two frictional levels**, τ_u and τ_f; it accounts for stress drop (or stress realease), but the process is instantaneous: there is a singularity at crack tip.
- **Cohesive zone models**: *Barenblatt (1959a, 1959b)*, *Ida (1972)*, *Andrews (1976a, 1976b)*. In these models the singularity is removed and the sress release occours over a breakdown zone distance X_b and in a breakdown zone time T_b.
- **Friction laws** (Rate and State dependent f. l.): *Dieterich (1976)*, *Ruina (1980, 1983)*. They accounts for fault spontaneous nucleation, re – strengthening, healing, etc..

CONSTITUTIVE LAW (continues)

- “ The central issue is *whether* faults obey simple friction laws, and if so, what is the friction coefficient associated with fault slip “ (*Scholz and Hanks, 2004*).

CONSTITUTIVE LAW (continues)

- In full of generality we can express the constitutive (or governing) as:

$$\tau = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\ \sigma_n^{eff}(\sigma_n, p_f)$$

where:

1st – order dependencies

u is the Slip (i. e. displ. disc.) modulus,

v is the Slip Velocity modulus (its time der.),

Ψ = ($\Psi_1, \ldots, \Psi_N$) is the State Variable vector,

T is the Temperature (accounting for Ductility, Plastic Flow, Melting and Vaporization),

H is the Humidity,

λ_c is the Characteristic Length of surface (accounting for Roughness and Topography of asperity contacts),

h is the Hardness,

g is the Gouge (accounting for Surface Consumption and Gouge formation),

C_e is the Chemical Environment

Strength & Constitutive Laws

1. *THE STRENGTH PARAMETER*

- **Hystorically introduced by *Das and Aki (1977a, 1977b)* to have a quantitative extimate of the ability to fracture for a fault**
- Its expression can be generalized as:

$$S = (\mu_u \sigma_n^{eff} - \tau_0) / (\tau_0 - \mu_f \sigma_n^{eff})$$

  where μ are the friction coefficient.

- We can also define

2. *THE FAULT STRENGTH*

- **Is the parameter that quantify the Strenght in the more general case, in which a fault is described by a rhealistic friction laws**

$$S^{fault} = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\, \sigma_n^{eff}(\sigma_n, p_{fluid})$$

Time - weakening Friction Law

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{(t - t_r)}{t_0} \right] \sigma_n^{\;eff} & , t - t_r < t_0 \\ \mu_f \sigma_n^{\;eff} & , t - t_r \geq t_0 \end{cases}$$

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ilaw = 11
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TW

$t_r = t_r(\xi)$ is the rupture onset time in every fault point ξ (when $u > 0$).

<u>*Andrews (1985)*</u>, *Bizzarri et al. (2001)* and other following Bizzarri' s papers

t_0 is the characteristic time – weakening duration.

Position - weakening Friction Law

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{x}{R_0} \right] \sigma_n^{eff} & , -R_0 < x < 0 \\ \mu_f \sigma_n^{eff} & , -L < x < -R_0 \end{cases}$$

PW

x is the position on the fault (extending up to – L).

Palmer and Rice (1973)

R_0 is the characteristic position – weakening distance.

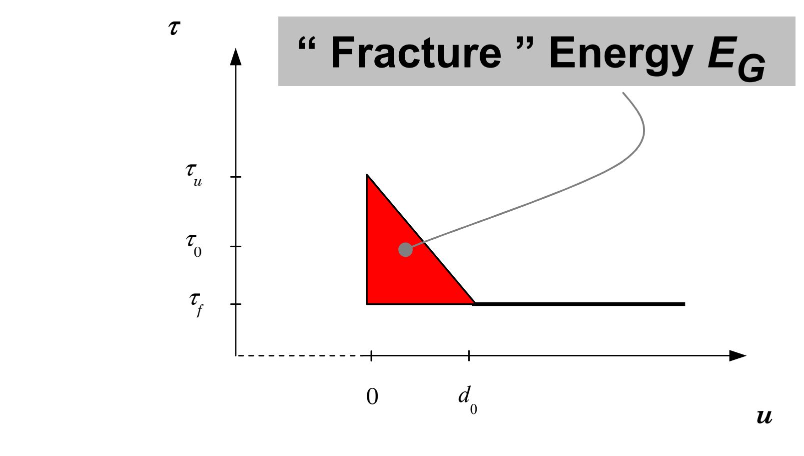
Slip - Dependent Friction Laws

1. LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{u}{d_0} \right] \sigma_n^{\ eff} & , u < d_0 \\ \mu_f \sigma_n^{\ eff} & , u \geq d_0 \end{cases}$$

ilaw = 21

SW



Barenblatt (1959a, 1959b), <u>Ida (1972)</u>, Andrews (1976a, 1976b), and many authors thereinafter

d_0 is the characteristic slip – weakening distance ilaw = 22

IW

2. NON – LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - \dfrac{\mu_u - \mu_f}{d_0} \left(u - \dfrac{(1 - p_{IW}) d_0}{2\pi} \sin\left(\dfrac{2\pi u}{d_0} \right) \right) \right] \sigma_n{}^{eff} & , u < d_0 \\ \mu_f \sigma_n{}^{eff} & , u \geq d_0 \end{cases}$$

Ionescu and Campillo (1999)

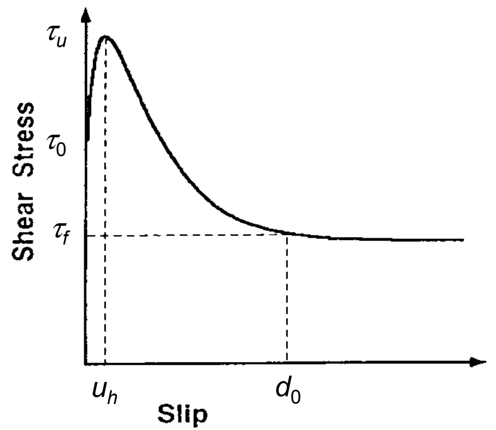
3. NON LINEAR SLIP – WEAKEING LAW WITH SLIP – HARDENING

$$\tau = \left\{ \left[\left(\frac{\tau_0}{\sigma_n^{\ eff}} - \mu_f \right) \left(1 + \alpha_{OW} \ln \left(1 + \frac{u}{\beta_{OW}} \right) \right) \right] e^{-\frac{u}{d_0}} + \mu_f \right\} \sigma_n^{\ eff}$$

ilaw = 23

OW

$$u_h : \left. \frac{\mathrm{d}\tau}{\mathrm{d}u} \right|_{u_h} = 0 ; \qquad \begin{cases} u_h = r d_0 \quad (\text{e.g. } r = 0.1) \\ \tau(u_h) = \tau_u \end{cases}$$



Ohnaka and Yamashita (1989) and the following papers by Ohnaka and coworkers

u_h is associated with the preparatory phase of the imminent macroscopic failure in the cohesive zone. It accounts for micro–cracking

Rate - and State - Dependent Friction Laws

1. DIETERICH IN REDUCED FORMULATION

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\frac{v_*}{v} \bullet \right) + b \ln\left(\frac{\Psi v_*}{L} \bullet \right) \right] \sigma_n^{eff} \\ \frac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \frac{\Psi v}{L} \end{cases}$$

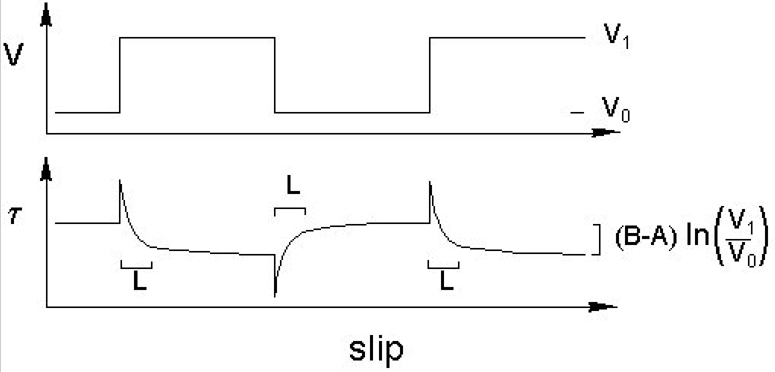
ilaw = 31

DR

However, while in velocity stepping experiments the traction response following the velocity variation is directly controlled by the parameter *L*, its effects are much less evident during the dynamic rupture propagation.

Bizzarri and Cocco (2005)

Response to an abrupt jump in load



2. RUINA – DIETERICH

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{\,eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = -\dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) \end{cases}$$

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ilaw = 32
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RD

Ruina (1980, 1983), Beeler et al. (1984), Roy and Marone (1996)

3. DIETERICH – RUINA WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{\,eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi\, v}{L} - \left(\dfrac{\alpha_{LD} \Psi}{b\, \sigma_n^{\,eff}} \right) \dfrac{\mathrm{d}}{\mathrm{d}t} \sigma_n^{\,eff} \end{cases}$$

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ilaw = 31
decis10=T
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DR

Linker and Dieterich (1992), Dieterich and Linker (1992), Bizzarri and Cocco (2006b, 2006c)

4. RUINA – DIETERICH WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v}\right) + b \ln\left(\dfrac{\Psi\, v_*}{L}\right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi = -\dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L}\right) - \left(\dfrac{\alpha_{LD}\Psi}{b\,\sigma_n^{eff}}\right) \dfrac{\mathrm{d}}{\mathrm{d}t}\sigma_n^{eff} \end{cases}$$

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ilaw = 32
decis10=T
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RD

<u>Linker and Dieterich (1992)</u>, Bizzarri and Cocco (2006b, 2006c)

5. DIETERICH IN REDUCED FORM REGULARIZED

$$
\begin{cases}
\tau = \left[\mu_* - a \ln\left(\dfrac{v + v_*}{v \boxed{+ v_r}} \right) + b \ln\left(\dfrac{\Psi \left(v \boxed{- v_r}\right)}{L} + 1 \right) \right] \sigma_n^{eff} \\
\dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi \left(v \boxed{+ v_r}\right)}{L}
\end{cases}
$$

`ilaw = 33`

DE

v_r *is a regularization fault slip velocity*

Perrin et al. (1995), Cocco et al. (2004)

6. RUINA REGULARIZED

$$
\begin{cases}
\tau = \left[\mu_* - a \ln\left(\dfrac{v_* \boxed{- v_r}}{v \boxed{+ v_r}} \right) + \dfrac{\Psi}{\sigma_n^{eff}} \right] \sigma_n^{eff} \\
\dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{v \boxed{+ v_r}}{L} \left(\Psi + b \ln\left(\dfrac{v \boxed{+ v_r}}{v_* \boxed{- v_r}} \right) \right)
\end{cases}
$$

ilaw = 34

RE

v_r is a regularization fault slip velocity

<u>Bizzarri (2002, unpublished work)</u>

7. DIETERICH IN REDUCED FORM WITH HEALING

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} + 1 \right) + b \ln\left(\dfrac{\Psi v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = \dfrac{\gamma_{fh} - \Psi}{\boxed{t_{fh}}} - \dfrac{\Psi v}{L} \end{cases}$$

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ilaw = 35
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DH

γ_{fh} = 1 s

t_{fh} is the time for healing (slip duration)

Evolution law proposed by *Nielsen et al. (2000)* and by *Nielsen and Carlson (2000)*. Used in this form by *Cocco et al. (2004)*

9. PRAKASH – CLIFTON

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi v_*}{L} \right) \right] \left(\dfrac{\mathrm{d}}{\mathrm{d}t}\Psi_1 + \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi_2 \right) \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi = 1 - \dfrac{\Psi v}{L} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi_1 = -\dfrac{v}{L_1}\left(\Psi_1 - \alpha_{PC_1} \sigma_n^{eff} \right) \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi_2 = -\dfrac{v}{L_2}\left(\Psi_2 - \alpha_{PC_2} \sigma_n^{eff} \right) \end{cases}$$

ilaw = 37

PC

Ψ_1 *and* Ψ_2 are additional state variables accountinf for the coupling with effective normal stress. The formulation of friction law is not based on the Amonton – Coulamb law.

Coupling with effective normal stress proposed by *Prakash and Clifton (1993)* and *Prakash (1998)*. Used in this form by *Bizzarri (2005, unpublished work)*

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Support Slides: Parameters, Notes, etc.

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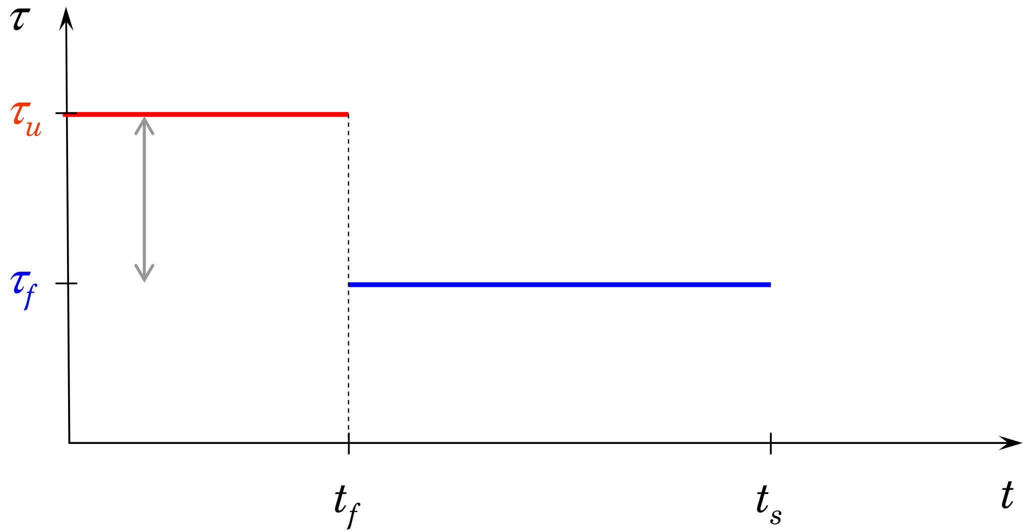
Simplest friction models

At a particular fault point ξ (following *Savage and Wood, 1971*; *Scholz, 1990*)

Maximum (or upper, or yield) stress

Kinetic (or frictional) stress



Strength excess: $\tau_u - \tau_0 = 0$

Dynamic stress drop: $\Delta\tau_d = \tau_0 - \tau_f$

Failure time (or rupture onset)

Rupture arrest

Simplest friction models

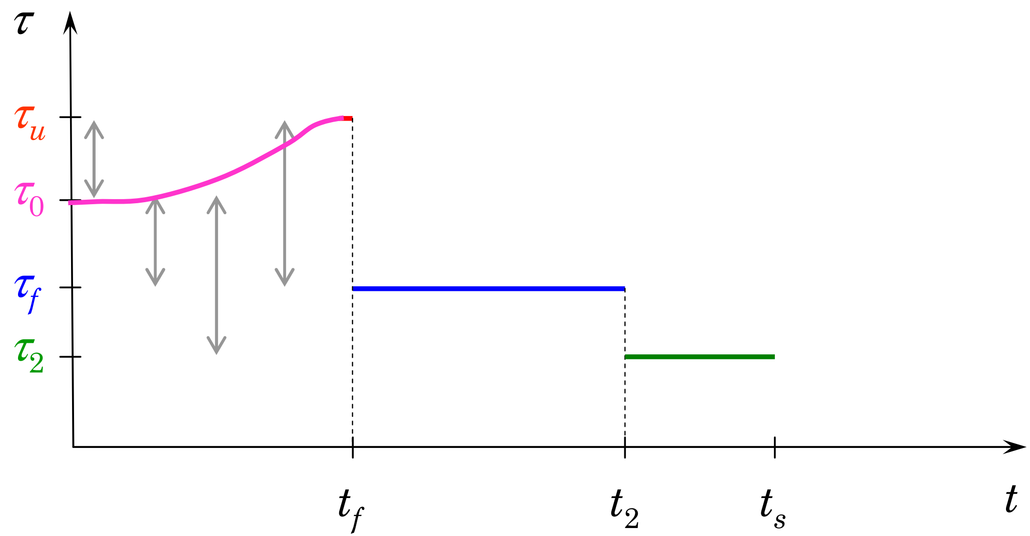
At a particular fault point ξ (following *Savage and Wood, 1971*; *Scholz, 1990*)

Maximum (or upper, or yield) stress

Initial stress

Kinetic (or frictional) stress

Residual stress



Strength excess: $\tau_u - \tau_0$

Dynamic stress drop: $\Delta\tau_d = \tau_0 - \tau_f$

Static stress drop: $\Delta\tau_s = \tau_0 - \tau_2$

Breakdown str. drop: $\Delta\tau_b = \tau_u - \tau_f$

Failure time (or rupture onset)

Dynamic overshoot

Rupture arrest

- *Savage and Wood (1971)* also define:

Mean stress: $<\tau> = ½ (\tau_u + \tau_2)$

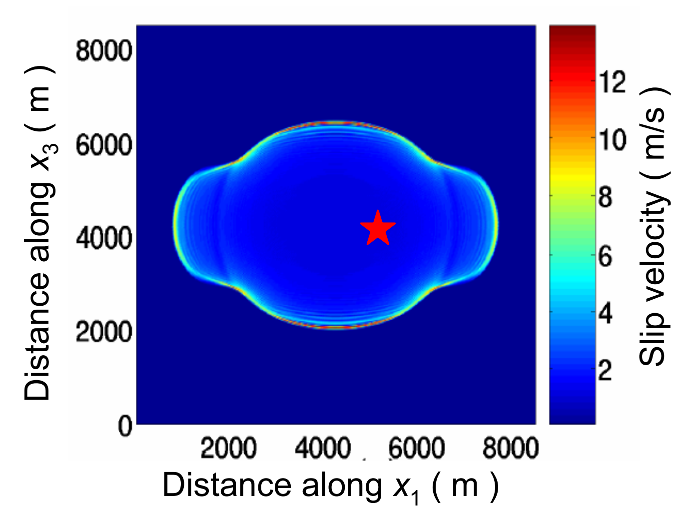
Seismic efficiency: $\eta = E_s/E$, where: E_s is the seismic energy
E is the total available energy

Apparent stress: $\tau_a = \eta <\tau>$

- Direct observation of the absolute stress near an earthquake is not feasible, but it is possible (*Wyss and Brune, 1968*) calculate τ_a and stress drop from physical observables.

The cohesive zone

Time snapshot (t = 0.8 s) - SW law



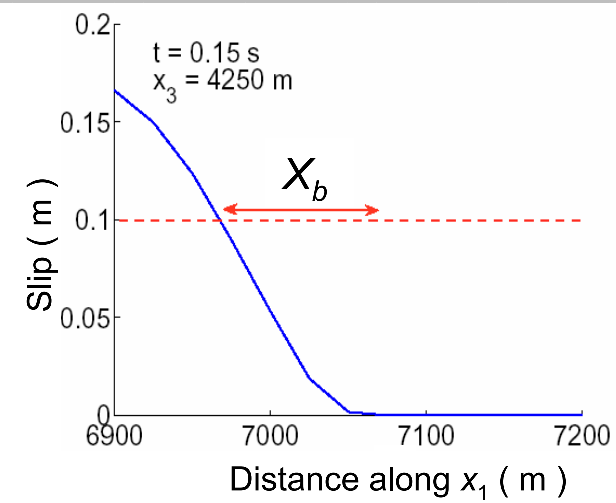
Local estimate

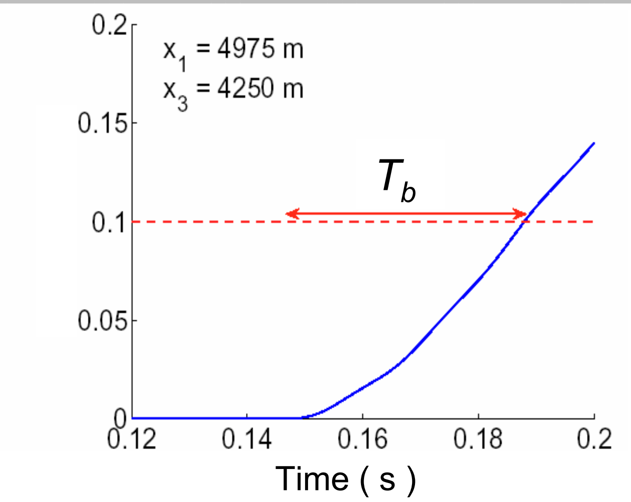
In the target location we can extimate:

X_b = 105 m T_b = 0.04 s

From these quantities:

$v_{rupt} = X_b/T_b$ = 2625 m/s





Slip - hardening effect

* The slip – hardening (SH) phenomenon has been also found in seismological inversion studies (e. g. *Quin, 1990*; *Miyatake, 1992*; *Mikumo and Miyatake, 1993*; *Beroza and Mikumo, 1996*; *Ide, 1997*; *Bouchon, 1997*).