# Rupture propagation in 2 – D fault models

# Numerical Method: BIE 2 - D

We solve the fundamental elastodynamic equation, neglecting body forces ***f***

$$\rho \ddot{U}_i = \sigma_{ij,j} + \cancel{f_i}$$

Source integral rapresentation ( *Betti*' s theorem, Integration in time limit in fault surface, Lamb' s problem ):

$$u_n(\mathbf{x},t) = \int_{-\infty}^{+\infty} \mathrm{d}t' \int_{\mathcal{S}(t')} \mathrm{d}\xi \, G_{n\alpha}(\mathbf{x}-\xi, t-t') \, \sigma_{\alpha\beta}{}^{p}(\xi, t') \; ; n=1,2,3; \alpha=1,2; \mathbf{x}, \xi \in \mathbb{R}^3$$

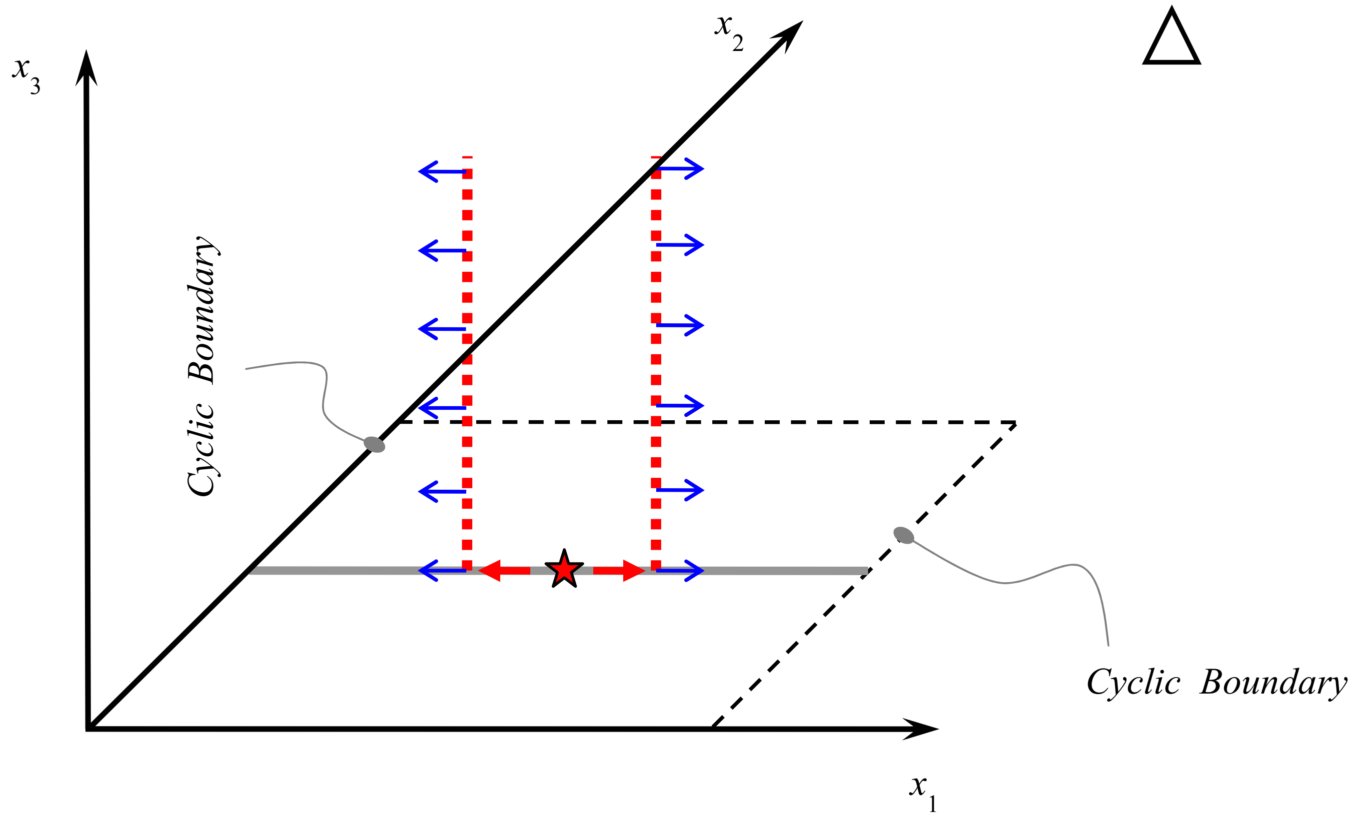
First neighbours decoupling ( in the case of a 2 – D, pure in – plane rupture ):

Traction

$$\begin{cases} u_1(x_1,t) + C\,\tau_1{}^{p}(x_1,t) = L_1(x_1,t) \\ \tau_{0_1} + \tau_1{}^{p}(x_1,t) = \mu\sigma_n{}^{eff} \end{cases}$$

Friction

# Numerical Method: FD 2 - D




$\boldsymbol{u} = (\ u_1(x_1, t), 0, 0\ )$

2-D

We solve the fundamental elastodynamic equation, neglecting body forces ***f***

$$\rho \ddot{u}_i = \sigma_{ij,j} + f_i$$

We discretize the $x_1x_2$ plane by using triangular cells ( better performances )

$$\rho \frac{\partial}{\partial t} \dot{u}_1 = \frac{\partial}{\partial x_1} \Sigma_{11} + \frac{\partial}{\partial x_2} \Sigma_{12}$$

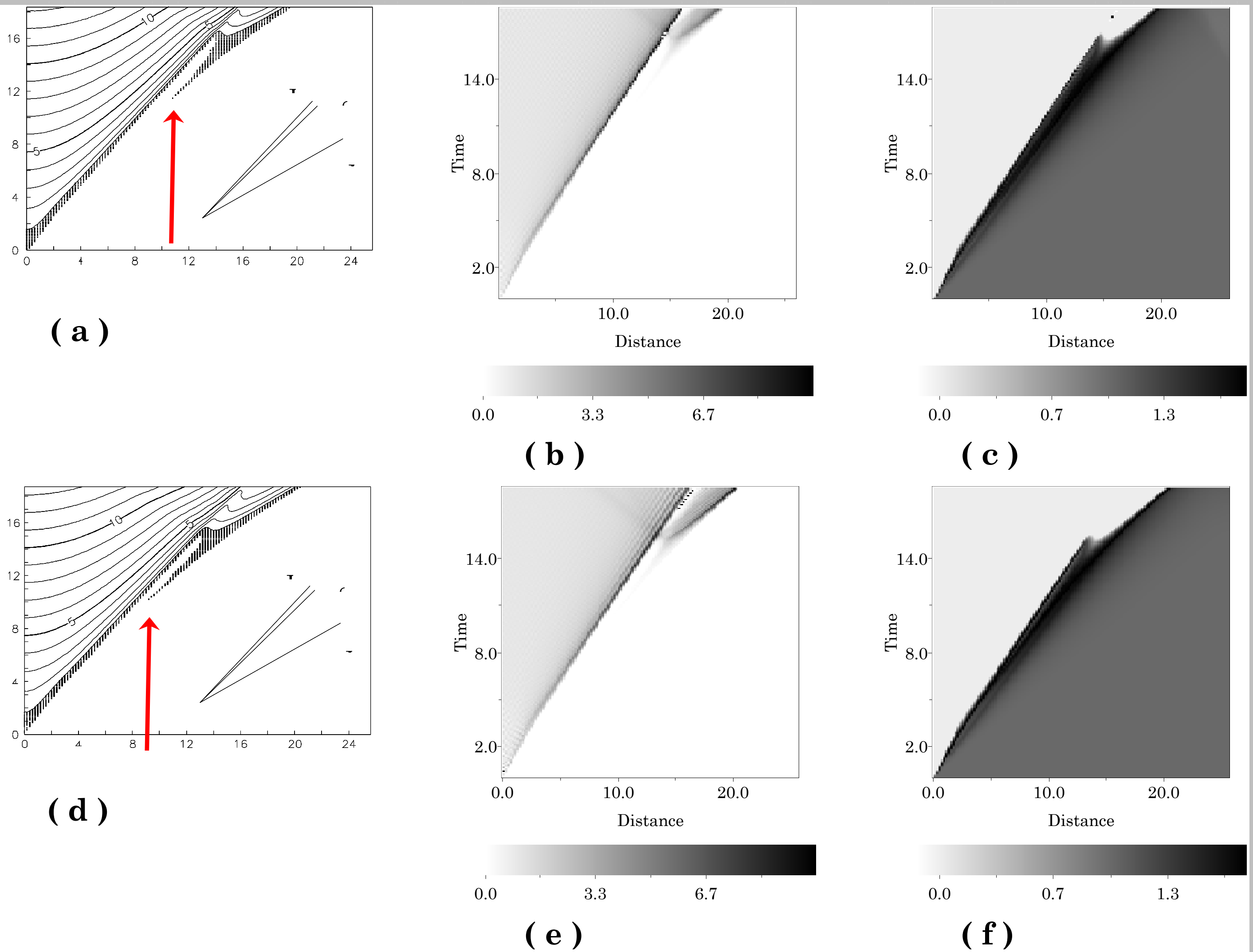
$$\rho \frac{\partial}{\partial t} \dot{u}_2 = \frac{\partial}{\partial x_1} \Sigma_{12} + \frac{\partial}{\partial x_2} \Sigma_{22}$$

The plane is linear and elastic except in the fault intersection line, where a Fault Boundary Condition ( TSN scheme ) is adopted. In tihs line a constitutiva law is assumed to relate staggered stress with observables ( slip, slip velocity, … )

# BIE vs. FD with SW #1

BIE

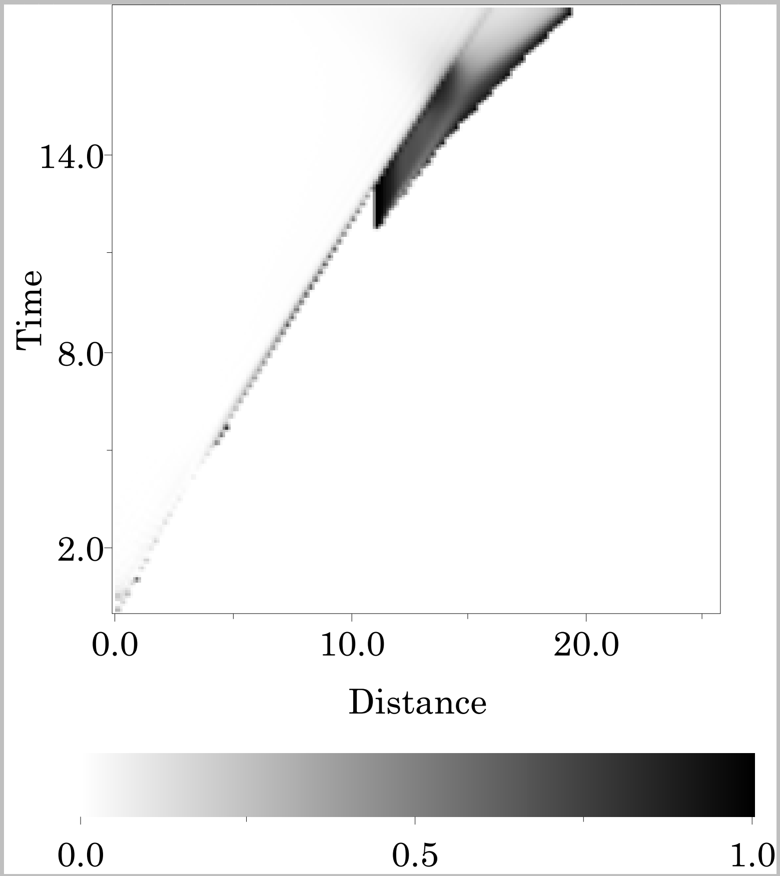
FD



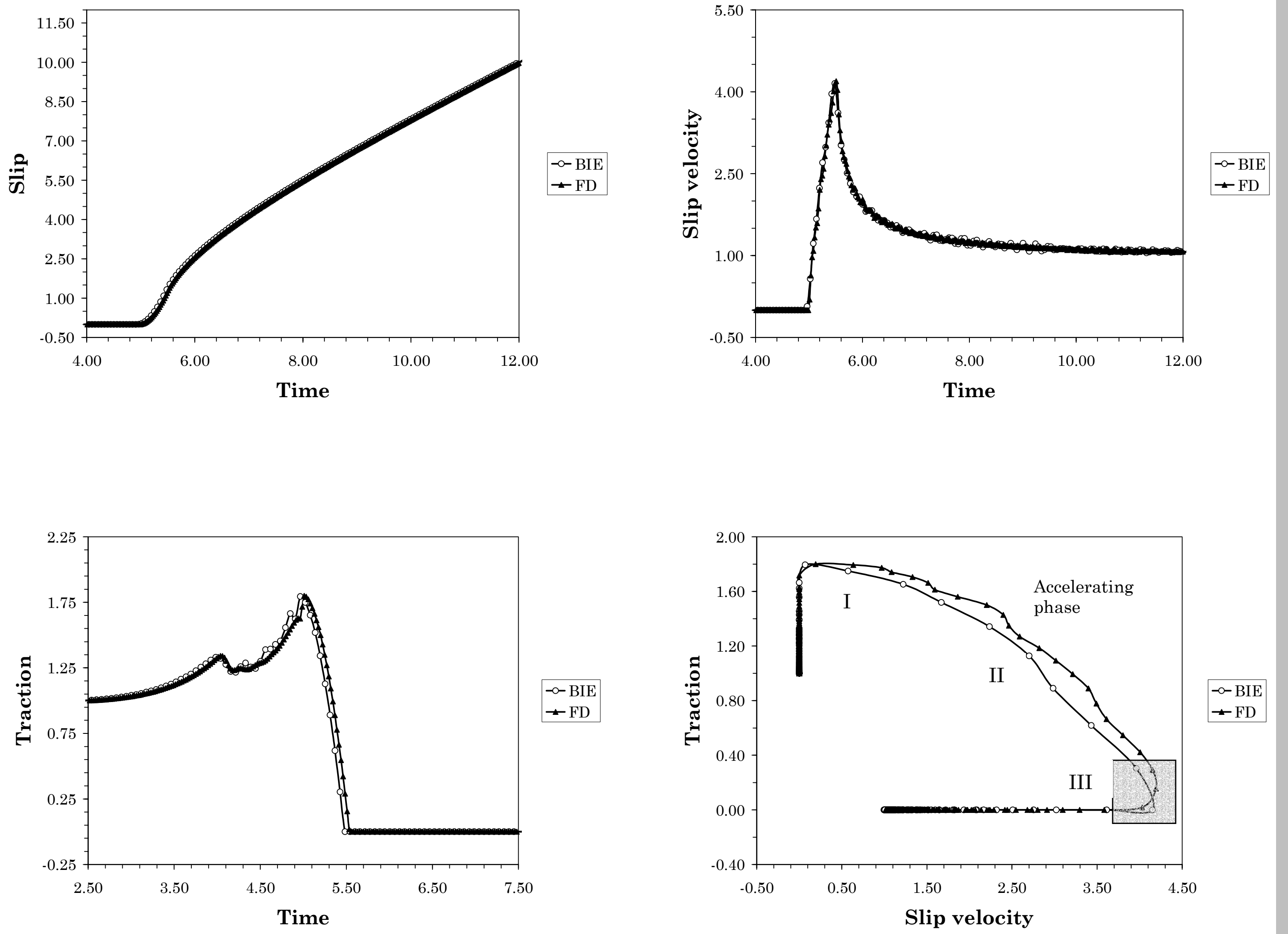
2-D

# Misfit between slip modeled with BIE and FD

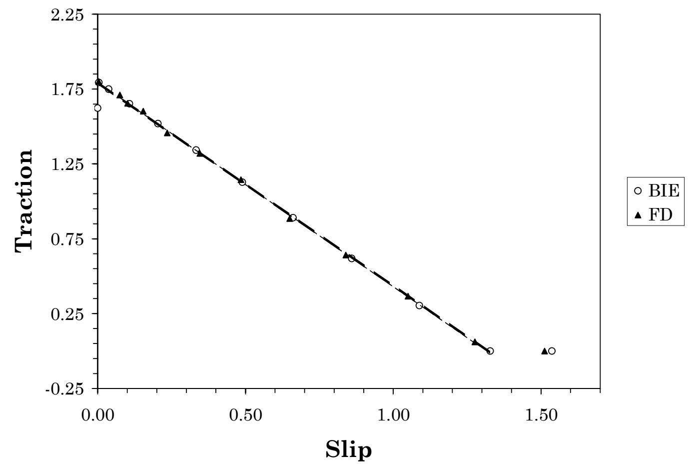
$$m(x_i, t_n) = \frac{\left| u^{(\mathrm{BIE})}(x_i, t_n) - \tilde{u}^{(\mathrm{FD})}(x_i, t_n) \right|}{\left| u^{(\mathrm{BIE})}(x_i, t_n) + \tilde{u}^{(\mathrm{FD})}(x_i, t_n) \right|}$$

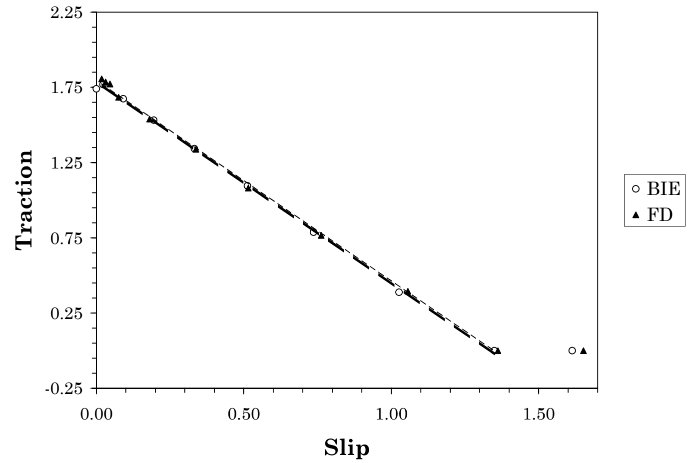


# BIE vs. FD with SW #2

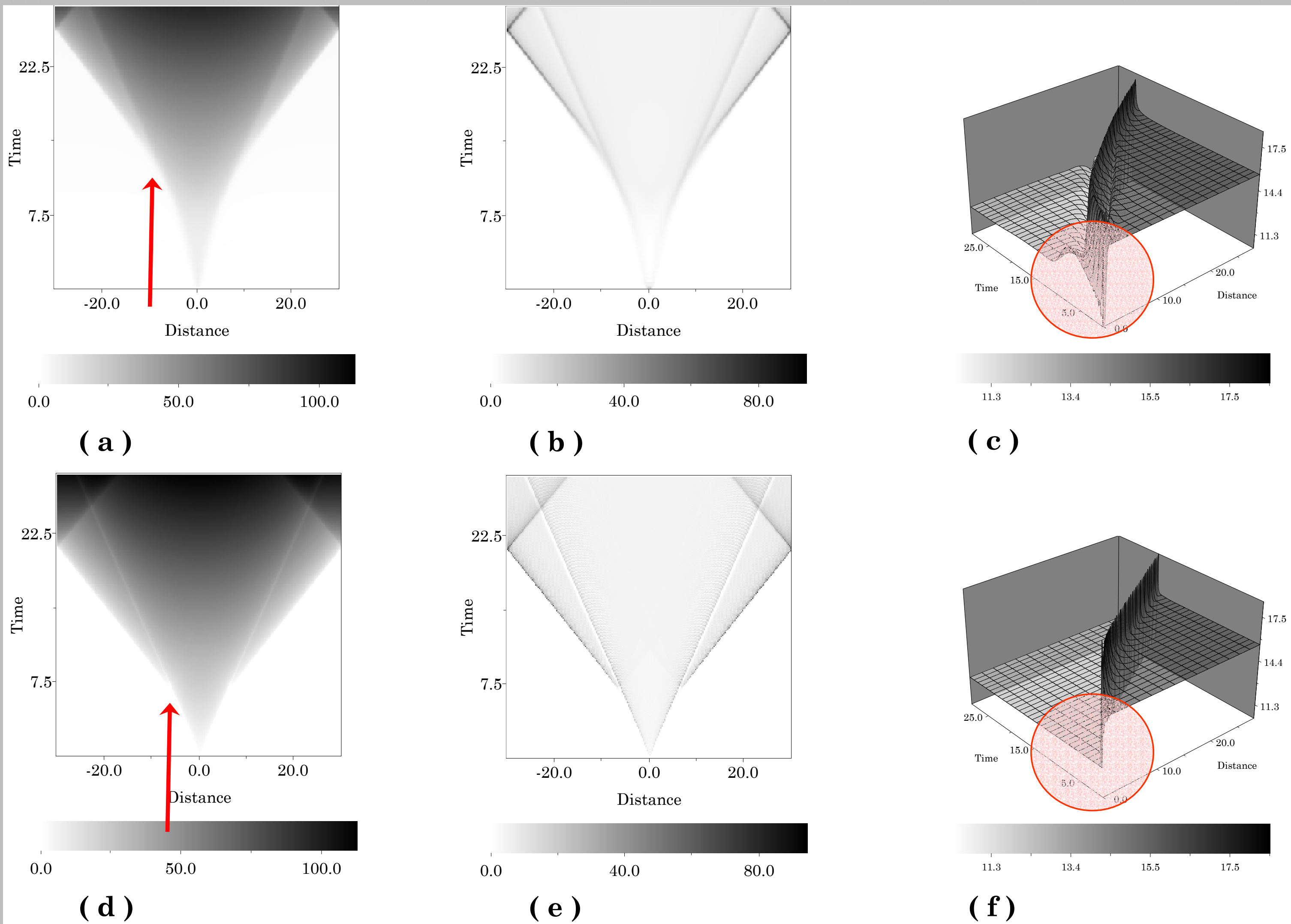


# BIE vs. FD with SW #3



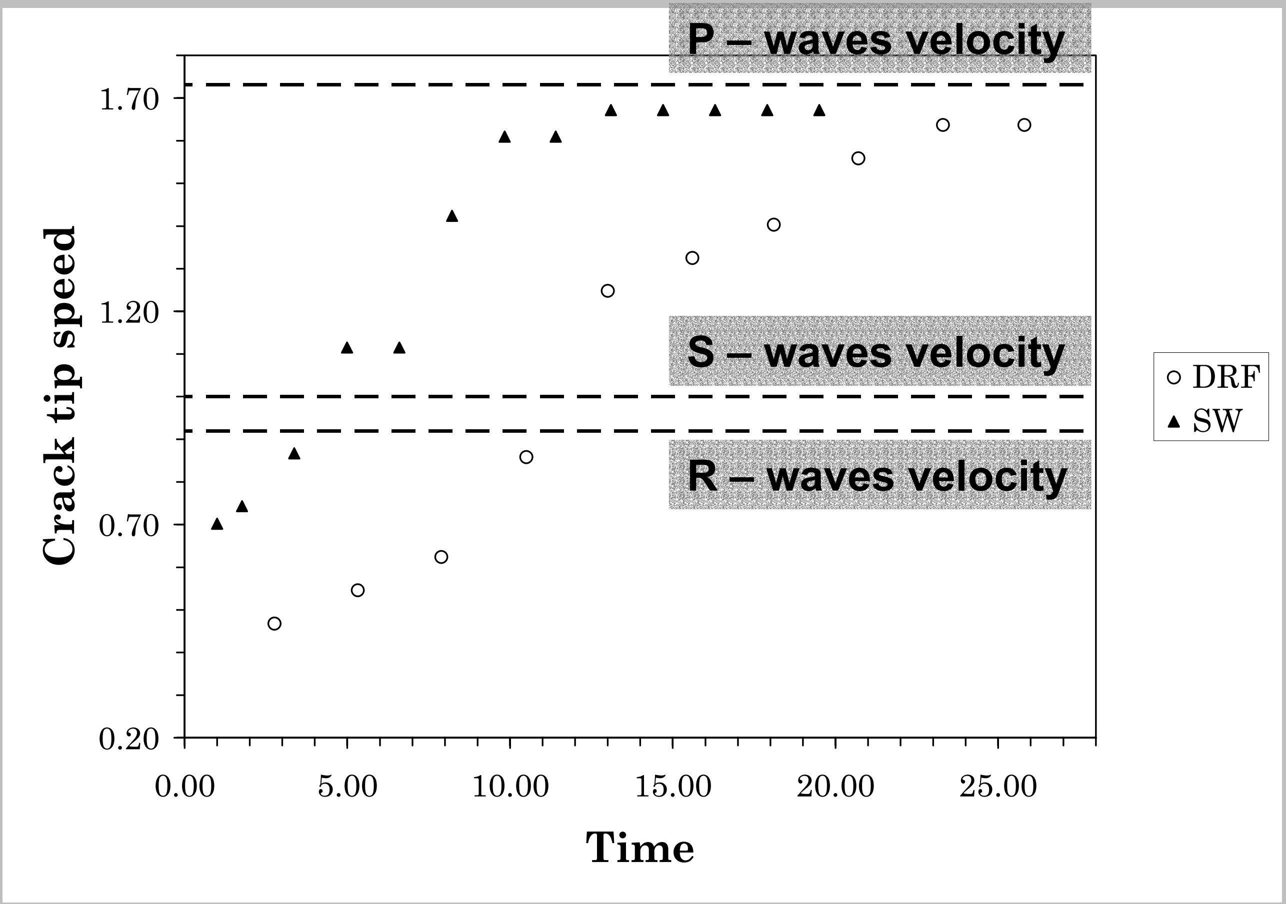
# SW vs. DR law #1




( a ) ( b ) ( c )

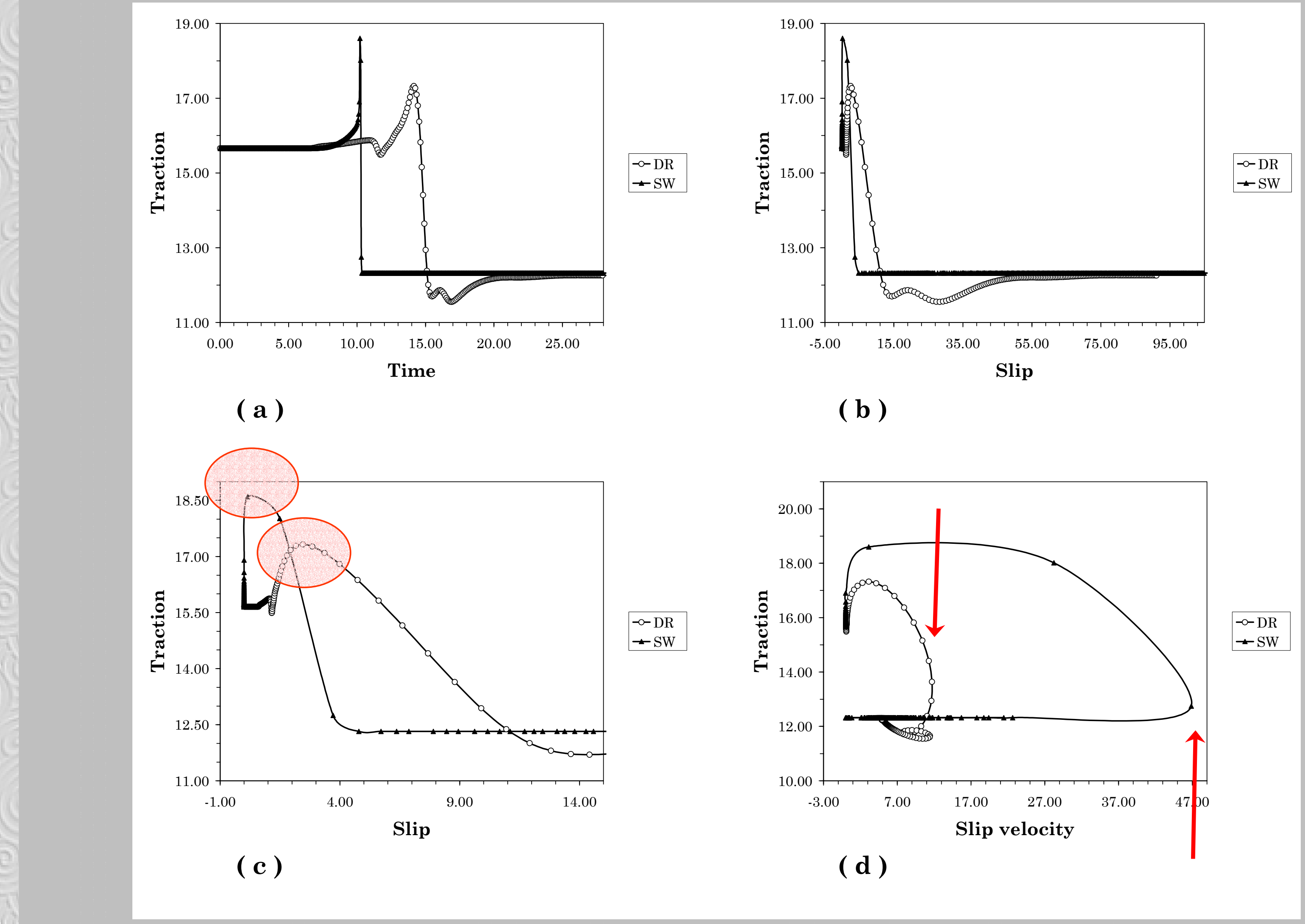
( d ) ( e ) ( f )

# SW vs. DR law #2

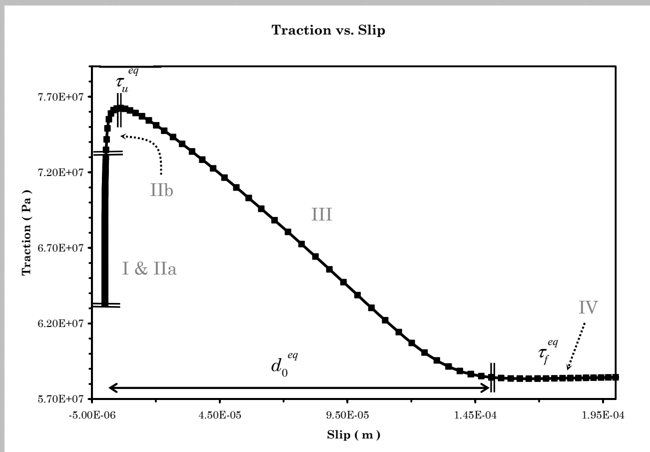


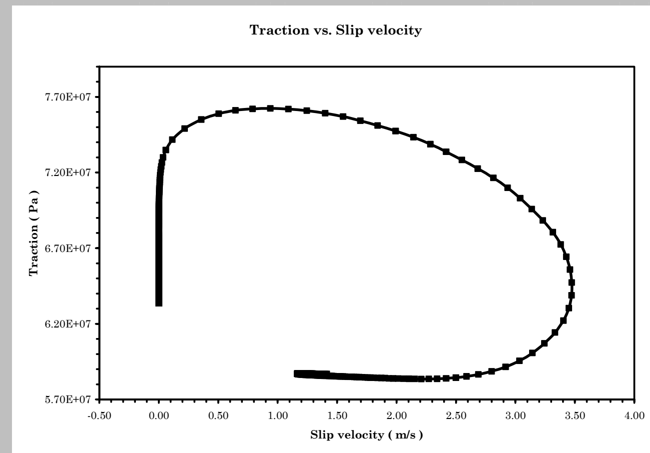
P – waves velocity
S – waves velocity
R – waves velocity
Crack tip speed
1.70
1.20
0.70
0.20
0.00
5.00
10.00
15.00
20.00
25.00
Time
DRF
SW

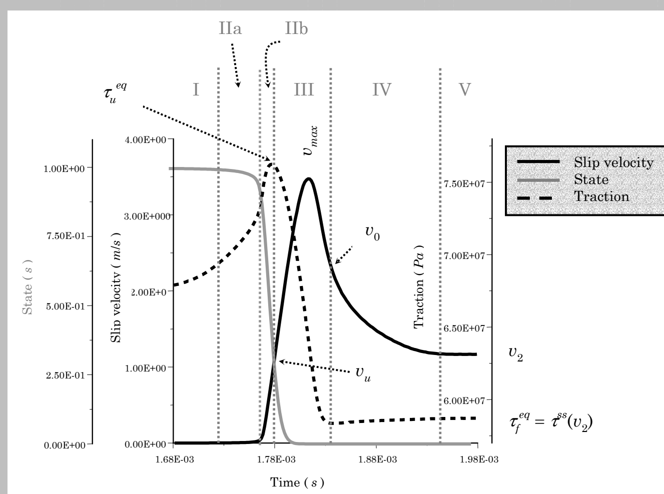
# SW vs. DR law #3


2-D
Traction
Time
Slip
Slip velocity
DR
SW
( a )
( b )
( c )
( d )
19.00
17.00
15.00
13.00
11.00
0.00
5.00
10.00
15.00
20.00
25.00
-5.00
35.00
55.00
75.00
95.00
18.50
15.50
14.00
12.50
-1.00
4.00
9.00
14.00
20.00
18.00
16.00
12.00
10.00
-3.00
7.00
17.00
27.00
37.00
47.00

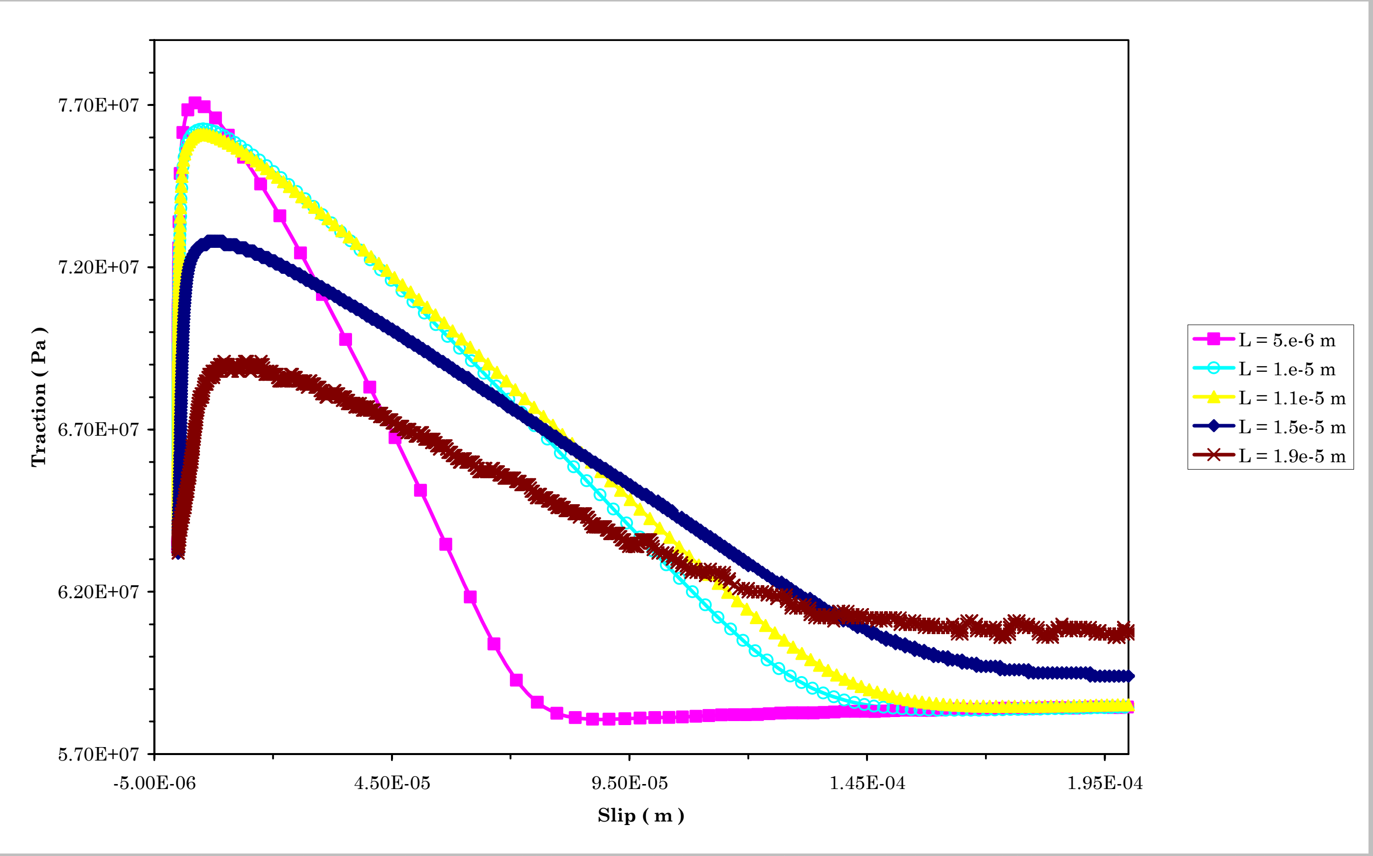
# The dynamic propagation. The cohesive zone and the breakdown



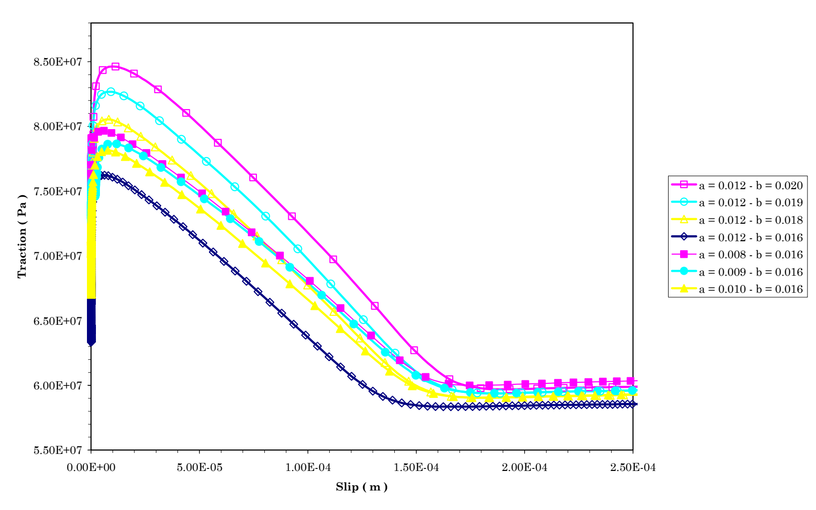



# Dependence on *L* parameter

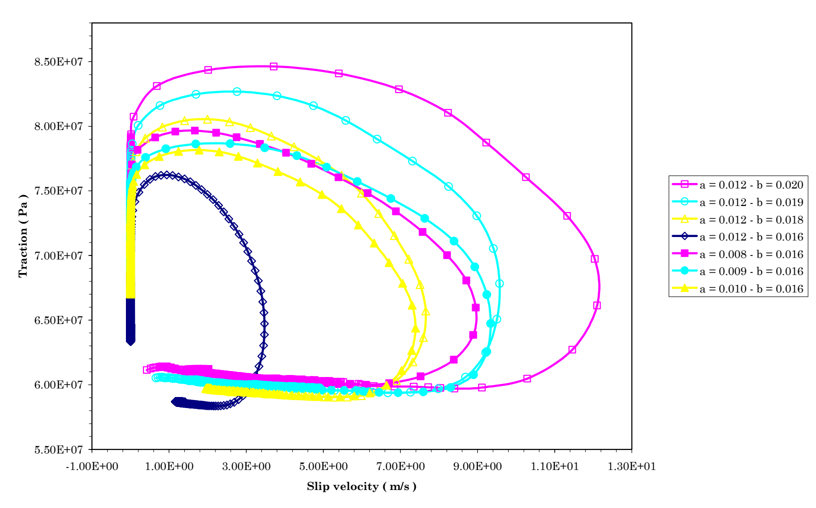


# Dependence on *a* and *b*

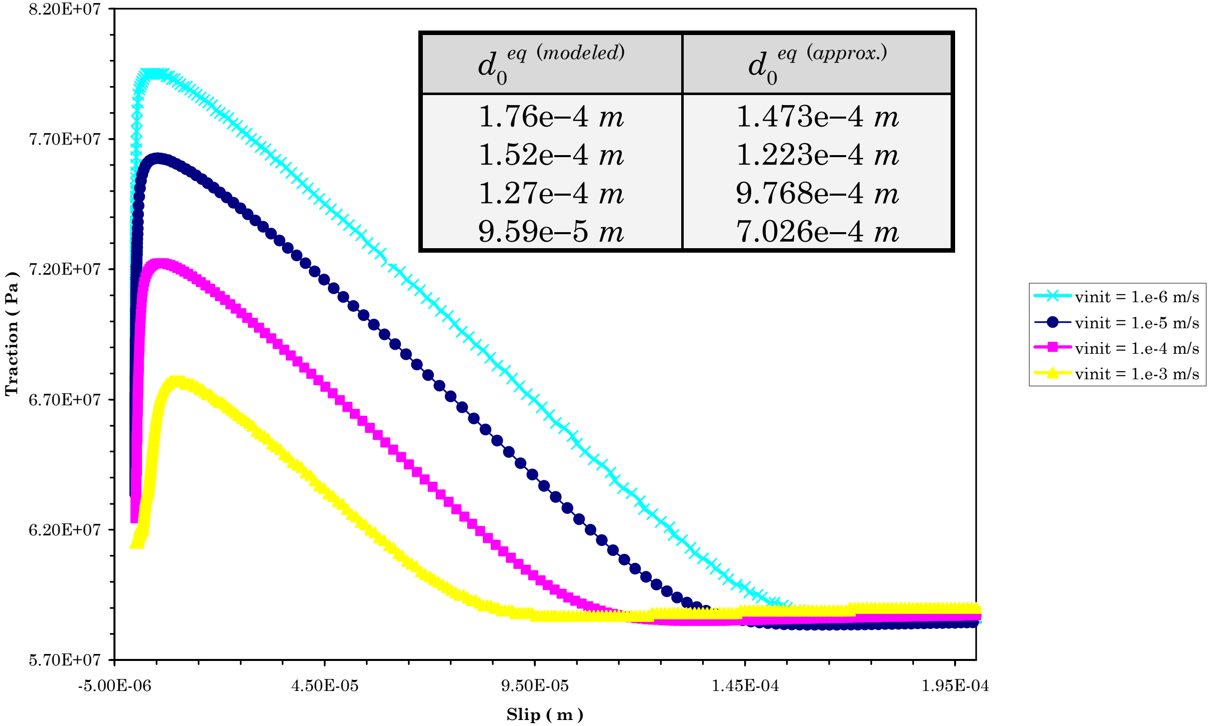
## Slip – weakening curves




## Phase portraits



# Dependence on the initial velocity




| $d_0^{eq}$ (modeled) | $d_0^{eq}$ (approx.) |
|---|---|
| 1.76e–4 $m$ | 1.473e–4 $m$ |
| 1.52e–4 $m$ | 1.223e–4 $m$ |
| 1.27e–4 $m$ | 9.768e–4 $m$ |
| 9.59e–5 $m$ | 7.026e–4 $m$ |

# Limitation for modeling dynamic Rupture and seismic vawe genration

- Because the initial slip velocity is totally arbitrary, it is difficult in the framework of R&S formulation to **prescribe** the traction evolution and the **SW** behavior within the cohesive zone.

- We can only infer an approximated value of the equivalent slip – weakening distance from the proposed scaling law. Moreover, the difference between $d_0^{eq}$ and $L$ depends on the adoption of a slowness (ageing) evolution equation.

# Theoretical interpretations

➢ We derived analytical expressions that relate the yield and the kinetic frictional stresses to the constitutive parameters and to slip:

$$\tau = \left[ \mu_* + a \ln\left(\frac{v}{v_*}\right) + b \ln\left(\frac{v_*}{v_{init}}\right) - b\frac{u}{L} \right] \sigma_n^{eff}$$

$$\tau_f^{eq} = \left[ \mu_* + (b-a)\ln\left(\frac{v_*}{v_2}\right) \right] \sigma_n^{eff}$$

$$D_0^{eq} = L\, ln\left(\frac{V_0}{V_i}\right) \approx \frac{\tau_u^{eq} - \tau_f^{eq}}{b\,\sigma_n} L$$

➢ These relations hold under the assumptions that and that slip velocity is large enough to neglect the term 1 / *v*. This yields

$$\phi(u) = \frac{L}{v_{init}} e^{-\frac{u}{L}}$$

# Numerical estimates of characteristic length

- Laboratory experiments:

**Laboratory scale** ⟺ **fault dimension ~ 20 *m*   $\Delta x$ ~ 0.01 m   L ~ $10^{-5}$ *m***

**$L \sim 10^{-5}$ *m*   $d_o^{eq} \sim 10^{-4}$ - $10^{-3}$ *m***

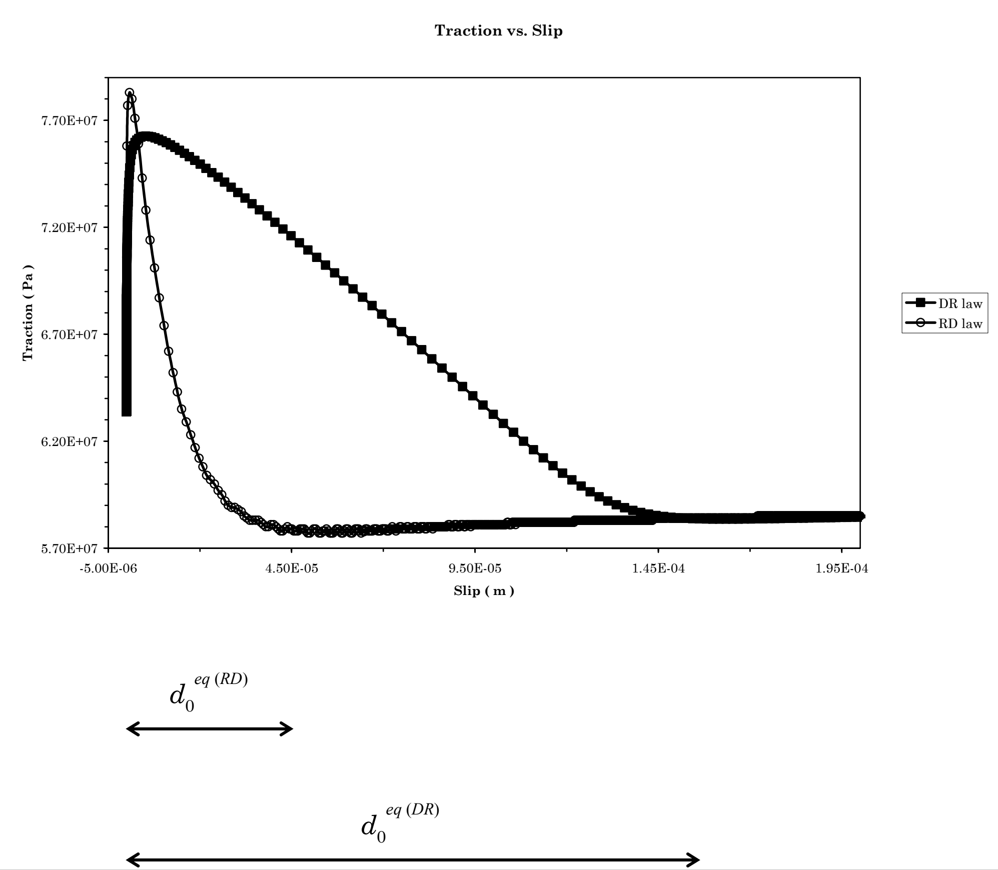
- Extending our calculations to real faults
  - ✓ Estimates of $D_o$ from ground motions or kinematic source models range within $0.5 \leq d_o \leq 1$ *m*.

    [ *Ide and Takeo*, 1997; *Olsen et al.*, 1997; *Guatteri and Spudich*, 2000 ]
  - ✓ There is a trade - off between strength - excess and the slip weakening distance $d_o$ [ $d_o < 0.3$ *m* are not resolved ]
  - ✓ Estimate of $d_o$ inferred from kinematic inversion models are biased due to smoothing constraints used in the inverse - problems formulation [ *Guatteri and Spudich*, 2000 ]

**Fault scale** ⟺ **fault dimension ~ 20 *km*   $\Delta x$ ~ 10 *m*   $L \sim 10^{-2}$ *m***

**$L \sim 10^{-2}$ *m* = 1 *cm*   $d_o^{eq} \sim 10^{-1}$ *m***

# Differences between DR and RD



**This slide is empty intentionally.**

# Support Slides:
# Parameters, Notes, etc.

*To not be displayed directly. Referenced above.*

# Remembering constitutive law...

***LINEAR SLIP – WEAKEING LAW***

$$\tau = \begin{cases} \left[ \mu_u - (\mu_u - \mu_f)\dfrac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

# Remembering constitutive law...

## *DIETERICH IN REDUCED FORMULATION*

$$\begin{cases} \tau = \left[ \mu_* - a \ln\left( \dfrac{v_*}{v} + 1 \right) + b \ln\left( \dfrac{\Psi\, v_*}{L} + 1 \right) \right] \sigma_n^{\,eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi = 1 - \dfrac{\Psi\, v}{L} \end{cases}$$

# Remembering constitutive law...

*RUINA – DIETERICH*

$$\begin{cases} \tau = \left[ \mu_* - a \ln\left( \dfrac{v_*}{v} \right) + b \ln\left( \dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{\Psi\, v}{L} \ln\left( \dfrac{\Psi\, v}{L} \right) \end{cases}$$