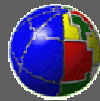


*Università degli Studi di Bologna
Dottorato di Ricerca in Geofisica – XXII Ciclo*

MODELLI DINAMICI DI ROTTURA SISMICA

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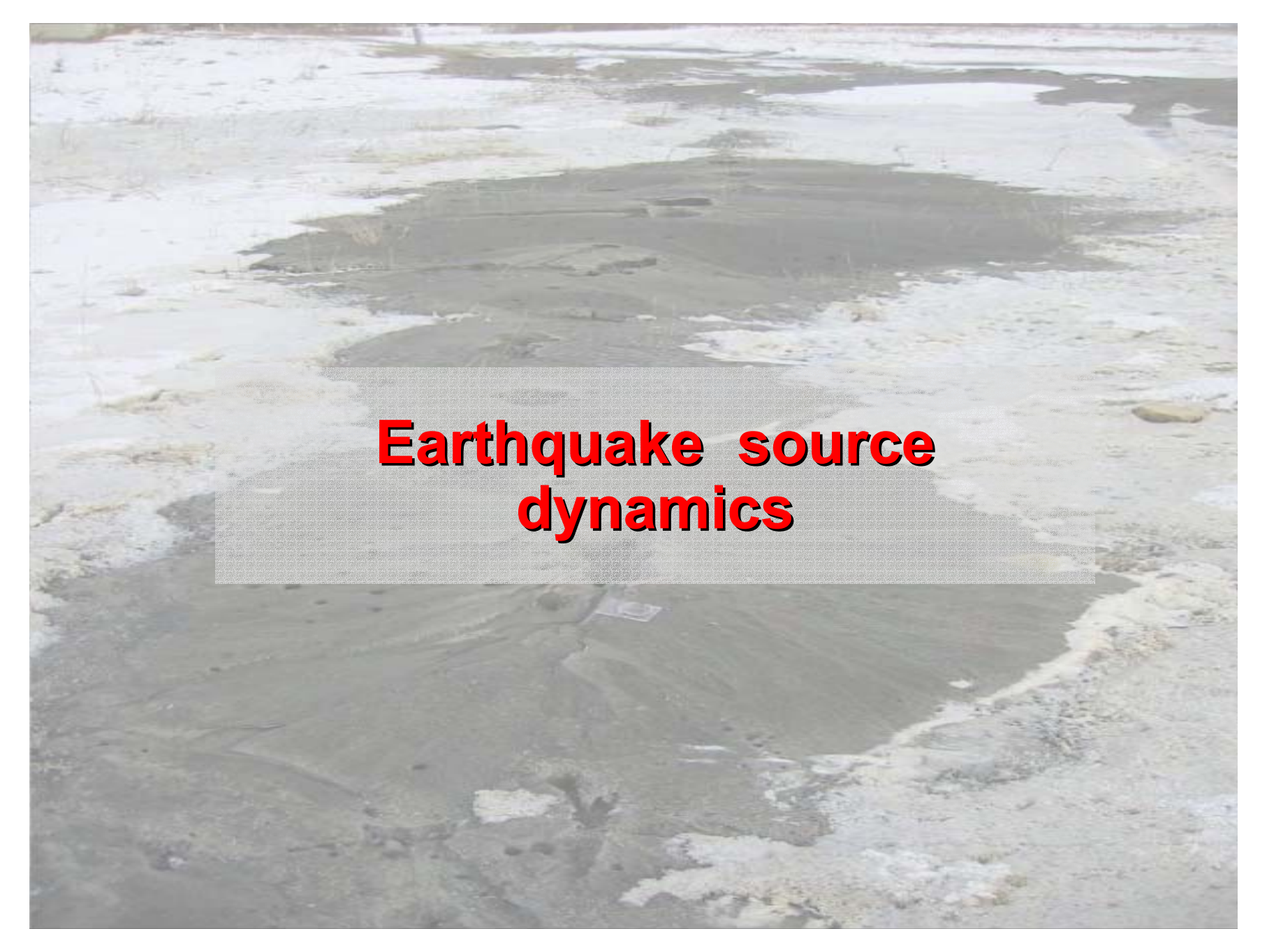
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An aerial photograph of a coastal or estuarine area. The central part of the image is dominated by a large, dark, irregularly shaped area, possibly a lagoon or a large pool of water, surrounded by lighter-colored sand and sparse vegetation. The overall scene suggests a natural or man-made water body in a coastal environment.

Earthquake source dynamics

Elasto - dynamic problem

- * **Solution of the fundamental elasto – dynamic equation (i. e. the II law of dynamic for continuum media):**

$$\rho(d^2/dt^2)U_i = \sigma_{ij,j} + f_i \quad ; i = 1, 2, 3$$

where:

ρ is the mass cubic density,

\mathbf{U} is the particle displacement vector ($\mathbf{U} = \mathbf{x}' - \mathbf{x}$),

$\{\sigma_{ij}\}$ is the stress tensor; $\sigma_{ij} = C_{ijkl} e_{kl}$; $i,j,k,l = 1, 2, 3$, where C_{ijkl} is the elastic constant tensor, accounting for the rheology of the medium and e_{kl} is the strain tensor ($e_{kl} = \frac{1}{2} (U_{k,l} + U_{l,k})$),

\mathbf{f} is the body force vector.

* **Choice of the dimensionality d of the problem**
($1 - D, 2 - D, 3 - D$).
($d = \text{rank of the } \mathbf{U} \text{ array, i. e. number of equations}$)

1. Wave propagation problem: Hyperbolic PDE
 D' Alembert wave equation:

$$\nabla^2 \mathbf{U} - (1/c_0) (\partial^2 / \partial t^2) \mathbf{U} = 0$$

where c_0 is the wave speed.

2. Rupture problem



Rupture Description

Following *Scholz (1990)* the rupture can be described by using:

- * ***CRACK MODELS:***

The energy dissipation at crack edge (or crack tip) is paramount. Describe explicitly the crack propagation.

- * ***FRICTION MODELS:***

The effects at the edges are not explicitly considered. Explicitly allow for the calculation of the evolution of stress tensor components in terms of material properties of the fault.

Dislocation vs. Crack Models

DISLOCATION MODELS

- * Study of **displacement discontinuity**
- * **Slip** is assumed to be constant on the fault;
The fault evolution is represented by unilateral or bilateral motion (rectangular dislocations: Haskell' s model)
- * **Kinematic description**: it accounts for time evolution of rupture front and it neglects dynamics of faulting

↑ **Long period** seismic waves modeling ($\lambda \geq L_{fault}$)

↓ **constant dislocation is inadmissible**;
strain **energy** at crack tip is **unbounded**;
stress drop is infinite

CRACK MODELS

- * Impose **finite energy flow** into the rupture
 - * **Slip is not prescribed**,
but it is calculated from the stress drop and from the fault strength S^{fault}
 - * **Dynamic description**: the shear stress drops inside the crack (after nucleation processes), increases the stress outside the crack (near the crack tip) and tends to facilitate further grow of the rupture
- ↑ The motion is determined by fracture criterion (and eventually by the assumed constitutive law on the fault)
- ↑ The problem is characterized by assuming the boundary conditions on the fault plane. It has mixed b. c.: slip assigned outside the crack tip and stress tensor components inside the crack tip

Forward modeling scheme

1. *Fault model:*

- **Fault geometry** (orientation, planar or non – planar, ...)
- **Fault system** (multiple segments, multiple faults, ...)



2. *Medium surrounding the fault surface(s)*

- **Properties of the medium** surrounding the fault(s): cubic mass density structure, velocity structure, anisotropy, attenuation



3. *Choice of the dimensionality d' of the problem (1 – D, 2 – D, 3 – D, 4 – D).*

(d' = number of the independent variables in the solutions)



4. *Choice of the representation*



5. *Choice of the numerical method*

- (FE, FD, BE, BIE, SE, hybrid)

6. *Specification of the Boundary Conditions*

- **Domain** Boundaries Conditions (DBCs)
- **Fault** Boundary Condition (FBCs)
- **Auxiliary** Conditions (ACs)



7. *Specification of the Initial Conditions*

- Initial conditions **on the fault**: (initial slip, slip velocity, state variable, pre – stress);
- Initial conditions **outside the fault**: (tectonic load, (state of neighbouring faults: the fault is not an isolated system))

8. *Evaluation of the solutions*

- Convergence analysis (**consistency + stability**)

Rupture stages

1. Nucleation (quasi – static to dynamic evolution)

- *How can we simulate nucleation?*
- *How can we promote fault instability?*

2. Propagation

- *What is the fault constitutive equation (governing law)?*

3. Healing

- *What type of healing occurs?*
- *What controls fault healing?*

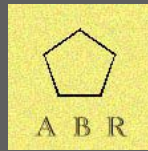
4. Rupture arrest

- *What is responsible of rupture arrest?*
- *How can we represent it? Earthquake energy balance?*

5. Fault re – strengthening

- *How can we model further instabilities episodes on the fault?*

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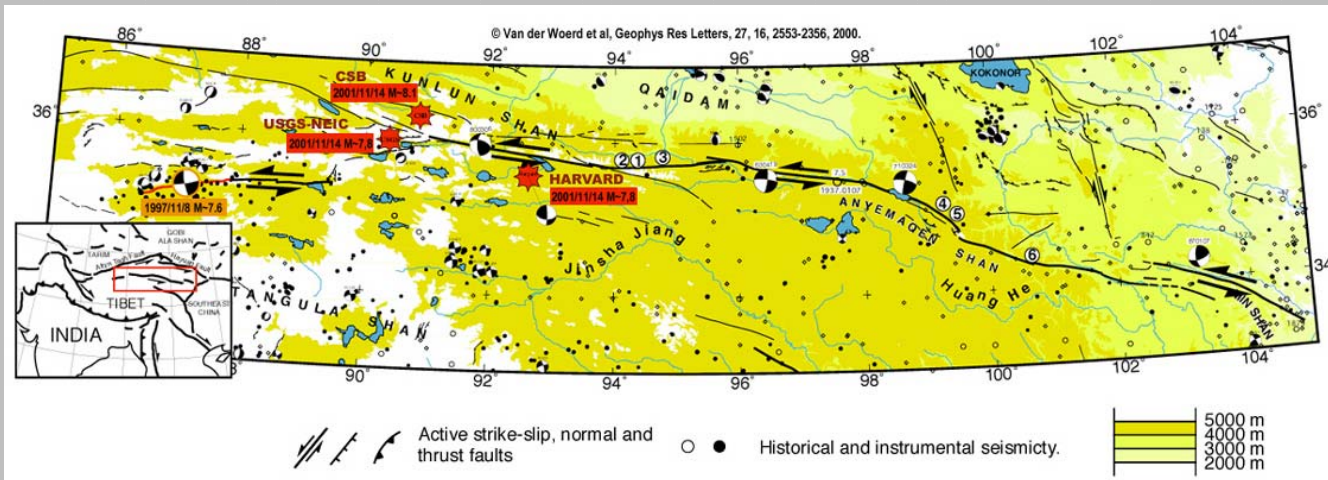
Support Slides: Parameters, Notes, etc.

To not be displayed directly. Referenced above.

Geometrical complexity



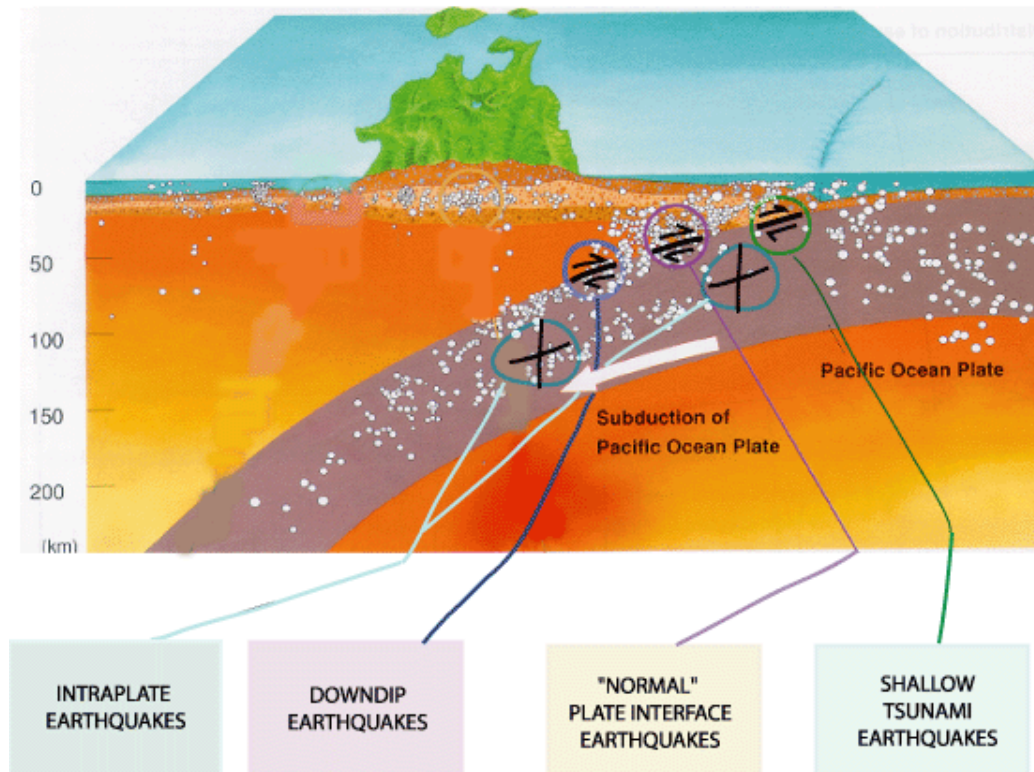
Kokoxili
 M_w 7.9
earthquake
(Qinghai
Province,
China)



Different types of earthquakes

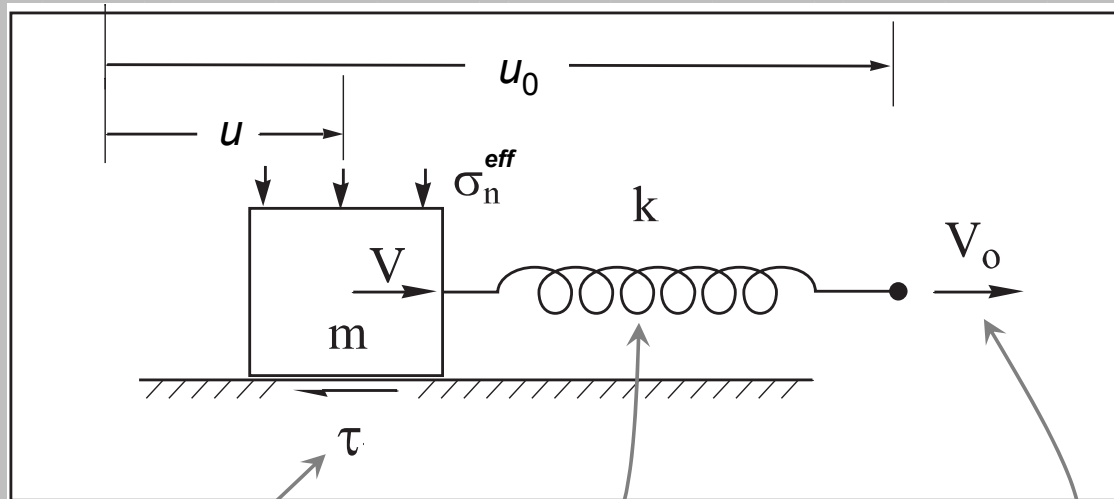


1. Interplate
2. Tsunami
3. Crustal
4. Downtip
5. Intraplate
6. Deep



Dimensionality d'

1 – D Spring – Slider (mass – spring) model



Frictional sliding

(\leftrightarrow rheological properties)

Elastic behaviour

(\leftrightarrow surrounding medium)

Loading velocity

(\leftrightarrow tectonic load)

Let us recall basic concepts on dislocation theory.

If $U_{i,j}$ is continuous in the integration domain $\int_{P_1}^{P_2} U_{i,j} dx_j$ then it does not depend on the integration path and therefore:

$$\oint_C dU_i = 0, \quad \forall C.$$

On the contrary, when $\oint_C dU_i = b_i$ the considered body contains a **dislocation** and the circuit C contains at least one curve (the **dislocation curve**) on which the tensor $U_{i,j}$ is not defined.

The vector $U_i(\mathbf{x})$ can be reduced to a one – valued function if we produce in the body and starting from the dislocation curve a cut (the **dislocation surface**), through which we assume an explicit discontinuity of \mathbf{U} . Being ζ the coordinate normal to the dislocation surface we can write:

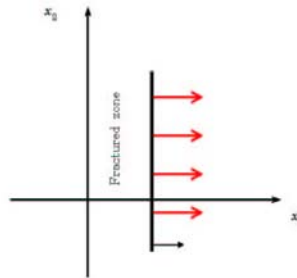
$$\Delta U_i \equiv u_i = \lim_{\zeta \rightarrow 0^+} U_i(\mathbf{x}) - \lim_{\zeta \rightarrow 0^-} U_i(\mathbf{x}) = b_i$$

where $\mathbf{b} = (b_1, b_2, b_3)$ is called Burgers' s vector.

In this framework the dislocation is described from a microscopic / crystallographic point of view.

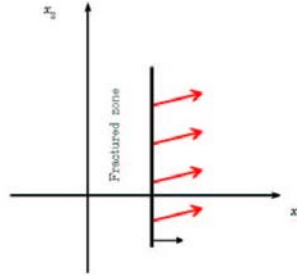
Shear rupture on a planar fault surface ($x_2 = 0$)
Snapshots at fixed time t

PURE MODE II



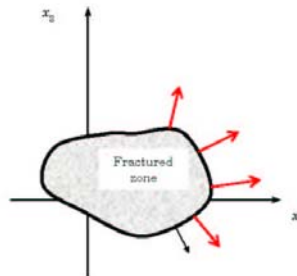
Dependence on x_1
Independence on x_2
 $\Rightarrow u_1(x_1, t)$

MIXED MODE



Dependence on x_1
Independence on x_2
 $\Rightarrow u_1(x_1, t)$
 $u_2(x_1, t)$

TRULY 3-D



Dependence on x_1
 Dependence on x_2
 $\Rightarrow u_1(x_1, x_2, t)$
 $u_2(x_1, x_2, t)$

- Crack tip
- Local crack enlargement direction
- Local displacement

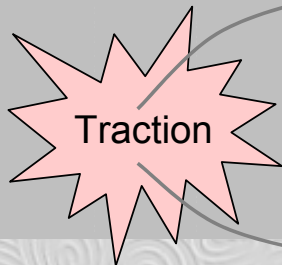
Representation

1. INTEGRAL REPRESENTATION

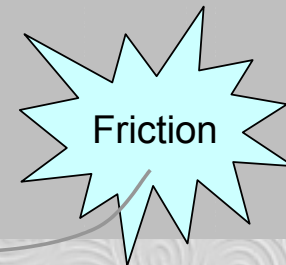
Source integral representation (*Betti*' s theorem, Integration in time (*Green – Volterra*' s relation), limit in fault surface, Lamb' s problem):

$$u_n(\mathbf{x}, t) = \int_{-\infty}^{+\infty} dt' \int_{S(t')} d\xi G_{n\alpha}(\mathbf{x} - \xi, t - t') \sigma_{\alpha\beta}^P(\xi, t'); n=1,2,3; \alpha=1,2; \mathbf{x}, \xi \in \mathbb{R}^3$$

First neighbours decoupling (in the case of a 2 – D, pure in – plane rupture):



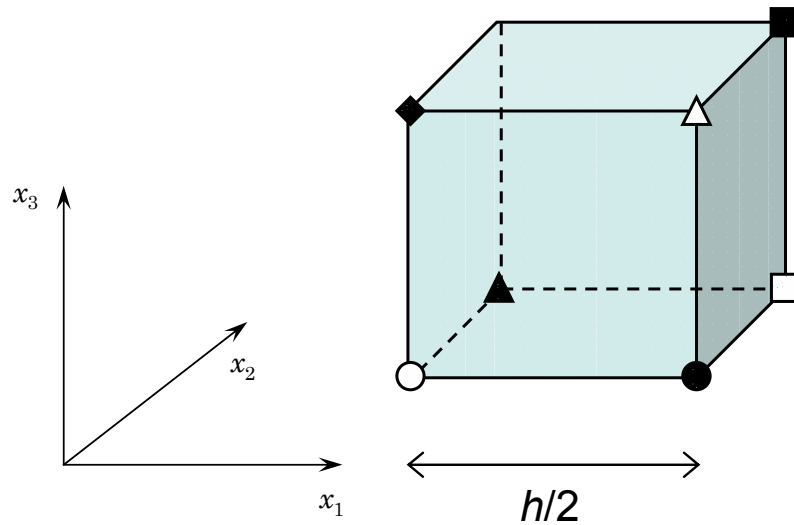
$$\begin{cases} u_1(x_1, t) + C \tau_1^P(x_1, t) = \mathcal{L}_1(x_1, t) \\ \tau_0 + \tau_1^P(x_1, t) = \mu \sigma_n^{eff} \end{cases}$$



2. DISCRETIZATION OF EQUATIONS (FE, FD APPROACHES)

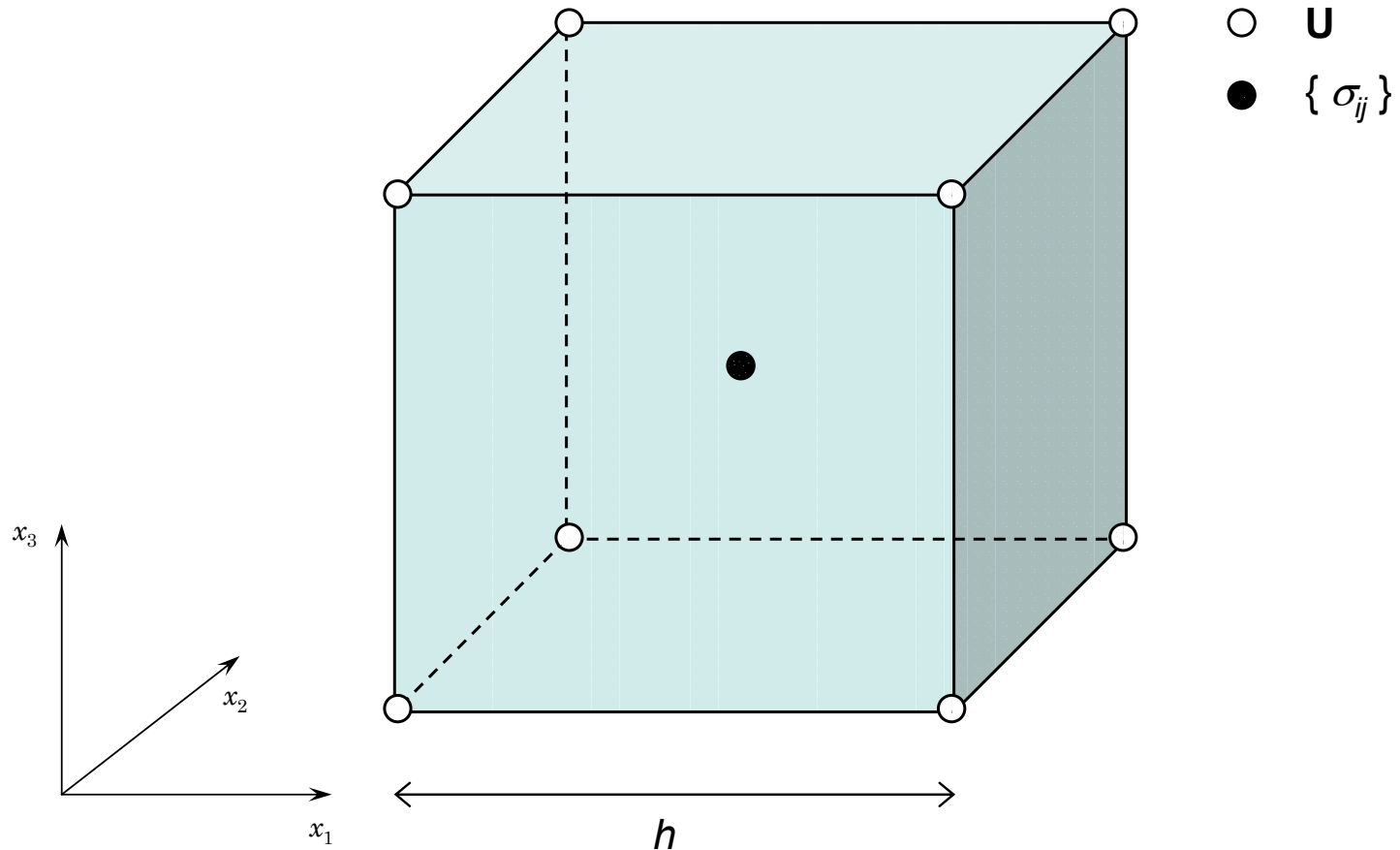
- Choice of the grid type

- **Staggered – grid (SG):**

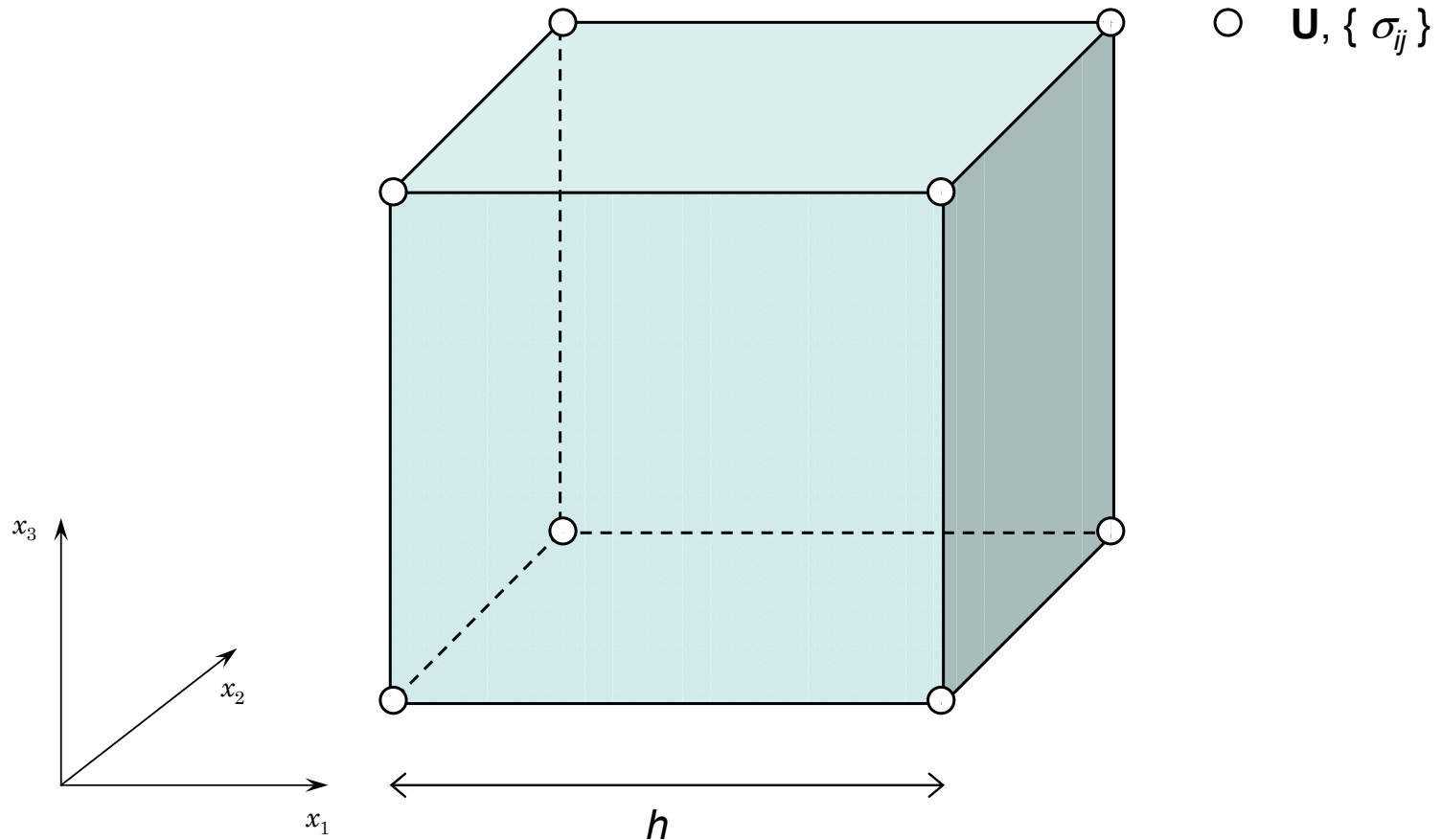


- U_1
- U_2
- △ U_3
- $\sigma_{11}, \sigma_{22}, \sigma_{33}$
- ▲ σ_{12}
- ◆ σ_{31}
- σ_{32}

- Partly Staggered – grid (PSG):



- **Conventional – grid (CG):**



Domain Boundaries Conditions

- * ***BOUNDARY:***
 - Bottom
 - Fixed
 - Absorbing
 - Top
 - Free surface
 - Topography
 - Coasts
 - Lateral
 - Cyclic
 - Absorbing

- Let us consider a boundary perpendicular to the i – axis. Indices i, j and k identify node location along x_1, x_2 and x_3 axes, respectively. Apex m indicates the actual time level, while index l stands for vector component ($l = 1, 2, 3$).

- Fixed Boundary (FB):**

$$U_{1jk_l}^m = 0, \quad \dot{U}_{1jk_l}^m = 0$$

$$U_{i_{end}jk_l}^m = 0, \quad \dot{U}_{i_{end}jk_l}^m = 0$$

(Conditions $\dot{U}_{1jk_l}^m = 0$ and $\dot{U}_{i_{end}jk_l}^m = 0$ represent a Dirichlet boundary condition).

- **Absorbing Boundary (AB):**

Left boundary:

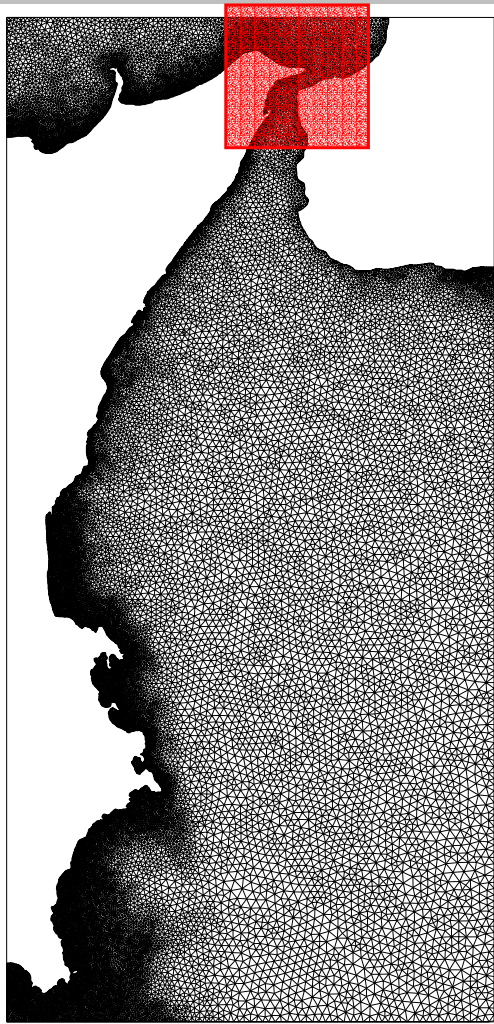
$$\begin{aligned}
 \dot{U}_{1jk_l}^m &= A_{01}\dot{U}_{2jk_l}^m + A_{02}\dot{U}_{3jk_l}^m \\
 &+ A_{10}\dot{U}_{1jk_l}^{m-1} + A_{11}\dot{U}_{2jk_l}^{m-1} + A_{12}\dot{U}_{3jk_l}^{m-1} \\
 &+ A_{20}\dot{U}_{1jk_l}^{m-2} + A_{21}\dot{U}_{2jk_l}^{m-2} + A_{22}\dot{U}_{3jk_l}^{m-2}
 \end{aligned}$$

Right boundary:

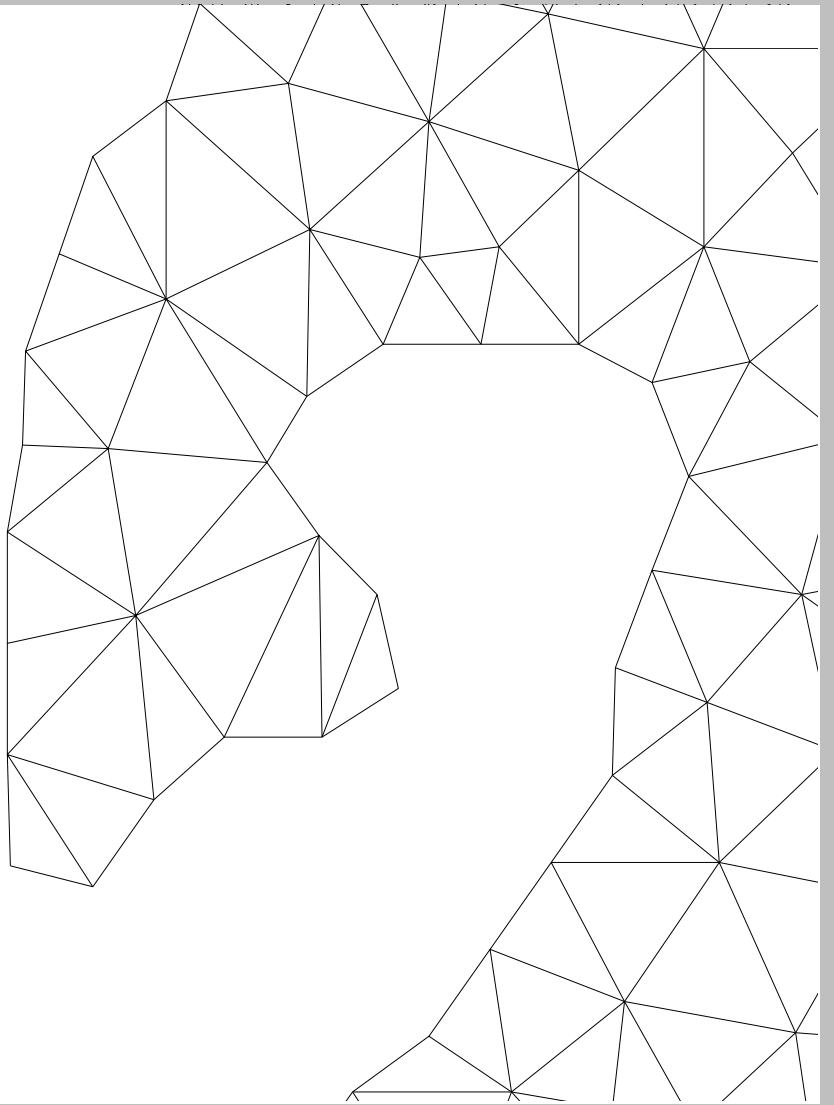
$$\begin{aligned}
 \dot{U}_{i_{end}jk_l}^m &= A_{01}\dot{U}_{i_{end}-1jk_l}^m + A_{02}\dot{U}_{i_{end}-2jk_l}^m \\
 &+ A_{10}\dot{U}_{i_{end}jk_l}^{m-1} + A_{11}\dot{U}_{i_{end}-1jk_l}^{m-1} + A_{12}\dot{U}_{i_{end}-2jk_l}^{m-1} \\
 &+ A_{20}\dot{U}_{i_{end}jk_l}^{m-2} + A_{21}\dot{U}_{i_{end}-1jk_l}^{m-2} + A_{22}\dot{U}_{i_{end}-2jk_l}^{m-2}
 \end{aligned}$$

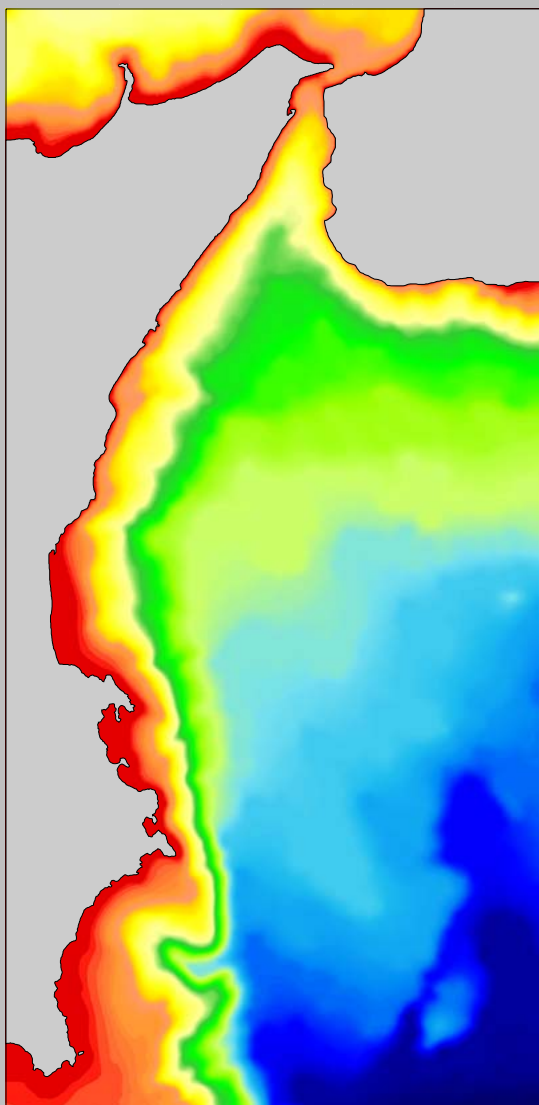
In the previous compact representation of ABCs (that follows *Moczo, 1998*):

- the coefficients $\{A_{pq}\}_{p,q=1,2,3}$ depend on the choice of ABC scheme (e. g. *Clayton and Engquist, 1977; Reynolds, 1978; Emerman and Stephen, 1983; Higdon, 1991; Peng and Toksöz, 1994, 1995; Liu and Archuleta, 2000, ...*);
- displacement components at actual time level m are derived by numerical integration from particle velocity components, after update;
- values in edges and in corners are derived from algebraic averaging of values of quantities belonging to walls;
- as it is a special boundary condition, there is no need to consider any rheology in the updated point.

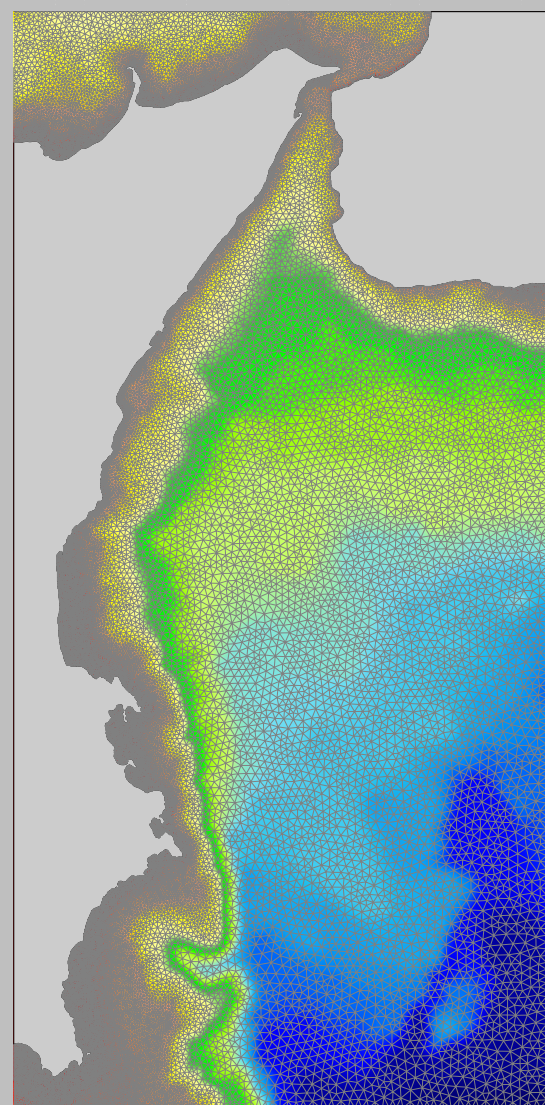
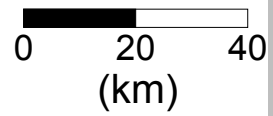
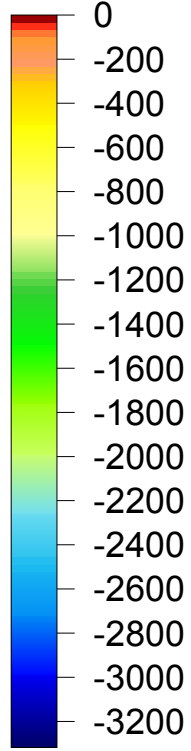


0 20 40 km





(m)





Number of nodes	30264
Number of elements	57733
Type of building block	Triangle
Minimum node distance	200 m
Maximum node distance	2000 m
References	Armigliato A., Tinti S., (2005), <i>EGU General Assebly</i> ; Tinti S., Armigliato A., Bortolucci E. (2001), <i>J. Seismol.</i> , 5 , 41-61.

Fault Boundary Conditions



1. TYPE

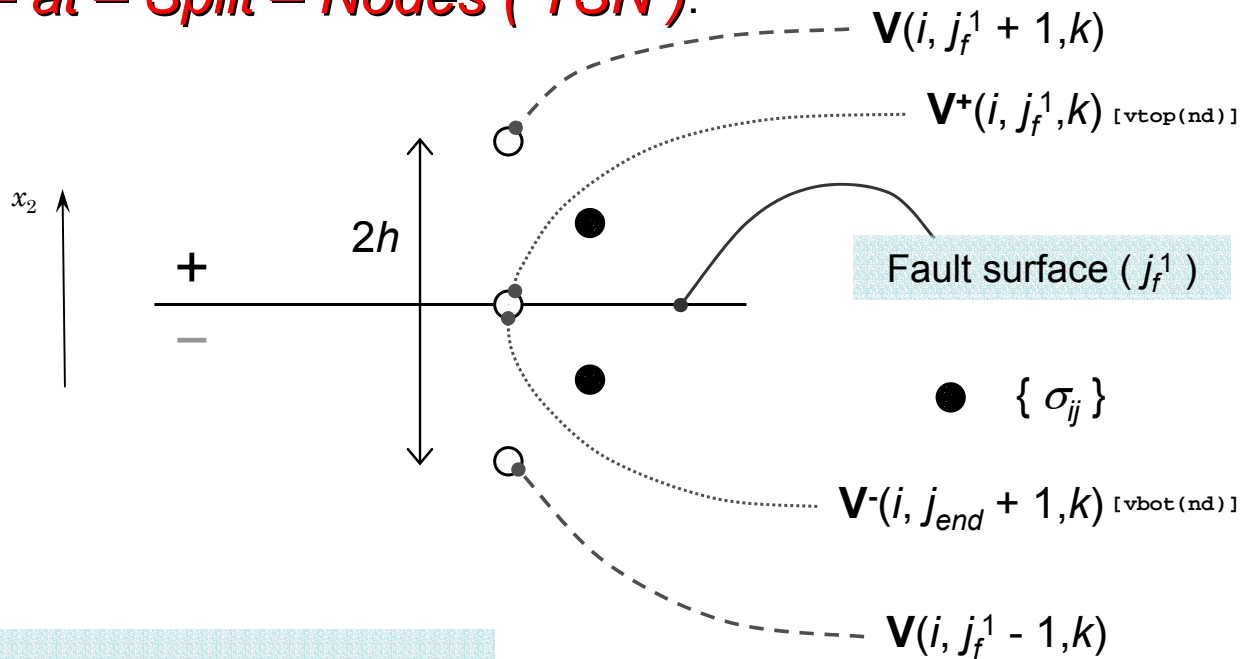
- Traction – at – Split – Nodes (**TSN**): in 2 – D by *Andrews (1973)*; in 3 – D by *Day (1977)*, *Archuleta and Day (1980)*, *Day (1982a, 1982b)*, *Andrews (1999)*, *Bizzarri (2003)*, *Bizzarri and Cocco (2005)*, *Day et al. (2005)*
- Stress – Glut (**SG**): *Backus and Mulchay (1976)*, *Andrews (1976)*
- Thin – zone (**TnZ**): *Virieux and Madariaga (1982)*
- Thick – zone (**TkZ**): *Madariaga et al. (1998)*



2. CONSTITUTIVE LAW

- Accounts for fault rheology
- and different physical phenomena occurring during the rupture process

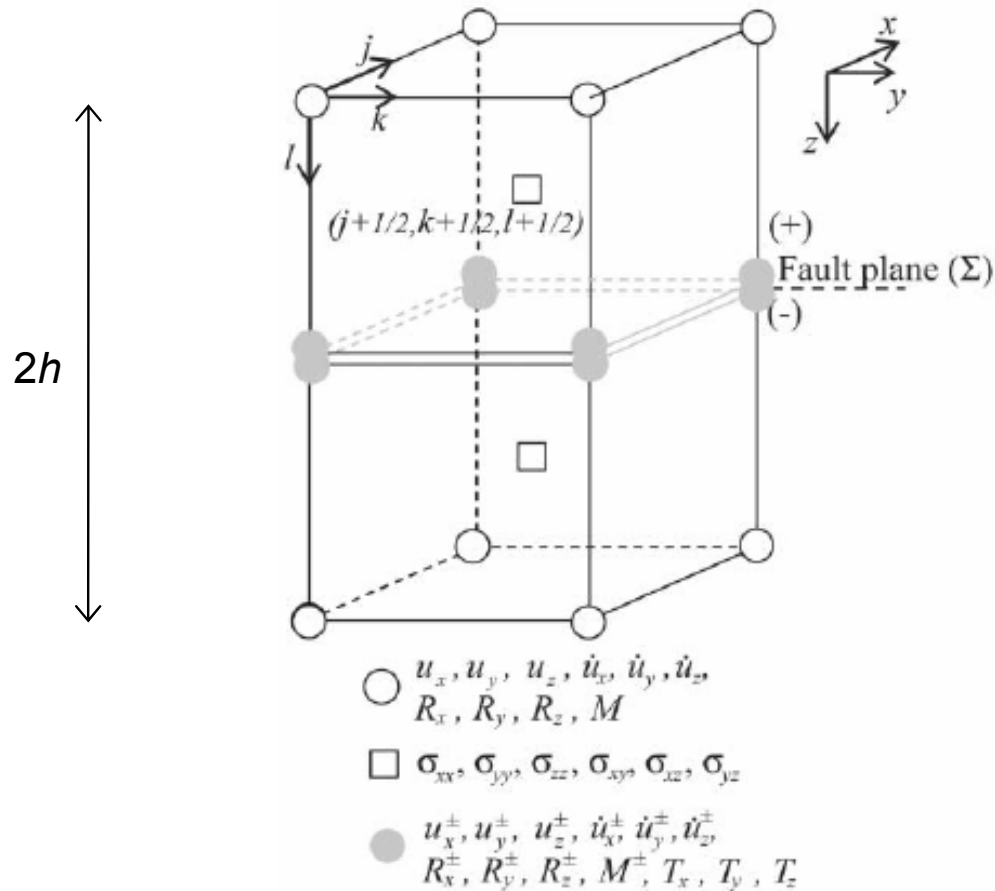
- Traction – at – Split – Nodes (TSN):**



Day (1982b)

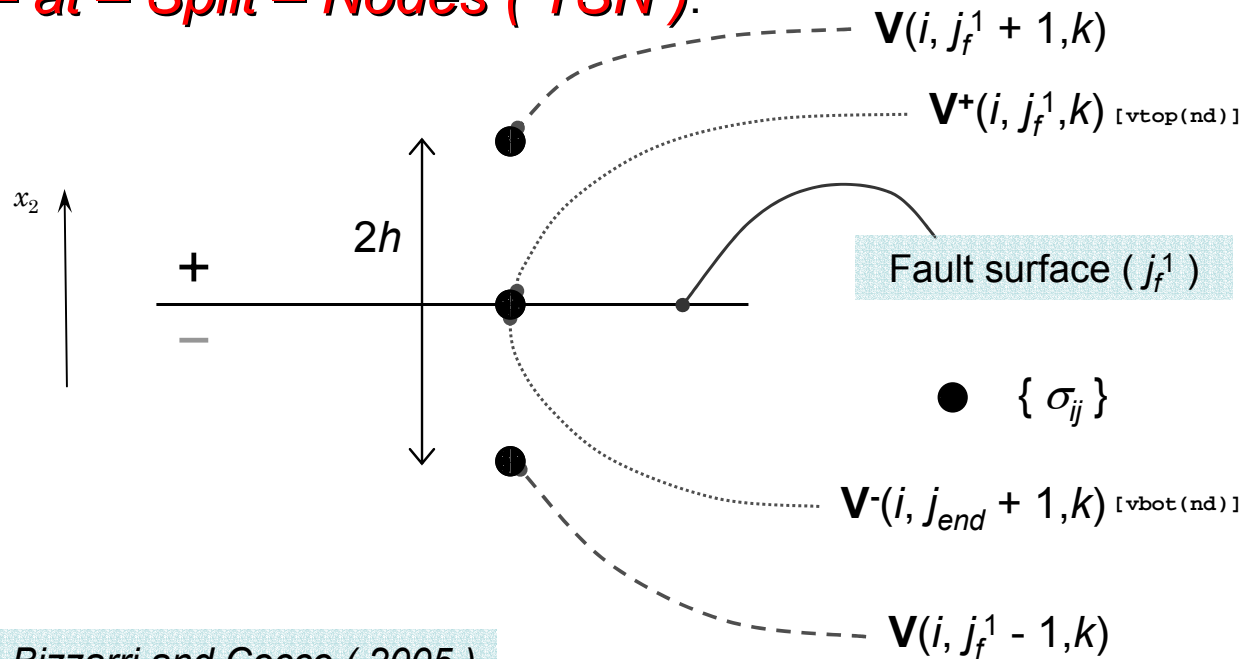
$$\Delta \mathbf{V} = \mathbf{V}^+ - \mathbf{V}^-$$

relative motion of the “ + ” side with respect to the “ - ” side. This value is used to calculate fault slip velocity \mathbf{v} , using the constitutive law



3 – D Partly Staggered – grid (SG) formulation

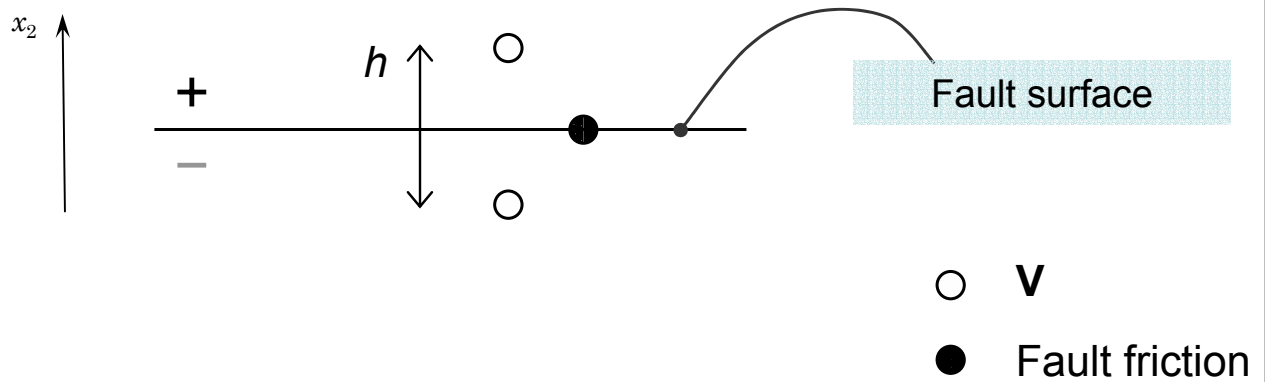
- Traction – at – Split – Nodes (TSN):**



Andrews (1999), Bizzarri and Cocco (2005)

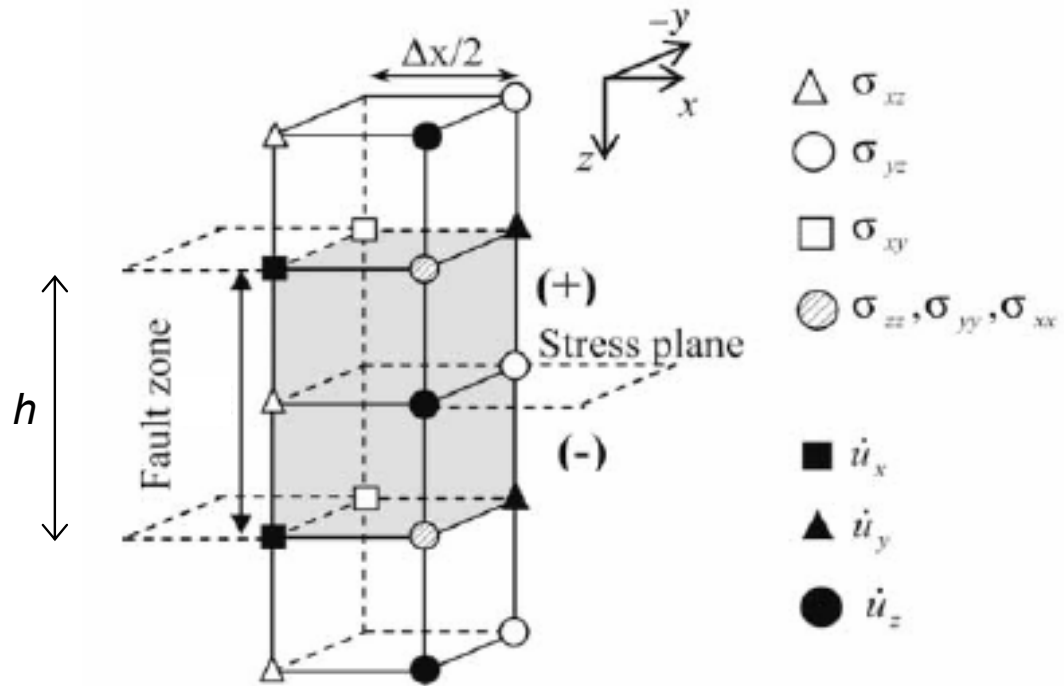
* **Discontinuum medium** (continuum mechanics equations of motion are applied to each half - space individually; the fault is an explicit discontinuity in displacement)

- **Thin – zone (TnZ):**



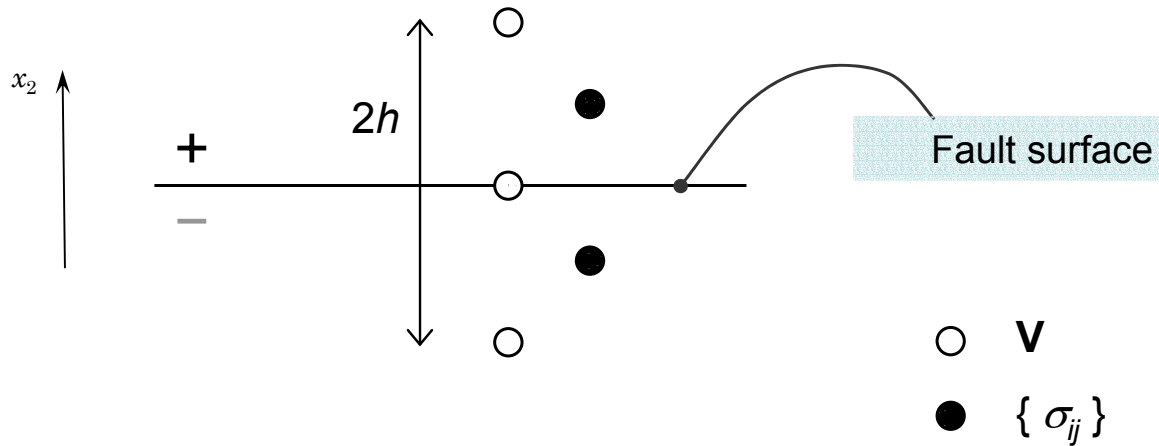
$\mathbf{v} = \mathbf{V}^+ - \mathbf{V}^-$ relative motion of the “+” side with respect to the “-” side.
It is calculated **over h**

* Continuum medium



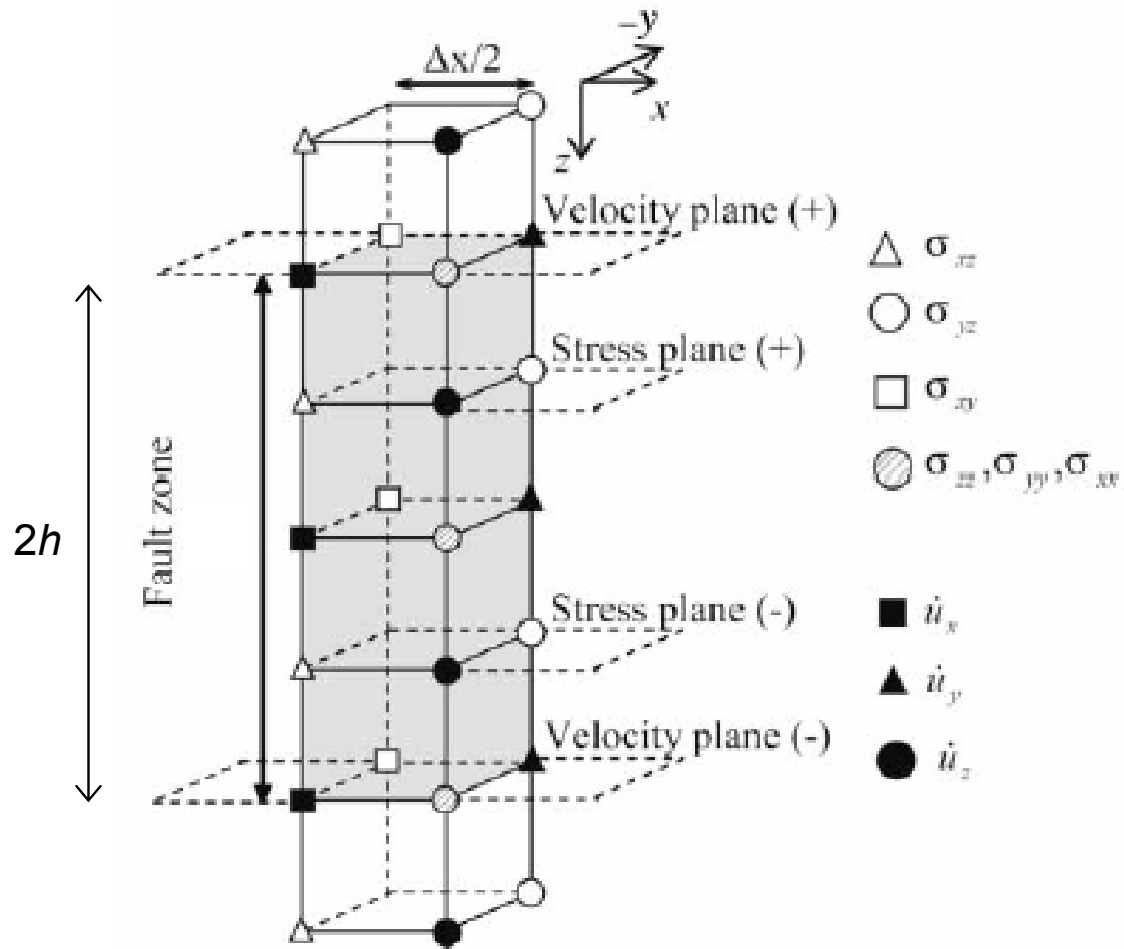
3 - D Velocity - Stress (VS) Staggered - grid (SG) formulation

- **Thick – zone (TkZ):**



$\mathbf{v} = \mathbf{V}^+ - \mathbf{V}^-$ relative motion of the “+” side with respect to the “-” side.
It is calculated **over $2h$**

* Continuum medium



3 - D Velocity - Stress (VS) Staggered - grid (SG) formulation

Auxiliary Conditions



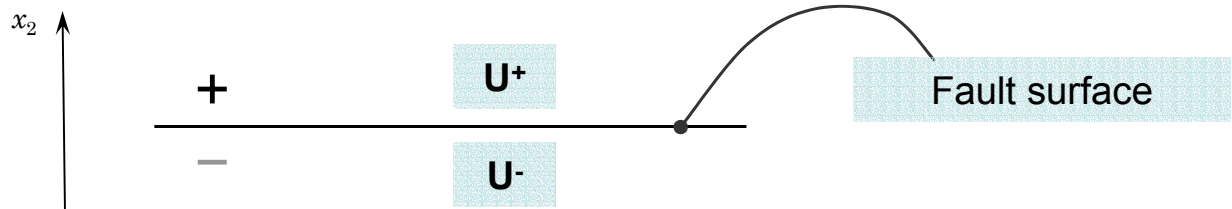
- * ***COLLINEARITY BETWEEN FAULT SHEAR TRACTION AND FAULT SLIP VELOCITY:***

$$\mathbf{T} \parallel \mathbf{v}$$

$$\text{(i. e. } \hat{\mathbf{T}} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \text{)}.$$

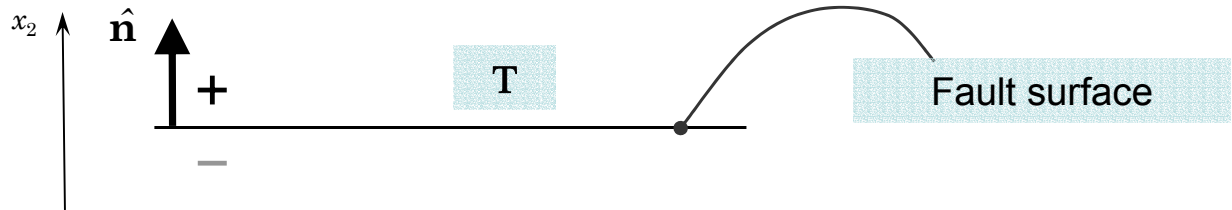
* **COLLINEARITY VS. ANTIPARALLELISM**

1. Definition of the fault slip u (i. e. displacement discontinuity):



$$u = U^+ - U^- \quad \text{relative motion of the " + " side with respect to the " - " side}$$

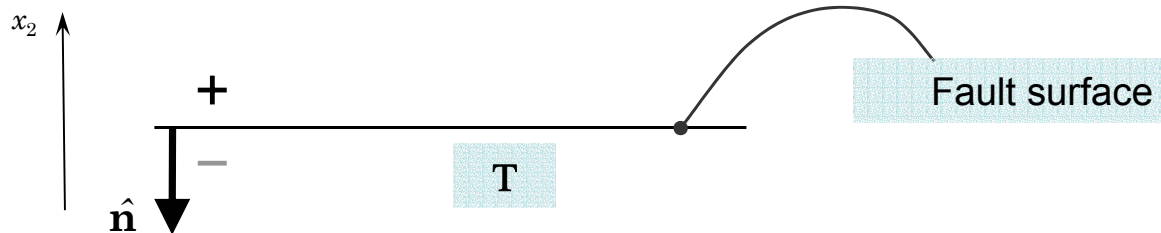
2. Fault surface orientation. Possibility A



T Shear traction that a particle located on the “+” side exercised on particle located in the “-” side

In this case the traction vector T is **collinear** to the direction of motion (namely to the fault slip vector u and therefore to the fault slip velocity vector v).

Possibility B



T Shear traction that a particle located on the “ - ” side exercised on particle located in the “ + ” side

In this case the traction vector T is antiparallel to the direction of motion (namely to the fault slip vector u and therefore to the fault slip velocity vector v).