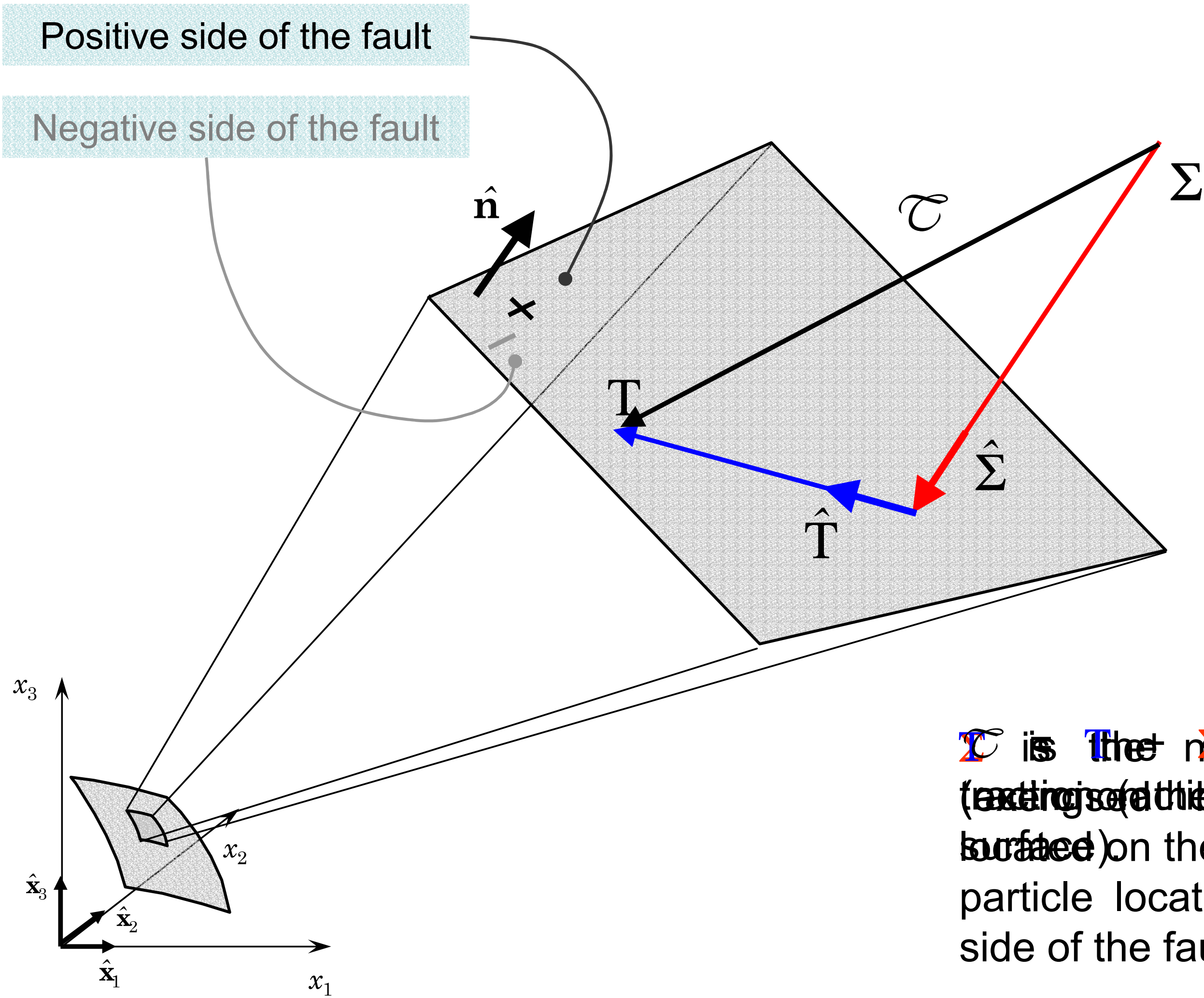

Fault governing laws
(constitutive equations)

Notations and symbols



Σ is the total traction (exerted by a particle located on the +ve side on a particle located on the –ve side of the fault surface)

$\mathcal{T}^{(\hat{\mathbf{n}})} = \mathbf{T}^{(\hat{\mathbf{n}})} + \Sigma^{(\hat{\mathbf{n}})}$ <u>total</u> traction (acting on the fault surface).

$\mathcal{T}_j^{(\hat{\mathbf{n}})} = n_i \sigma_{ij}^{eff}$ *Cauchy*' s formula, where $\mathcal{T}^{(\hat{\mathbf{n}})} = (\mathcal{T}_1^{(\hat{\mathbf{n}})}, \mathcal{T}_2^{(\hat{\mathbf{n}})}, \mathcal{T}_3^{(\hat{\mathbf{n}})})$,

$\mathbf{n} = (n_1, n_2, n_3)$ and

$$\sigma_{ij}^{eff} = \sigma_{ij} + p_{fluid}\,\delta_{ij} = \begin{bmatrix} -\sigma_{n_1}^{eff} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & -\sigma_{n_2}^{eff} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{n_3}^{eff} \end{bmatrix}$$

where: $\sigma_{n_i}^{eff} = \sigma_{n_i} - p_{fluid} = -\sigma_{ii} - p_{fluid}$ *and stresses are assumed to be negative for compression*

$T_j^{(\hat{\mathbf{n}})} = n_i \sigma_{ij}^{eff} - n_j (n_i \sigma_{ik}^{eff} n_k)$ <u>shear</u> traction

$\Sigma_j^{(\hat{\mathbf{n}})} = n_j (n_i \sigma_{ik}^{eff} n_k)$ <u>normal</u> traction

Fracture Criteria & Constitutive Laws

1. FRACTURE CRITERION

- Condition that specify, at a given fault point and at a given time, if there is a rupture or not.
- It can be expressed in terms of **energy**, in terms of **maximum frictional resistence**, and so on.
- It is based on (*i*) the *Benioff (1951)* hypothesis: The fracture occours when the stress in a volume reaches the rock strength

  or, analogoulsy,

  (*ii*) the *Reid (1910)* statement: The fracture takes place when the stress attains a value greater than the rock can endure.

2. CONSTITUTIVE LAW

- Analytical relation existing between the components of the stress tensor and physical observable(s), like the slip, the slip velocity, the state variable, etc..
- From a mathematical point of view it is a **Fault Boundary Condition** (**FBC**) that controls earthquake dynamics and its complexity in space and in time.
- Its simplest form consider only **two frictional levels**, τ_u and τ_f; it accounts for stress drop (or stress realease), but the process is instantaneous: there is a singularity at crack tip.
- **Cohesive zone models**: *Barenblatt (1959a, 1959b), Ida (1972), Andrews (1976a, 1976b)*. In these models the singularity is removed and the sress release occours over a breakdown zone distance X_b and in a breakdown zone time T_b.
- **Friction laws** (Rate and State dependent f. l.): *Dieterich (1976), Ruina (1980, 1983)*. They accounts for fault spontaneous nucleation, re – strengthening, healing, etc..

CONSTITUTIVE LAW (continues)

- “ The central issue is *whether* faults obey simple friction laws, and if so, what is the friction coefficient associated with fault slip “ (*Scholz and Hanks, 2004*).

CONSTITUTIVE LAW (continues)

- In full of generality we can express the constitutive (or governing) as:

$$\tau = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\, \sigma_n^{eff}(\sigma_n, p_f)$$

where:

1st – order dependencies

u is the Slip (i. e. displ. disc.) modulus,

v is the Slip Velocity modulus (its time der.),

Ψ = (Ψ_1, …, Ψ_N) is the State Variable vector,

T is the Temperature (accounting for Ductility, Plastic Flow, Melting and Vaporization),

H is the Humidity,

λ_c is the Characteristic Length of surface (accounting for Roughness and Topography of asperity contacts),

h is the Hardness,

g is the Gouge (accounting for Surface Consumption and Gouge formation),

C_e is the Chemical Environment

Strength & Constitutive Laws

1. THE STRENGTH PARAMETER

- **Hystorically introduced by *Das and Aki (1977a, 1977b)* to have a quantitative extimate of the ability to fracture for a fault**
- Its expression can be generalized as:

$$S = (\mu_u \sigma_n^{eff} - \tau_0) / (\tau_0 - \mu_f \sigma_n^{eff})$$

  where μ are the friction coefficient.

- We can also define

2. THE FAULT STRENGTH

- **Is the parameter that quantify the Strenght in the more general case, in which a fault is described by a rhealistic friction laws**

$$S^{fault} = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\ \sigma_n^{eff}(\sigma_n, p_{fluid})$$

Time - weakening Friction Law

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{(t - t_r)}{t_0} \right] \sigma_n^{\ eff} & , t - t_r < t_0 \\ \mu_f \sigma_n^{\ eff} & , t - t_r \geq t_0 \end{cases}$$

ilaw = 11

TW

$t_r = t_r(\xi)$ is the rupture onset time in every fault point ξ (when $u > 0$).

Andrews (1985), *Bizzarri et al. (2001)* and other following Bizzarri' s papers

t_0 is the characteristic time – weakening duration.

Position - weakening Friction Law

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{x}{R_0} \right] \sigma_n^{\,eff} & , -R_0 < x < 0 \\ \mu_f \sigma_n^{\,eff} & , -L < x < -R_0 \end{cases}$$

PW

x is the position on the fault (extending up to – L).

Palmer and Rice (1973)

R_0 is the characteristic position – weakening distance.

Slip - Dependent Friction Laws

1. LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{u}{d_0} \right] \sigma_n^{\,eff} & , u < d_0 \\ \mu_f \sigma_n^{\,eff} & , u \geq d_0 \end{cases}$$

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ilaw = 21

SW
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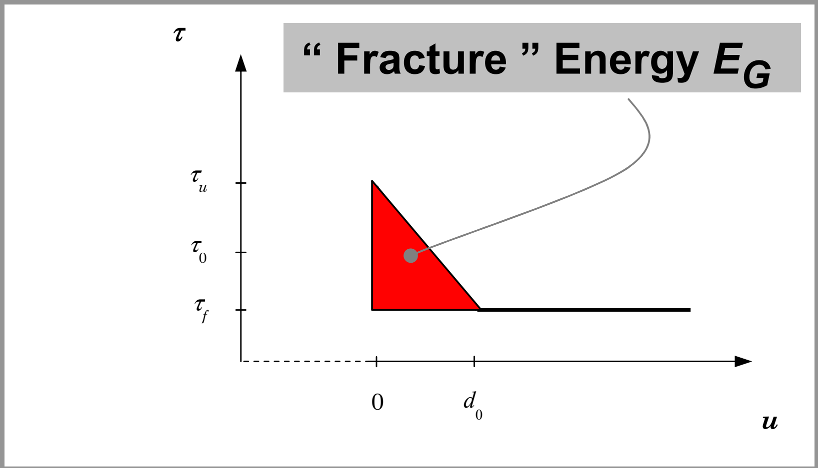

Barenblatt (1959a, 1959b), Ida (1972), Andrews (1976a, 1976b), and many authors thereinafter

d_0 is the characteristic slip – weakening distance

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ilaw = 22
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IW

2. NON – LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - \dfrac{\mu_u - \mu_f}{d_0} \left(u - \dfrac{(1 - p_{IW}) d_0}{2\pi} \sin\left(\dfrac{2\pi u}{d_0} \right) \right) \right] \sigma_n{}^{eff} & , u < d_0 \\ \mu_f \sigma_n{}^{eff} & , u \geq d_0 \end{cases}$$

Ionescu and Campillo (1999)

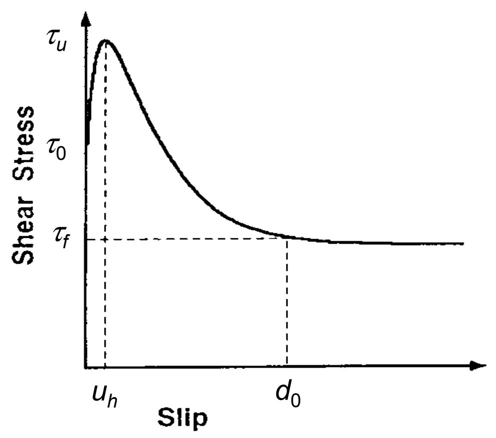
3. NON LINEAR SLIP – WEAKEING LAW WITH SLIP – HARDENING

$$\tau = \left\{ \left[\left(\frac{\tau_0}{\sigma_n^{\,eff}} - \mu_f \right) \left(1 + \alpha_{OY} \ln \left(1 + \frac{u}{\beta_{OY}} \right) \right) \right] e^{-\frac{u}{d_0}} + \mu_f \right\} \sigma_n^{\,eff}$$

ilaw = 23

OW

$$u_h : \left. \frac{\mathrm{d}\tau}{\mathrm{d}u} \right|_{u_h} = 0 \, ; \qquad \begin{cases} u_h = r d_0 & (\mathrm{e.g.}\ r = 0.1) \\ \tau(u_h) = \tau_u \end{cases}$$



Ohnaka and Yamashita (1989) and the following papers by Ohnaka and coworkers

u_h is associated with the preparatory phase of the imminent macroscopic failure in the cohesive zone. It accounts for micro–cracking

4. NON LINEAR SLIP – WEAKEING LAW WITH EXPONENTIAL DECAY

$$\tau = \left[\left(\mu_u - \mu_f \right) e^{-\frac{u}{d_0}} + \mu_f \right] \sigma_n^{eff}$$

ilaw = 24

EW

5. POWER LAW SLIP – WEAKEING

$$\tau = \left\{ \mu_u - \left(\mu_u - \mu_f\right)\left[\left(\frac{p_{PW}}{p_{PW}+1}\right)\frac{u}{d_0} \right]^{p_{PW}} \right\} \sigma_n{}^{eff}$$

ilaw = 25

PW

Rate - Dependent Friction Law

$$\tau = \frac{v_*}{v + v_*} \mu_u \sigma_n^{\,eff}$$

Burridge and Knopoff (1967), *Carlson and Langer (1989)*, Madariaga and Cochard (1994), Cochard and Madariaga (1994)

Rate - and State - Dependent Friction Laws

1. DIETERICH IN REDUCED FORMULATION

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi v_*}{L} \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi v}{L} \end{cases}$$

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ilaw = 31
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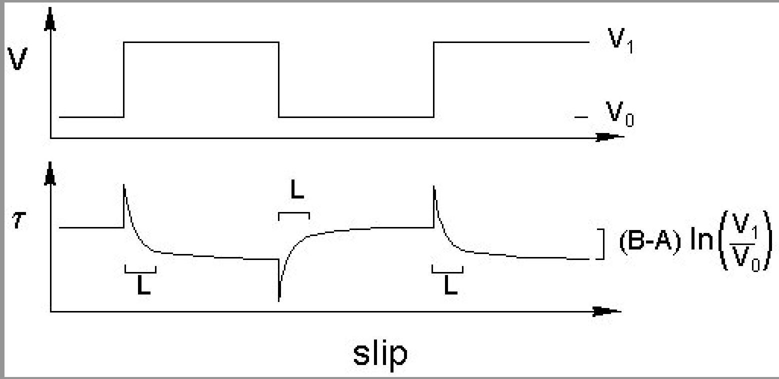
DR

However, while in velocity stepping experiments the traction response following the velocity variation is directly controlled by the parameter *L*, its effects are much less evident during the dynamic rupture propagation.

Bizzarri and Cocco (2005)

Response to an abrupt jump in load



2. RUINA – DIETERICH

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = -\dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) \end{cases}$$

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ilaw = 32
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RD

Ruina (1980, 1983), Beeler et al. (1984), Roy and Marone (1996)

3. DIETERICH – RUINA WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\frac{v_*}{v} \right) + b \ln\left(\frac{\Psi\, v_*}{L} \right) \right] \sigma_n^{eff} \\ \frac{\mathrm{d}}{\mathrm{d}\, t} \Psi = 1 - \frac{\Psi\, v}{L} - \left(\frac{\alpha_{LD} \Psi}{b\, \sigma_n^{eff}} \right) \frac{\mathrm{d}}{\mathrm{d}\, t} \sigma_n^{eff} \end{cases}$$

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ilaw = 31
decis10=T
```

DR

Linker and Dieterich (1992), Dieterich and Linker (1992), Bizzarri and Cocco (2006b, 2006c)

4. RUINA – DIETERICH WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{\,eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) - \left(\dfrac{\alpha_{LD} \Psi}{b\, \sigma_n^{\,eff}} \right) \dfrac{\mathrm{d}}{\mathrm{d}t} \sigma_n^{\,eff} \end{cases}$$

```
ilaw = 32
decis10=T
```

RD

Linker and Dieterich (1992), Bizzarri and Cocco (2006b, 2006c)

5. DIETERICH IN REDUCED FORM REGULARIZED

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v + v_*}{v \boxed{+ v_r}} \right) + b \ln\left(\dfrac{\Psi \left(v \boxed{- v_r}\right)}{L} + 1 \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi \left(v \boxed{+ v_r}\right)}{L} \end{cases}$$

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ilaw = 33

DE
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v_r is a regularization fault slip velocity

Perrin et al. (1995), Cocco et al. (2004)

6. RUINA REGULARIZED

$$
\begin{cases}
\tau = \left[\mu_* - a \ln\left(\dfrac{v_* \boxed{-v_r}}{v \boxed{+v_r}} \right) + \dfrac{\Psi}{\sigma_n^{eff}} \right] \sigma_n^{eff} \\
\dfrac{\mathrm{d}}{\mathrm{d}t}\Psi = -\dfrac{v \boxed{+v_r}}{L} \left(\Psi + b \ln\left(\dfrac{v \boxed{+v_r}}{v_* \boxed{-v_r}} \right) \right)
\end{cases}
$$

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ilaw = 34
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RE

v_r is a regularization fault slip velocity

Bizzarri (2002, unpublished work)

7. DIETERICH IN REDUCED FORM WITH HEALING

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} + 1 \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi = \dfrac{\gamma_{fh} - \Psi}{\boxed{t_{fh}}} - \dfrac{\Psi\, v}{L} \end{cases}$$

ilaw = 35

DH

γ_{fh} = 1 s

t_{fh} is the time for healing (slip duration)

Evolution law proposed by *Nielsen et al. (2000)* and by *Nielsen and Carlson (2000)*. Used in this form by *Cocco et al. (2004)*

9. PRAKASH – CLIFTON

$$
\begin{cases}
\tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \left(\dfrac{\mathrm{d}}{\mathrm{d}\,t} \Psi_1 + \dfrac{\mathrm{d}}{\mathrm{d}\,t} \Psi_2 \right) \\
\dfrac{\mathrm{d}}{\mathrm{d}\,t} \Psi = 1 - \dfrac{\Psi\, v}{L} \\
\dfrac{\mathrm{d}}{\mathrm{d}\,t} \Psi_1 = - \dfrac{v}{L_1} \left(\Psi_1 - \alpha_{PC_1} \sigma_n{}^{eff} \right) \\
\dfrac{\mathrm{d}}{\mathrm{d}\,t} \Psi_2 = - \dfrac{v}{L_2} \left(\Psi_2 - \alpha_{PC_2} \sigma_n{}^{eff} \right)
\end{cases}
$$

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ilaw = 37
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PC

Ψ_1 *and* Ψ_2 are additional state variables accountinf for the coupling with effective normal stress. The formulation of friction law is not based on the Amonton – Coulamb law.

Coupling with effective normal stress proposed by *Prakash and Clifton (1993)* and *Prakash (1998)*. Used in this form by *Bizzarri (2005, unpublished work)*

Slip - and State - Dependent Friction Law

$$\begin{cases} \tau = \begin{cases} \left[(\mu_u - \Delta\mu)\left(1 - \dfrac{u}{d_1}\right)\right] \sigma_n^{eff} & , u < d_1, \Psi \geq \Psi_1 \\ 0 & , u \geq d_1, \Psi \geq \Psi_0 \\ \mu_{sp}\left(1 - \dfrac{\Psi}{\Psi_0}\right)\sigma_n^{eff} & , \Psi < \Psi_0, \Psi < \Psi_1 \end{cases} \\ \dfrac{\mathrm{d}}{\mathrm{d}t}\Psi = -\dfrac{\beta_{CM}}{d_0}(\Psi - v) \end{cases}$$

CM

Cochard and Madariaga (1994)

$\Delta\mu$ is an initial artificial stress drop

$\Psi_1 \equiv \Psi_0 \, (u - u_1)/(d_1 - u_1)$

$U_1 \equiv - d_1 \, (\mu_{sp} - \mu_u + \Delta\mu)/(\mu_u - \Delta\mu)$

d_0 and d_1 are characteristic lengths

$\mu_{sp} = 0 \Rightarrow$ linear SW with d_1 as characteristic length

How to relate relevant quantities to contitutive parameters

Dynamic Parameters

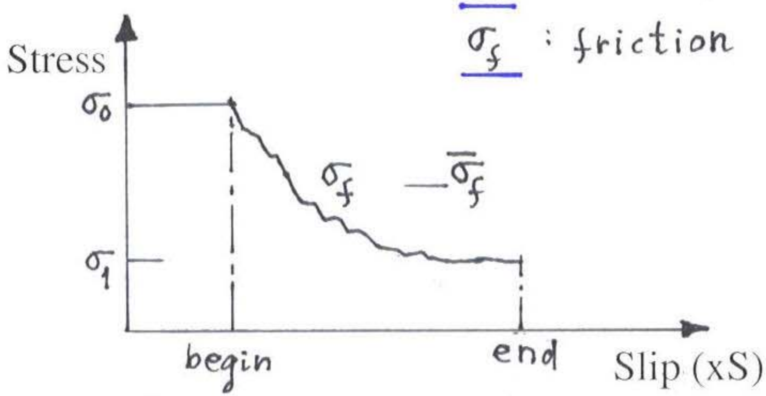
E_R : radiated energy

ruputre speed, $V \approx 0.8$ to $0.9\ \beta$

$E_{NR} = E_F + E_G + \ldots$: non-radiated energy

E_F : friction (heat), E_G : fracture energy

σ_f : friction



$\Delta\sigma_s = \sigma_0 - \sigma_1$: static stress drop

$\Delta\sigma_d = \sigma_0 - \bar{\sigma}_f$: dynamic stress drop

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A B R

Support Slides: Parameters, Notes, etc.

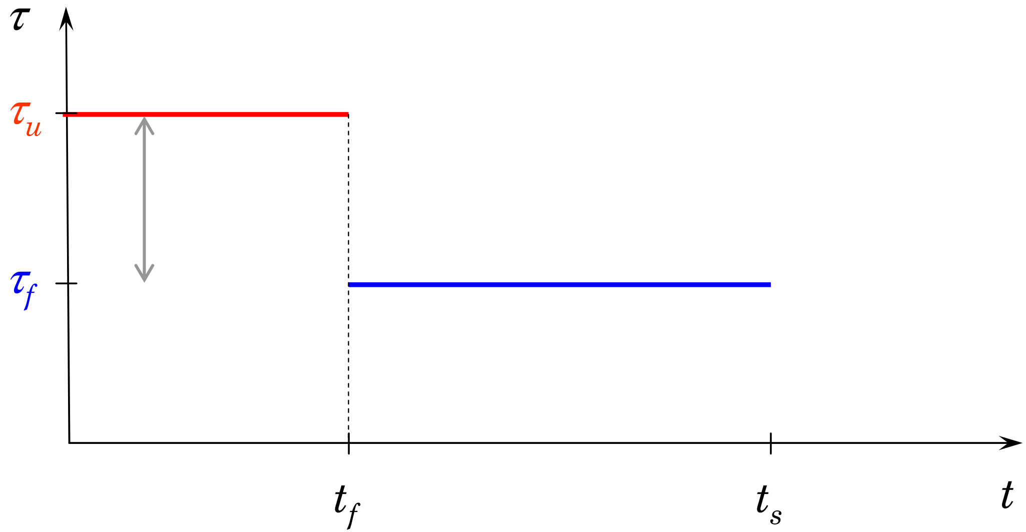
To not be displayed directly. Referenced above.

Simplest friction models

At a particular fault point ξ (following *Savage and Wood, 1971*; *Scholz, 1990*)

Maximum (or upper, or yield) stress

Kinetic (or frictional) stress



Failure time (or rupture onset)

Rupture arrest

Strength excess: $\tau_u - \tau_0 = 0$

Dynamic stress drop: $\Delta\tau_d = \tau_0 - \tau_f$

In the Dugdale’ s model (*Dugdale, 1960*; *Barenblatt, 1962*) the drop occurs when $u = d_0$.

Simplest friction models

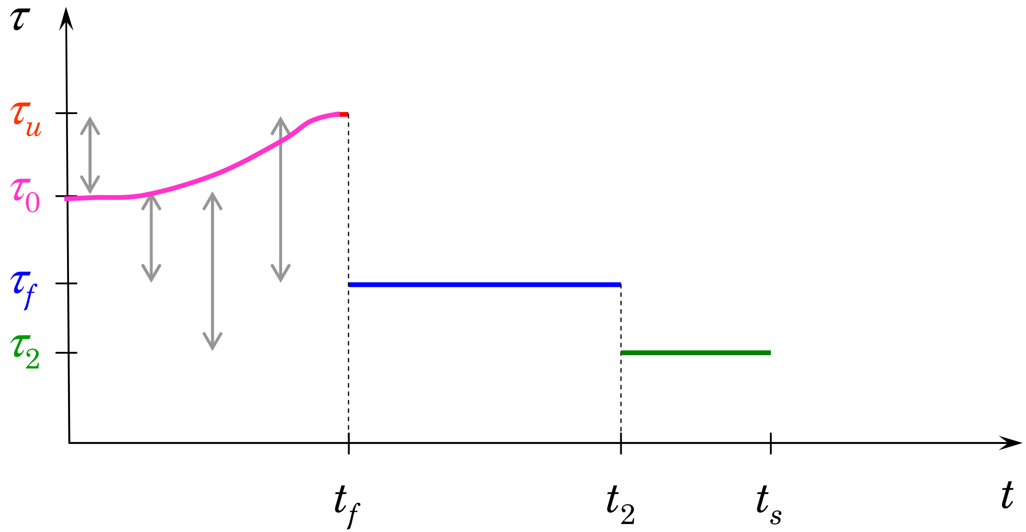
At a particular fault point ξ (following *Savage and Wood, 1971*; *Scholz, 1990*)

Maximum (or upper, or yield) stress

Initial stress

Kinetic (or frictional) stress

Residual stress



Strength excess: $\tau_u - \tau_0$

Dynamic stress drop: $\Delta\tau_d = \tau_0 - \tau_f$

Static stress drop: $\Delta\tau_s = \tau_0 - \tau_2$

Breakdown str. drop: $\Delta\tau_b = \tau_u - \tau_f$

Failure time (or rupture onset)

Dynamic overshoot

Rupture arrest

- *Savage and Wood (1971)* also define:

Mean stress: $<\tau> = ½ (\tau_u + \tau_2)$

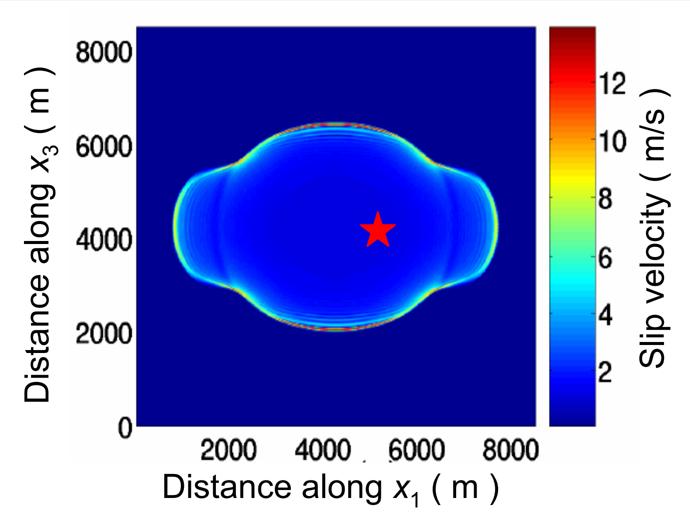
Seismic efficiency: $\eta = E_s/E$, where: E_s is the seismic energy
E is the total available energy

Apparent stress: $\tau_a = \eta <\tau>$

- Direct observation of the absolute stress near an earthquake is not feasible, but it is possible (*Wyss and Brune, 1968*) calculate τ_a and stress drop from physical observables.

The cohesive zone

Time snapshot (t = 0.8 s) - SW law



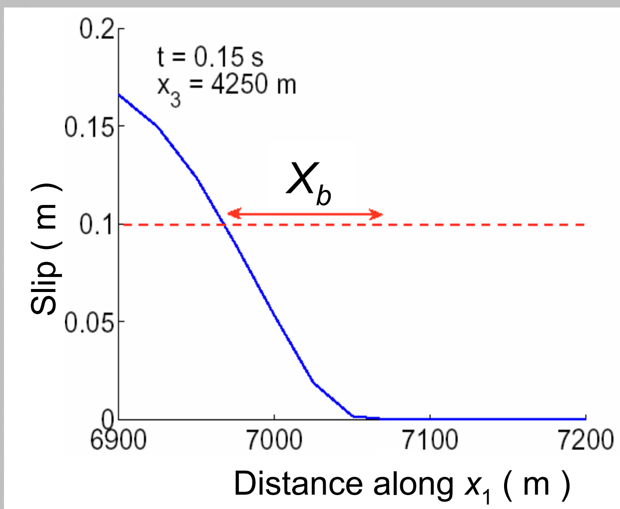
In the target location we can extimate:

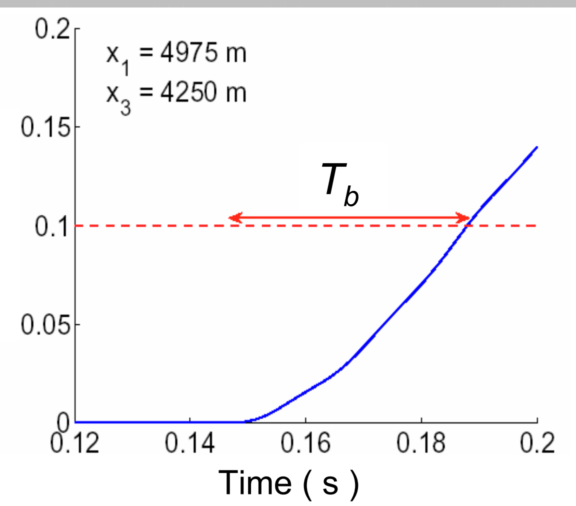
X_b = 105 m T_b = 0.04 s

From these quantities:

$v_{rupt} = X_b / T_b$ = 2625 m/s

Local estimate





Slip - hardening effect

* The slip – hardening (**SH**) phenomenon has been also found in seismological inversion studies (e. g. *Quin, 1990*; *Miyatake, 1992*; *Mikumo and Miyatake, 1993*; *Beroza and Mikumo, 1996*; *Ide, 1997*; *Bouchon, 1997*).

Interpretation of the state variable