Rupture propagation in 2 – D fault models

Earthquake parameters

Observed Parameters

Seismic Moment: M_0

Radiated Energy: E_R

Rupture Speed: v_r

Source Duration (Corner Frequency): T (f_c)

Inferred (sometimes Observed) Parameters

Static Stress Drop: $\Delta\sigma_s$

Particle Motion Velocity: $\dot{U}$

Source Dimension: L

Critical Slip: D_c

Numerical Method: BIE 2 - D

We solve the fundamental elastodynamic equation, neglecting body forces ***f***

$$\rho \ddot{U}_i = \sigma_{ij,j} + f_i$$

Source integral rapresentation (*Betti*' s theorem, Integration in time limit in fault surface, Lamb' s problem):

$$u_n(\mathbf{x},t) = \int_{-\infty}^{+\infty} \mathrm{d}t' \int_{S(t')} \mathrm{d}\xi\, G_{n\alpha}(\mathbf{x}-\xi, t-t')\, \sigma_{\alpha\beta}{}^{p}(\xi, t')\ ; n=1,2,3; \alpha=1,2; \mathbf{x}, \xi \in \mathbb{R}^3$$

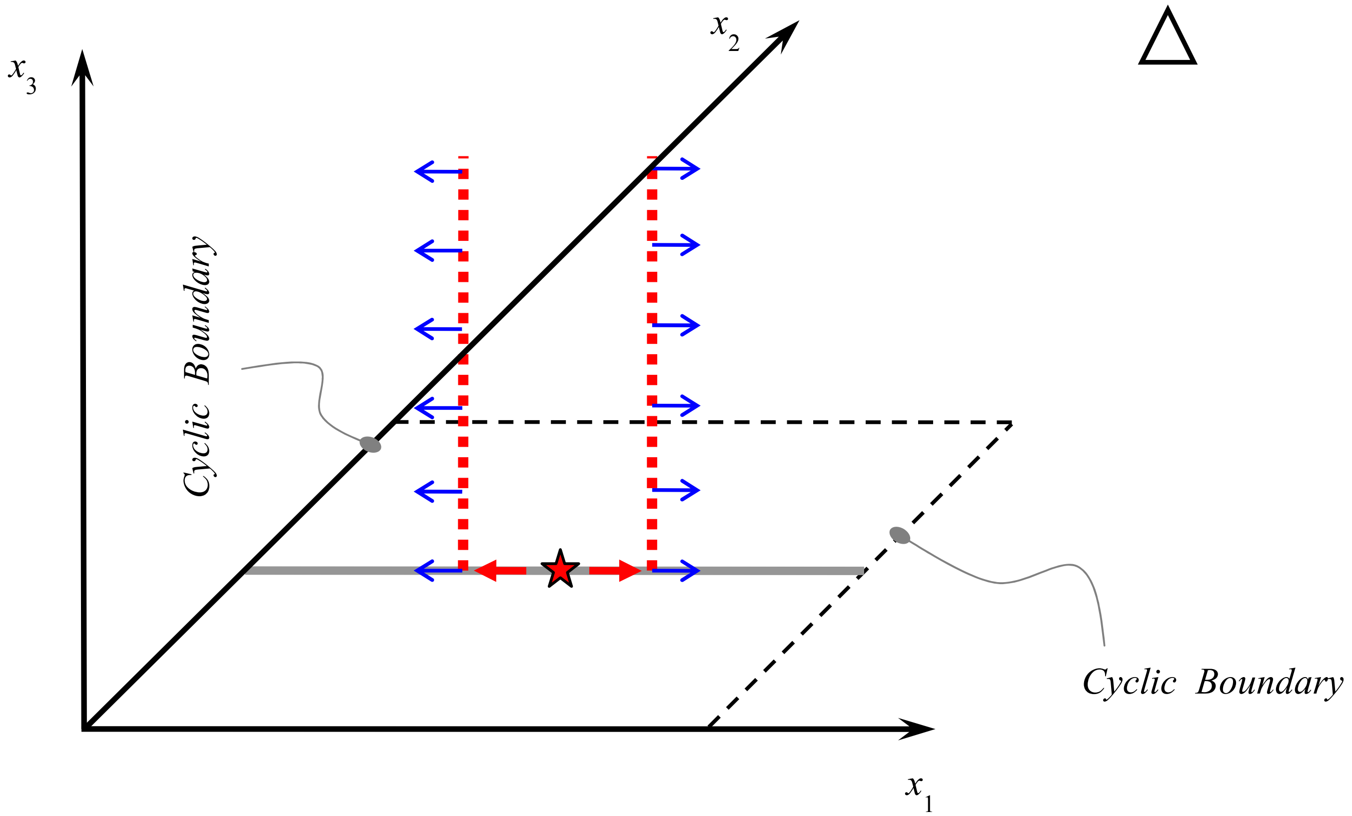
First neighbours decoupling (in the case of a 2 – D, pure in – plane rupture):

Traction

$$\begin{cases} u_1(x_1,t) + C\, \tau_1{}^{p}(x_1,t) = L_1(x_1,t) \\ \tau_{0_1} + \tau_1{}^{p}(x_1,t) = \mu \sigma_n{}^{eff} \end{cases}$$

Friction

Numerical Method: FD 2 - D



$$\boldsymbol{u} = (\ u_1(x_1, t), 0, 0\ )$$

2-D

We solve the fundamental elastodynamic equation, neglecting body forces ***f***

$$\rho \ddot{u}_i = \sigma_{ij,j} + f_i$$

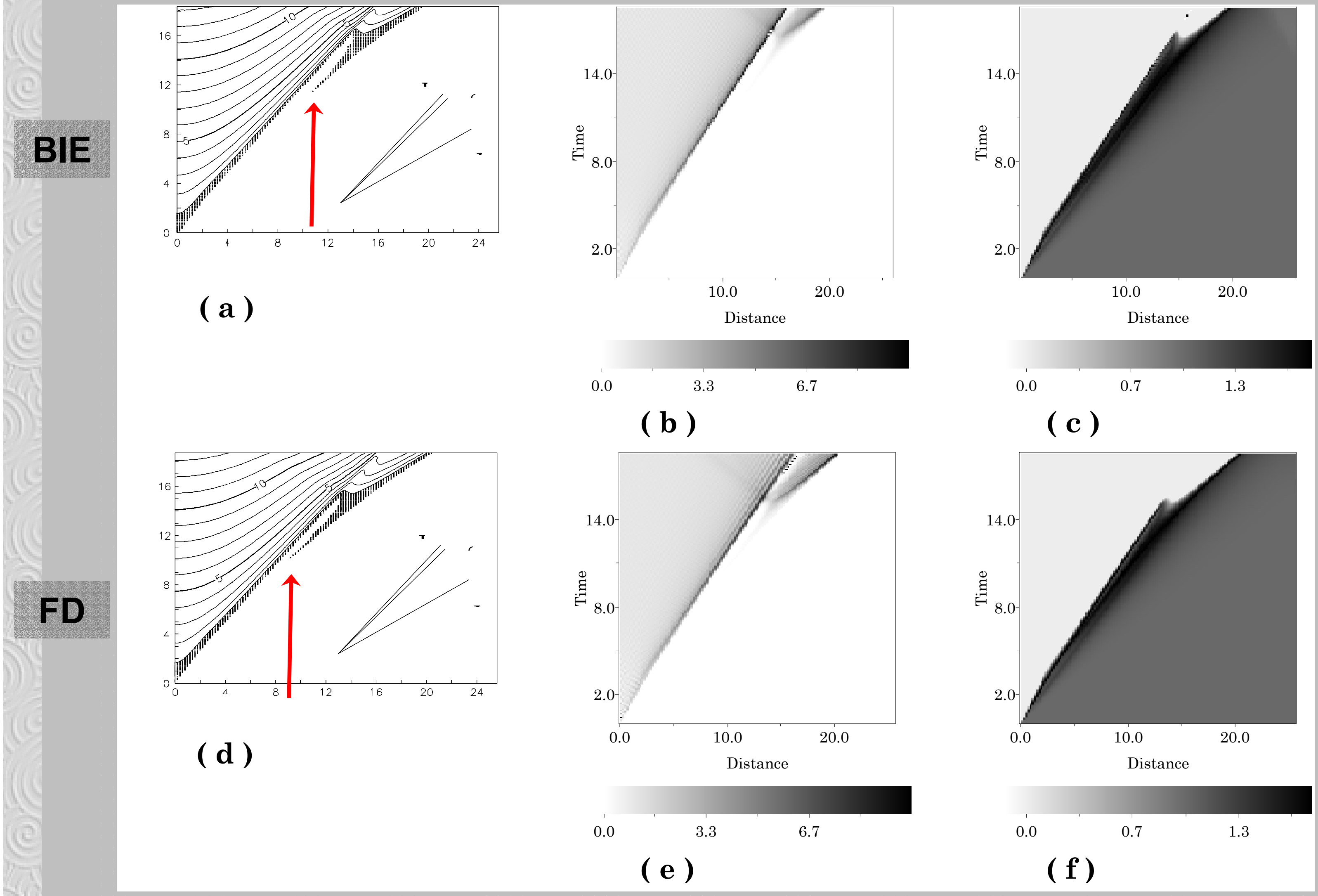
We discretize the $x_1 x_2$ plane by using triangular cells (better performances)

$$\rho \frac{\partial}{\partial t} \dot{u}_1 = \frac{\partial}{\partial x_1} \Sigma_{11} + \frac{\partial}{\partial x_2} \Sigma_{12}$$

$$\rho \frac{\partial}{\partial t} \dot{u}_2 = \frac{\partial}{\partial x_1} \Sigma_{12} + \frac{\partial}{\partial x_2} \Sigma_{22}$$

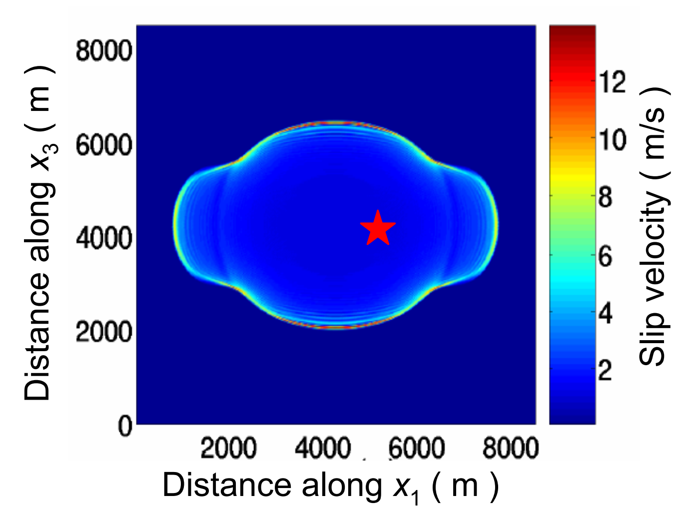
The plane is linear and elastic except in the fault intersection line, where a Fault Boundary Condition (TSN scheme) is adopted. In tihs line a constitutiva law is assumed to relate staggered stress with observables (slip, slip velocity, …)

BIE vs. FD with SW #1



The cohesive zone

Time snapshot (t = 0.8 s) - SW law



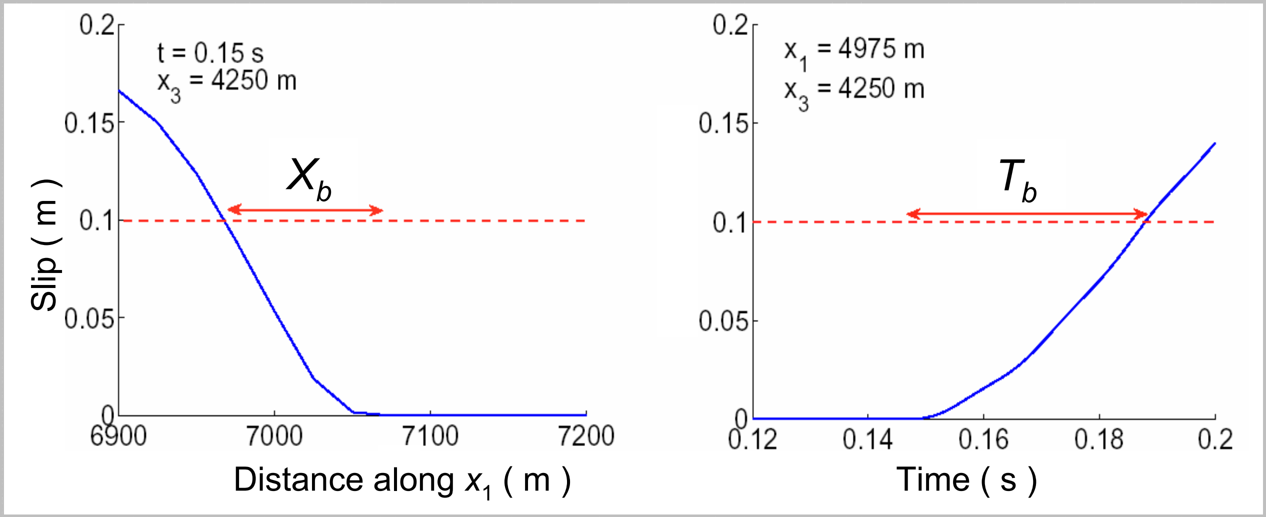
In the target location we can extimate:

X_b = 105 m T_b = 0.04 s

From these quantities:

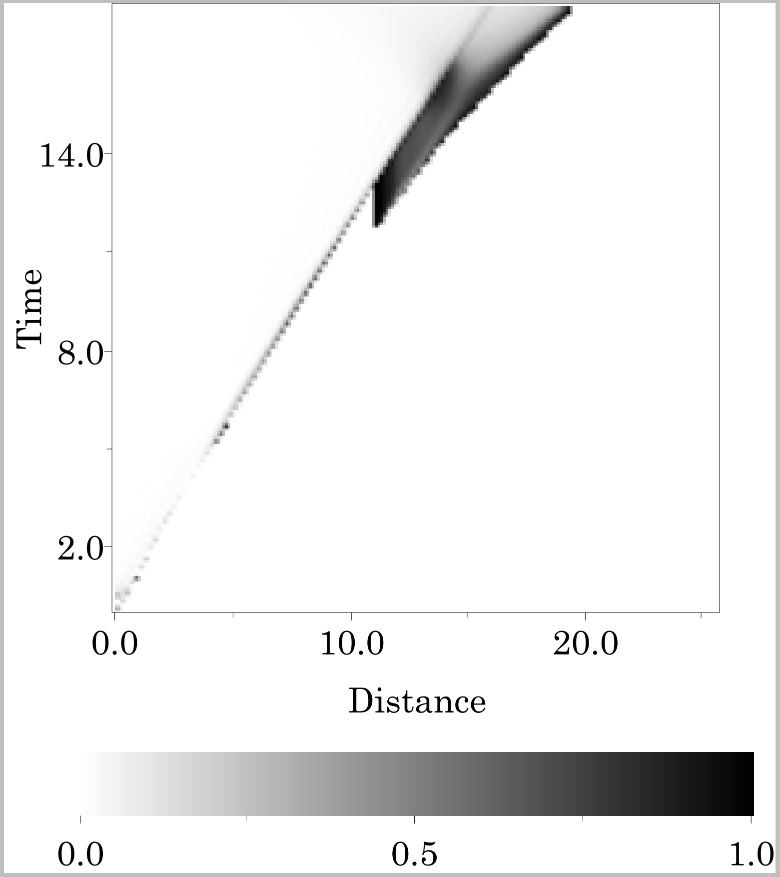
$v_{rupt} = X_b/T_b$ = 2625 m/s

Local estimate

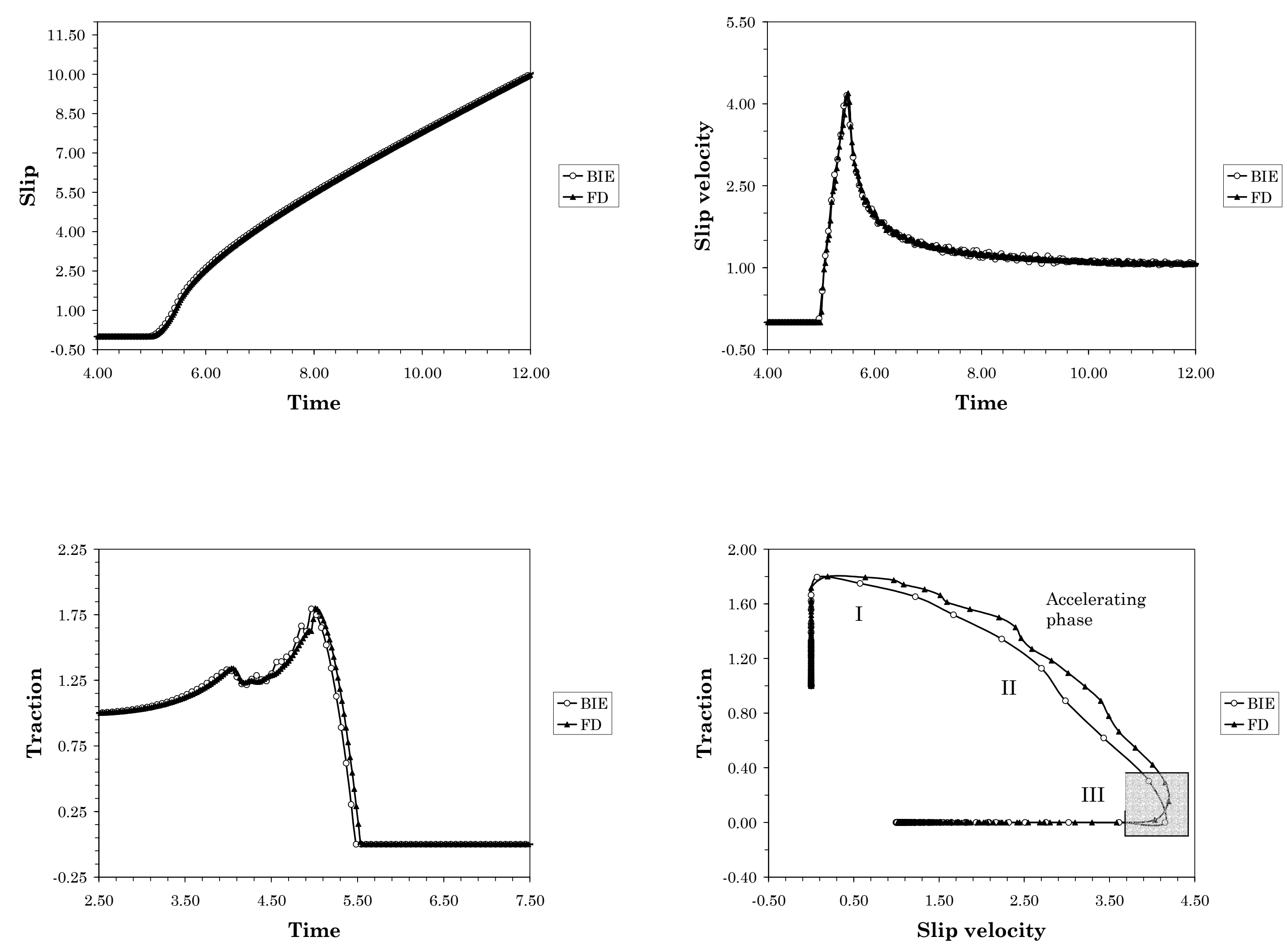


Misfit between slip modeled with BIE and FD

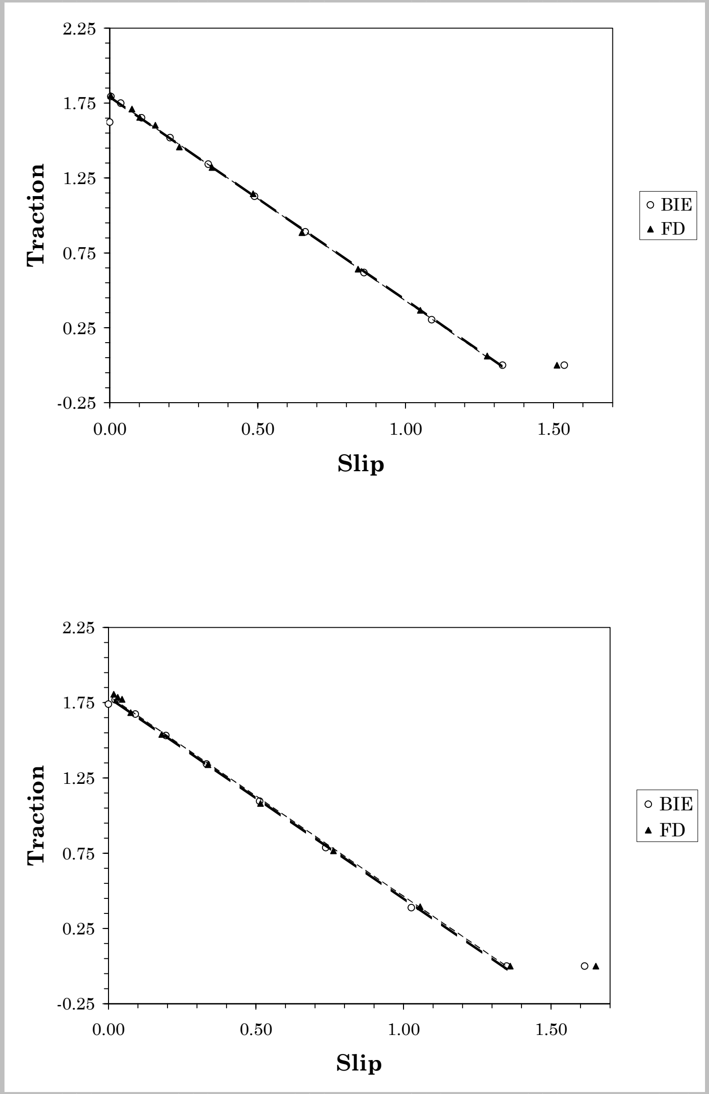
$$m(x_i,t_n)=\frac{\left|u^{(\mathrm{BIE})}(x_i,t_n)-\tilde{u}^{(\mathrm{FD})}(x_i,t_n)\right|}{\left|u^{(\mathrm{BIE})}(x_i,t_n)+\tilde{u}^{(\mathrm{FD})}(x_i,t_n)\right|}$$



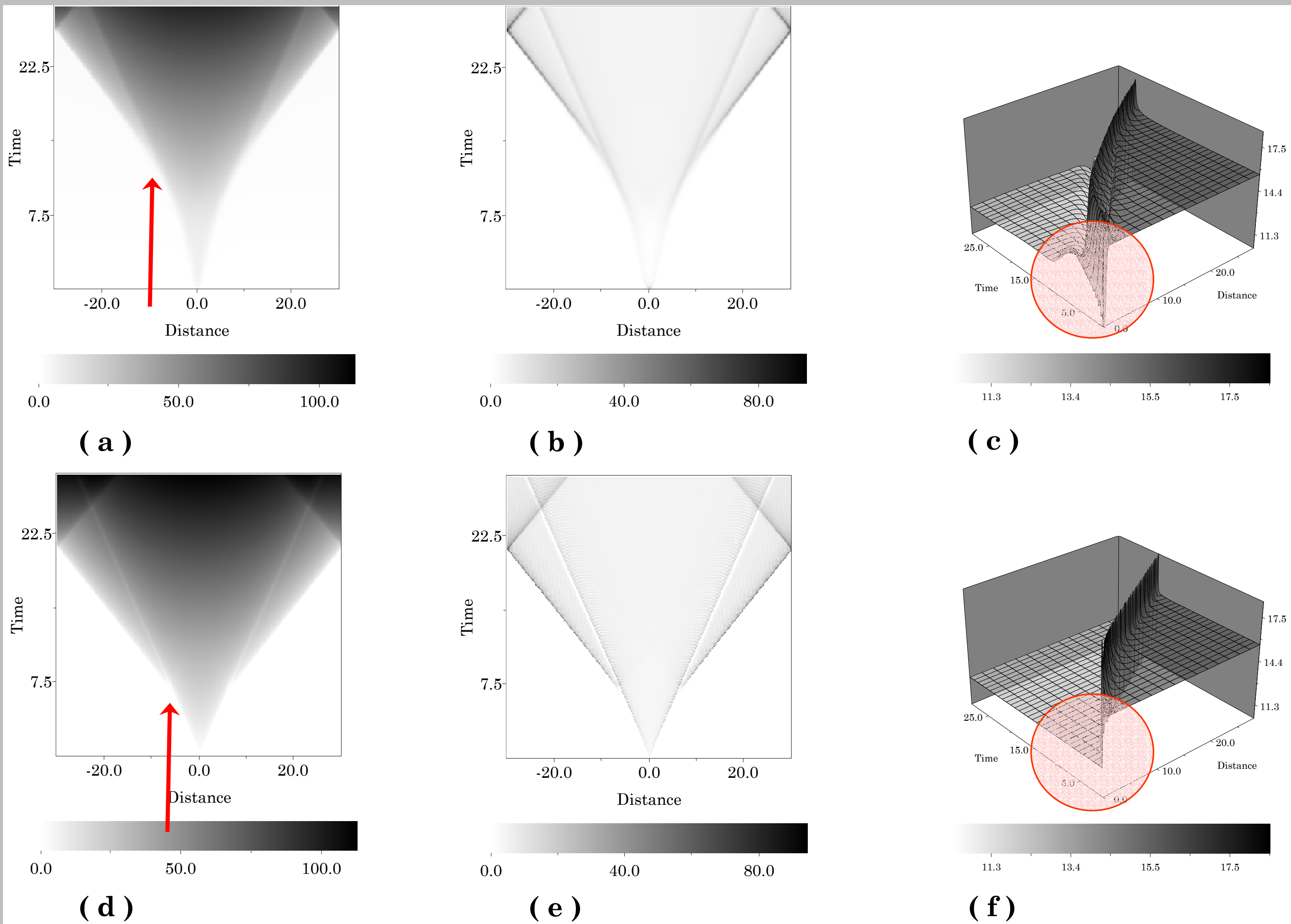
BIE vs. FD with SW #2



BIE vs. FD with SW #3

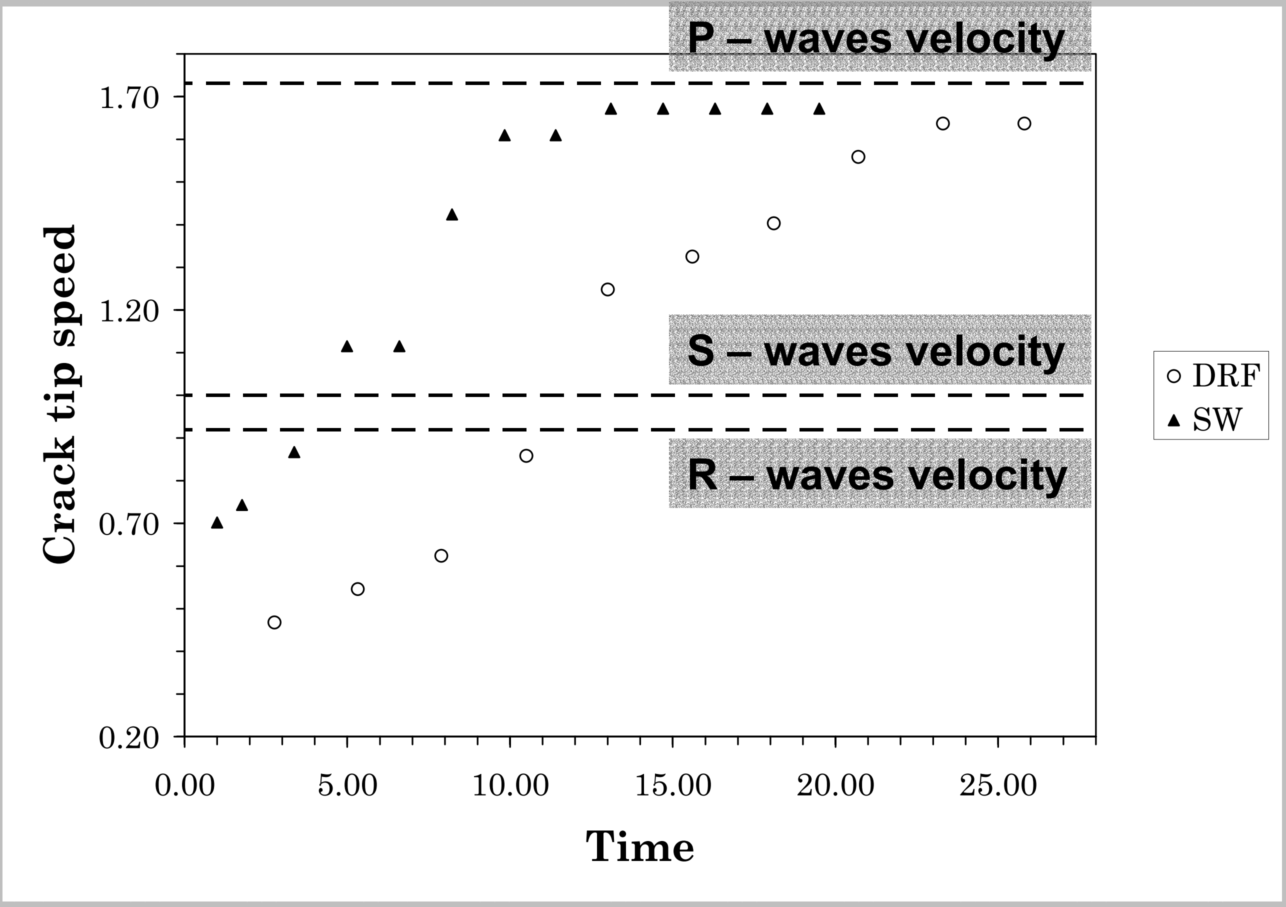


SW vs. DR law #1



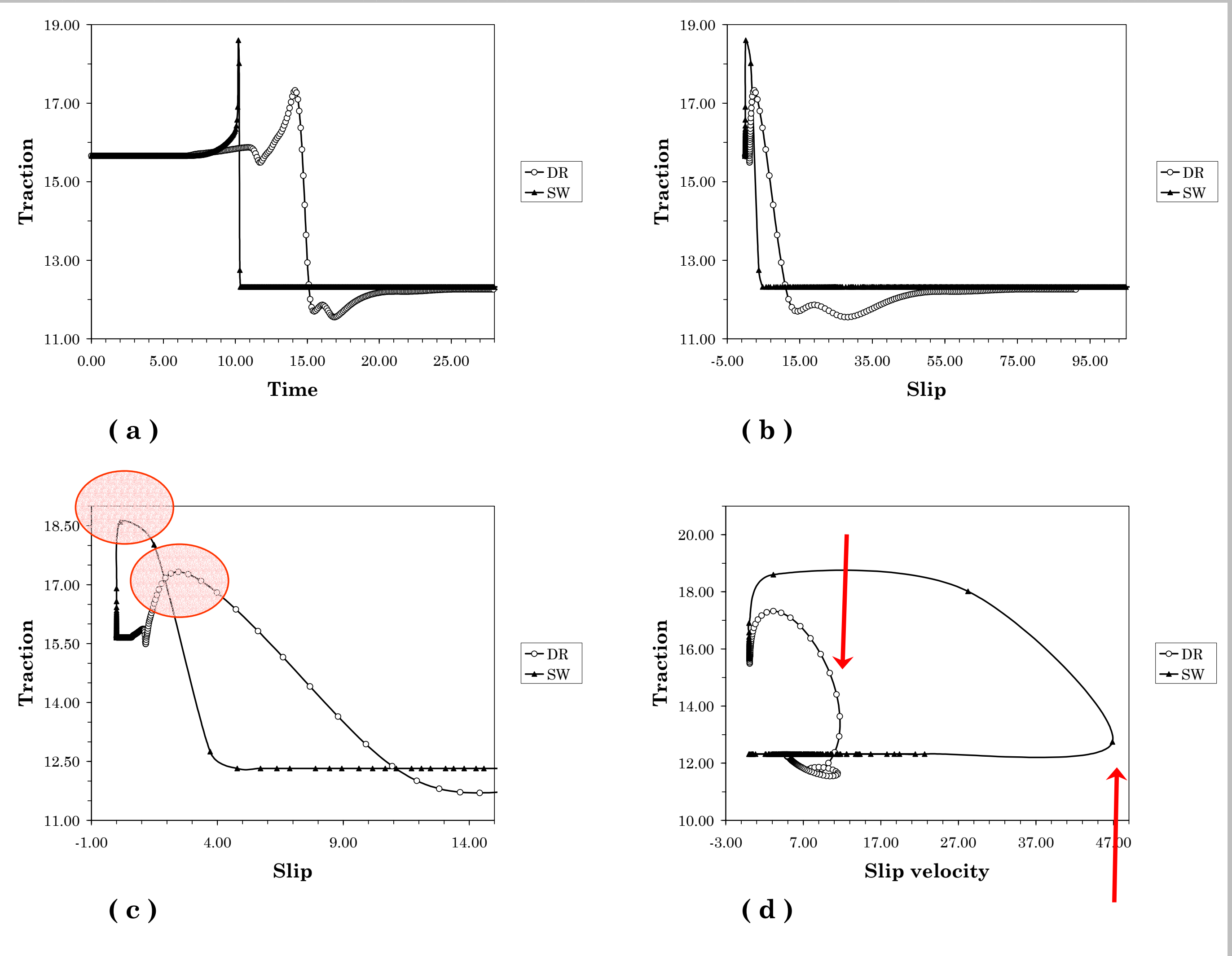
Time
22.5
7.5
-20.0
0.0
20.0
Distance
0.0
50.0
100.0
(a)
40.0
80.0
(b)
25.0
15.0
5.0
0.0
10.0
20.0
17.5
14.4
11.3
11.3
13.4
15.5
17.5
(c)
(d)
(e)
(f)

SW vs. DR law #2



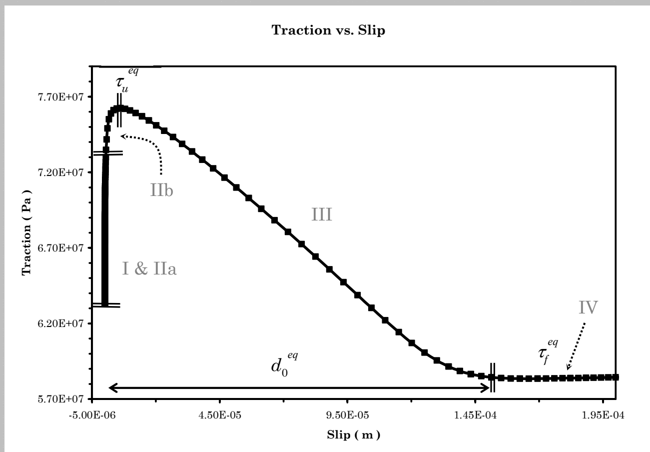
P – waves velocity
S – waves velocity
R – waves velocity
Crack tip speed
1.70
1.20
0.70
0.20
0.00
5.00
10.00
15.00
20.00
25.00
Time
DRF
SW

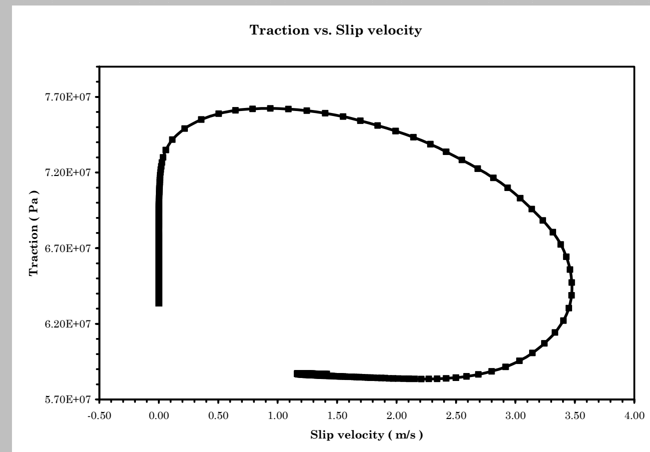
SW vs. DR law #3

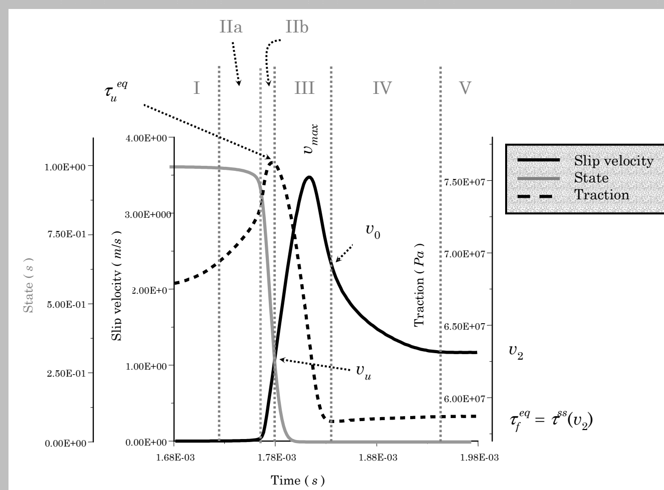


19.00
17.00
15.00
13.00
11.00
Traction
0.00
5.00
10.00
15.00
20.00
25.00
Time
DR
SW
(a)
-5.00
15.00
35.00
55.00
75.00
95.00
Slip
(b)
18.50
17.00
15.50
14.00
12.50
11.00
-1.00
4.00
9.00
14.00
(c)
20.00
18.00
16.00
14.00
12.00
10.00
-3.00
7.00
17.00
27.00
37.00
47.00
Slip velocity
(d)

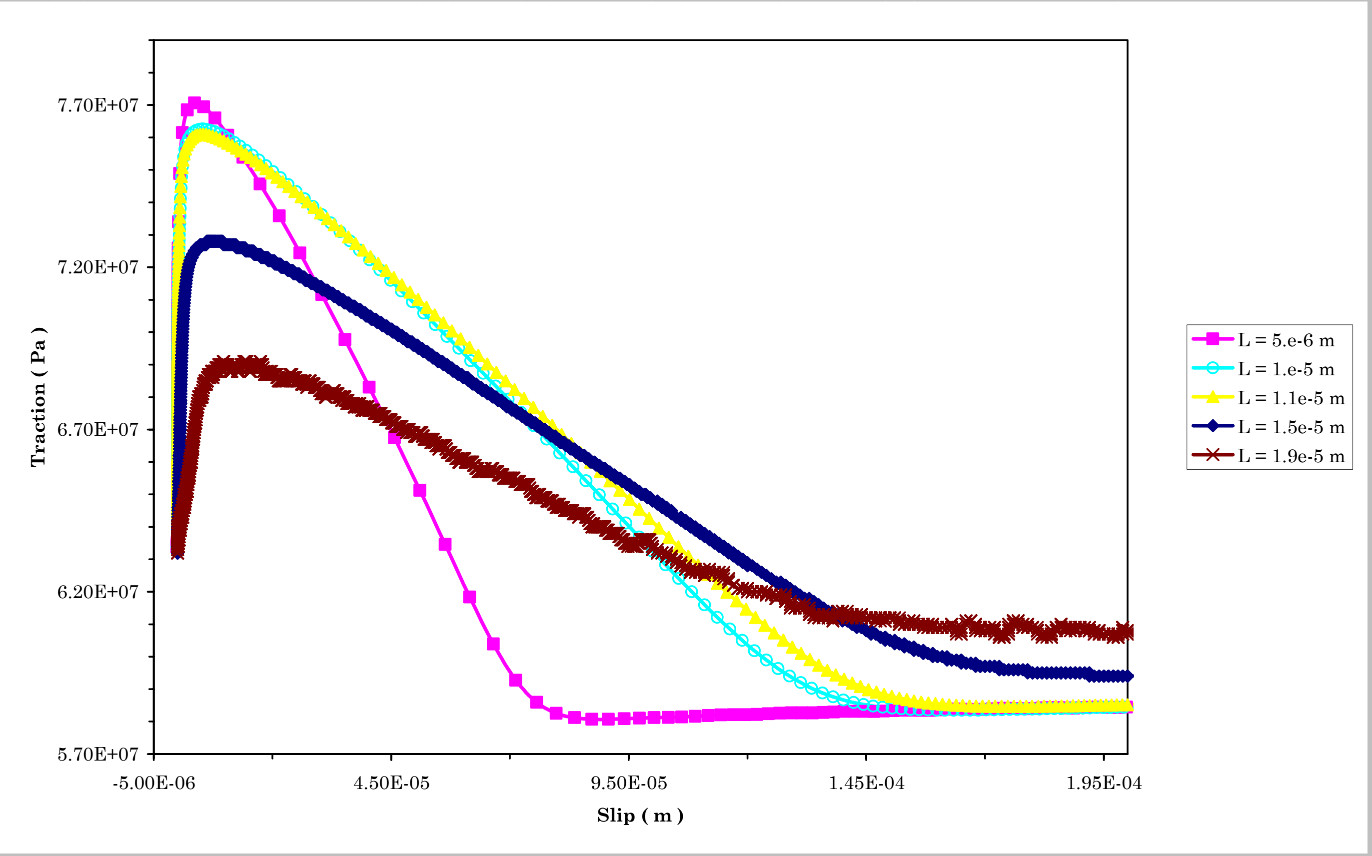
The dynamic propagation. The cohesive zone and the breakdown





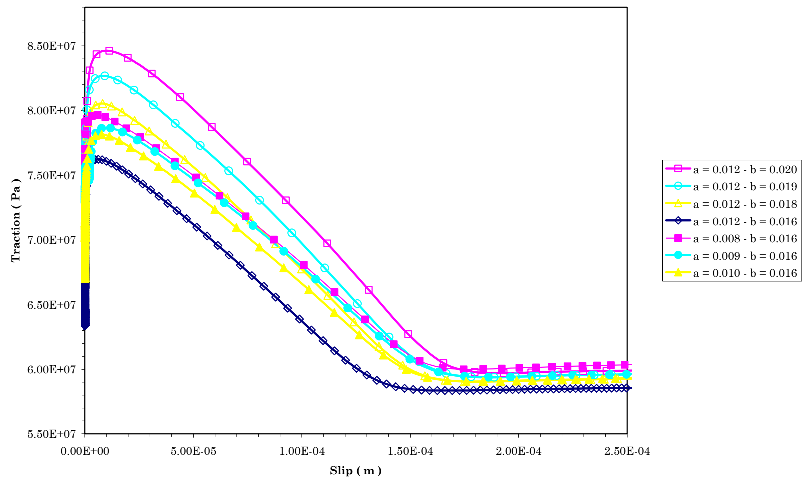


Dependence on *L* parameter

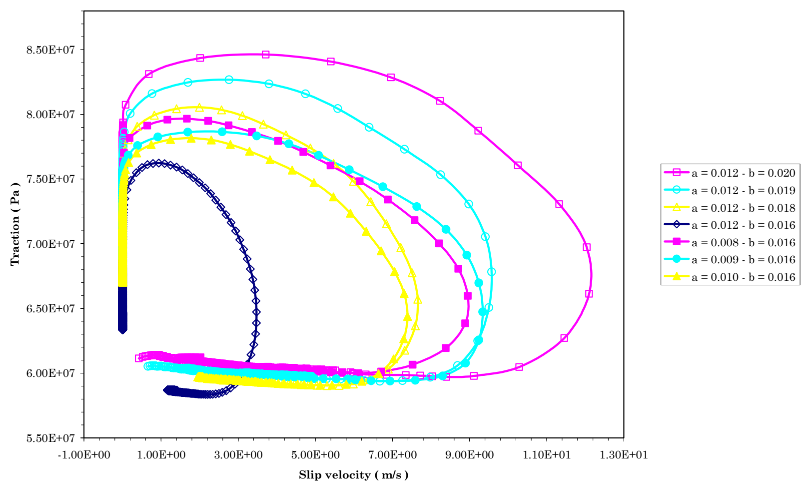


Dependence on *a* and *b*

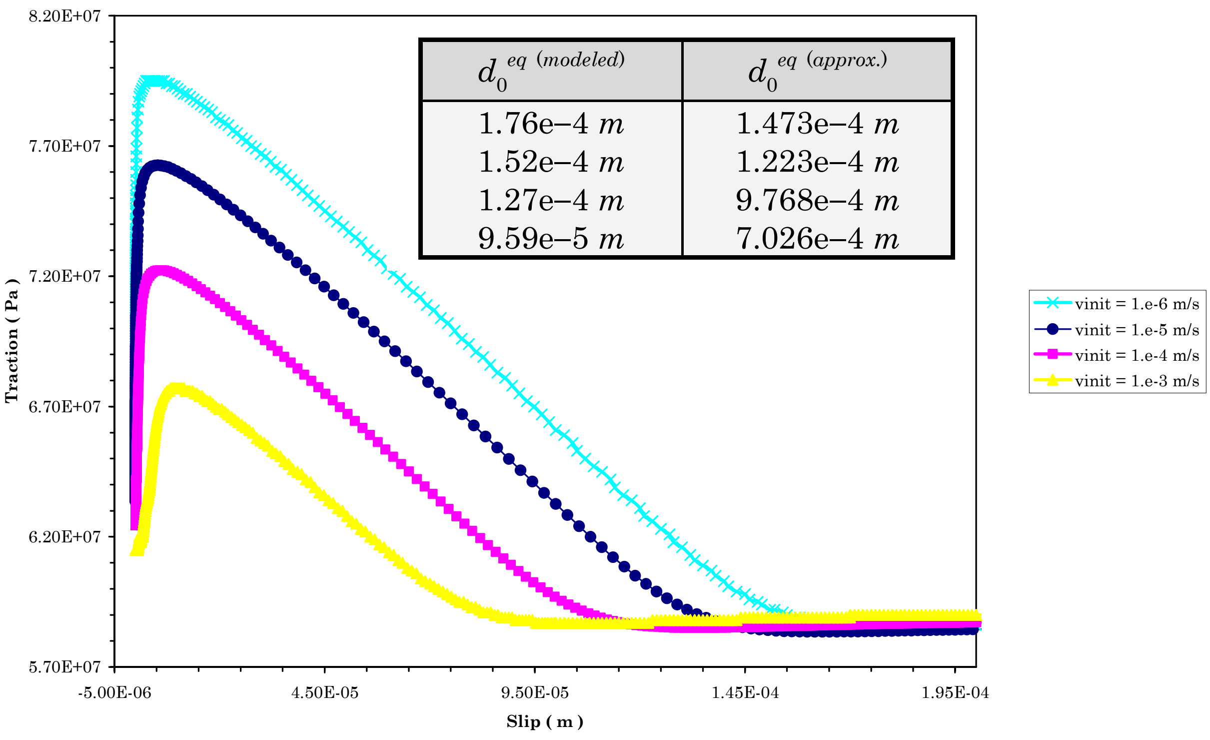
Slip – weakening curves



Phase portraits



Dependence on the initial velocity



d_0^{eq} (modeled)	d_0^{eq} (approx.)
1.76e–4 m	1.473e–4 m
1.52e–4 m	1.223e–4 m
1.27e–4 m	9.768e–4 m
9.59e–5 m	7.026e–4 m

Limitation for modeling dynamic Rupture and seismic vawe genration

- Because the initial slip velocity is totally arbitrary, it is difficult in the framework of R&S formulation to **prescribe** the traction evolution and the **SW** behavior within the cohesive zone.

- We can only infer an approximated value of the equivalent slip – weakening distance from the proposed scaling law. Moreover, the difference between d_0^{eq} and L depends on the adoption of a slowness (ageing) evolution equation.

Theoretical interpretations

➢ We derived analytical expressions that relate the yield and the kinetic frictional stresses to the constitutive parameters and to slip:

$$\tau = \left[\mu_* + a \ln\left(\frac{v}{v_*}\right) + b \ln\left(\frac{v_*}{v_{init}}\right) - b \frac{u}{L} \right] \sigma_n^{eff}$$

$$\tau_f^{eq} = \left[\mu_* + (b-a) \ln\left(\frac{v_*}{v_2}\right) \right] \sigma_n^{eff}$$

$$D_0^{eq} = L \, ln\left(\frac{V_0}{V_i}\right) \approx \frac{\tau_u^{eq} - \tau_f^{eq}}{b\,\sigma_n} L$$

➢ These relations hold under the assumptions that and that slip velocity is large enough to neglect the term 1 / *v*. This yields

$$\phi(u) = \frac{L}{v_{init}} e^{-\frac{u}{L}}$$

Numerical estimates of characteristic length

- Laboratory experiments:

Laboratory scale ⟺ **fault dimension ~ 20 *m* Δx ~ 0.01 m   L ~ 10 $^{-5}$ *m***

$L \sim 10^{-5}$ *m*   **$d_o^{eq} \sim 10^{-4}$ - 10^{-3} *m***

- Extending our calculations to real faults

  ✓ **Estimates of D_o from ground motions or kinematic source models range within $0.5 \leq d_o \leq 1$ *m*.**

  [*Ide and Takeo*, 1997; *Olsen et al.*, 1997; *Guatteri and Spudich*, 2000]

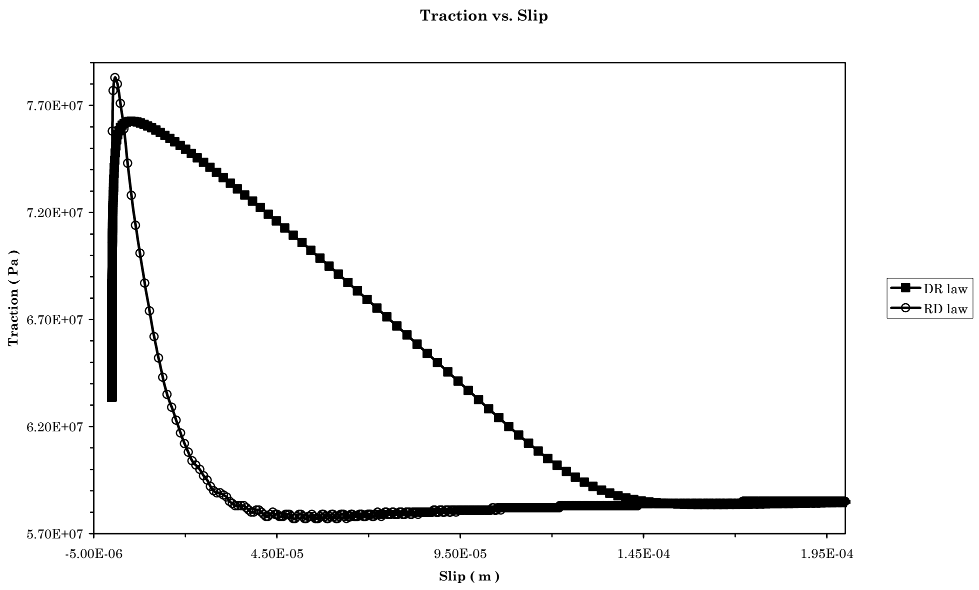
  ✓ **There is a trade - off between strength - excess and the slip weakening distance d_o [$d_o < 0.3$ *m* are not resolved]**

  ✓ **Estimate of d_o inferred from kinematic inversion models are biased due to smoothing constraints used in the inverse - problems formulation [*Guatteri and Spudich*, 2000]**

Fault scale ⟺ **fault dimension ~ 20 *km* Δx ~ 10 *m* $L \sim 10^{-2}$ *m***

$L \sim 10^{-2}$ *m* = 1 *cm*   **$d_o^{eq} \sim 10^{-1}$ *m***

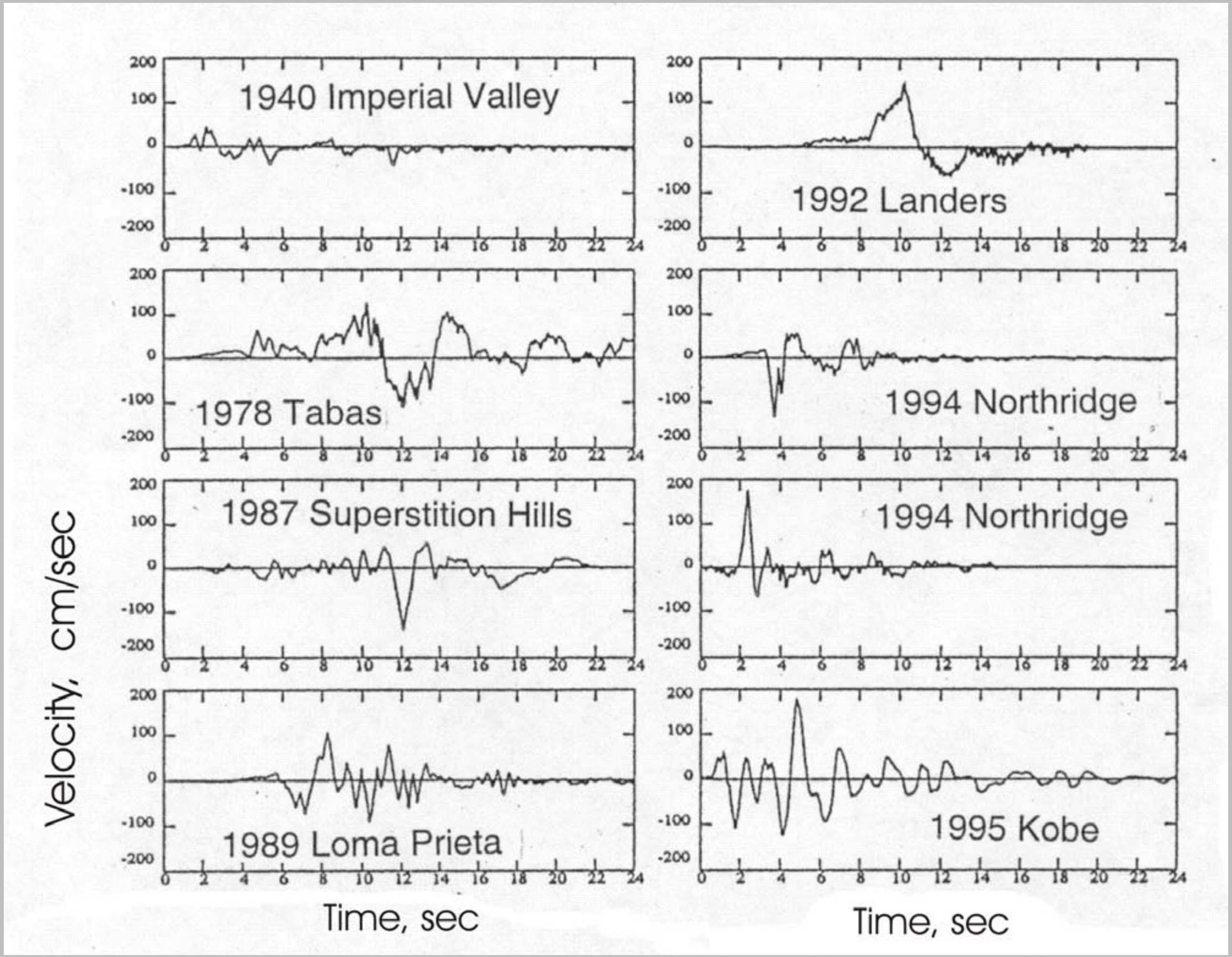
Differences between DR and RD



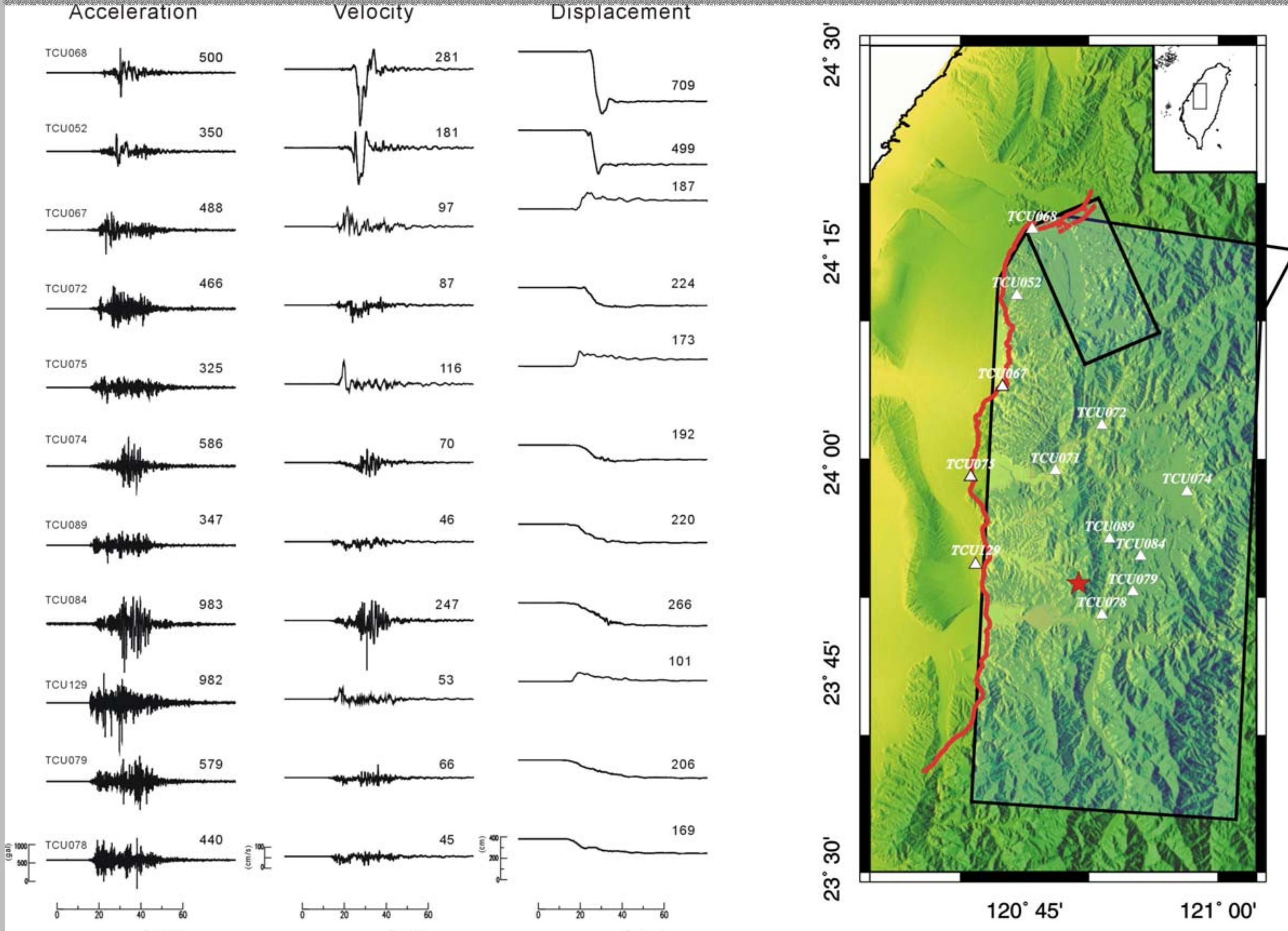
$d_0^{eq\,(RD)}$

$d_0^{eq\,(DR)}$

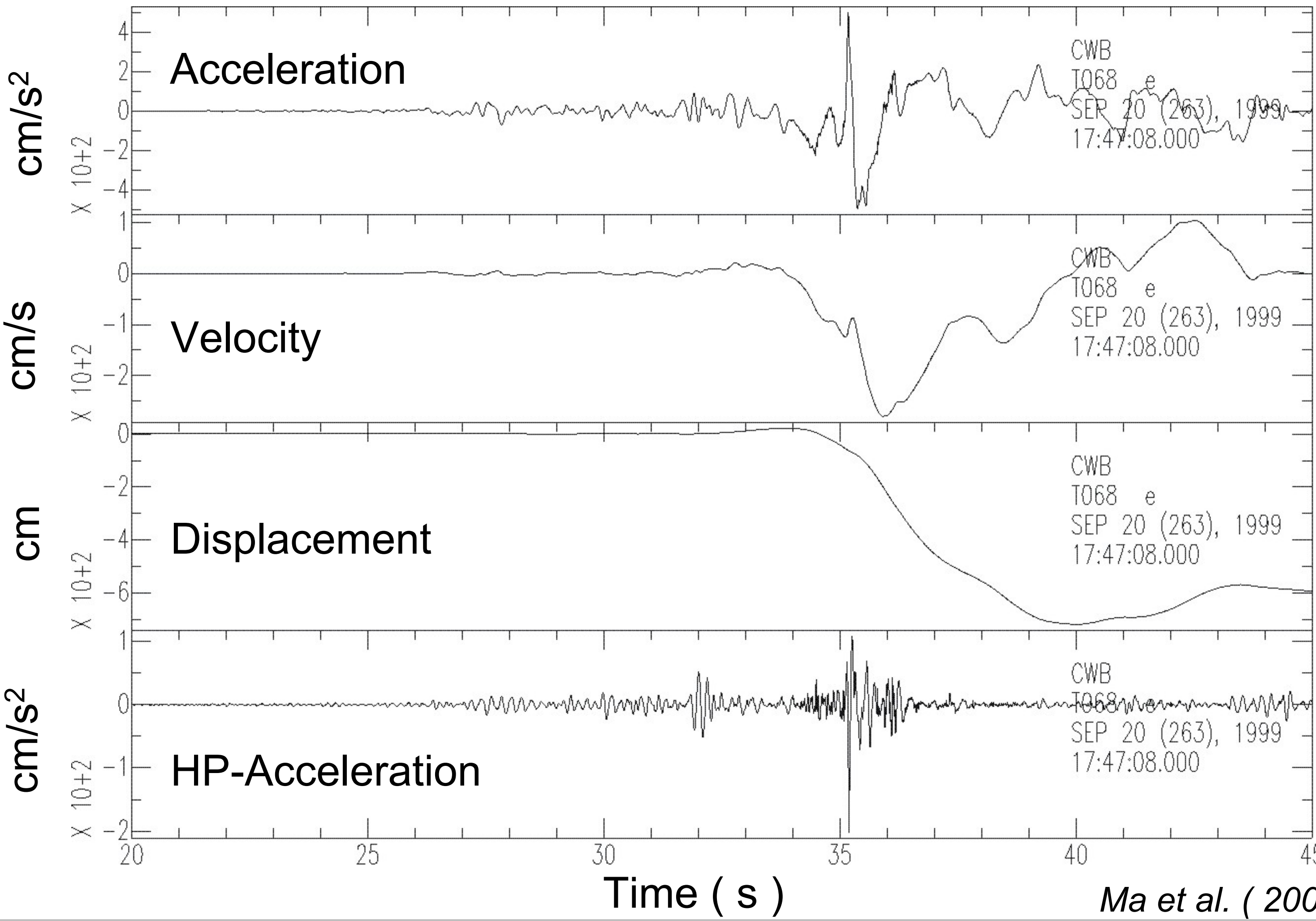
How to relate model results to physical observables



Ground motion from Chi – Chi, Taiwan, EQ



Chi – Chi, Taiwan, EQ; CWBT068



Ma et al. (2002)

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Support Slides:
Parameters, Notes, etc.

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Remembering constitutive law...

LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f)\dfrac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

Remembering constitutive law...

DIETERICH IN REDUCED FORMULATION

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} + 1 \right) + b \ln\left(\dfrac{\Psi v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi v}{L} \end{cases}$$

Remembering constitutive law...

RUINA – DIETERICH

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) \end{cases}$$