Rupture propagation in 2 – D fault models

Earthquake parameters

Observed Parameters

Seismic Moment: M_0
Radiated Energy: E_R
Rupture Speed: v_r
Source Duration (Corner Frequency): T (f_c)

Inferred (sometimes Observed) Parameters

Static Stress Drop: $\Delta\sigma_s$
Particle Motion Velocity: $\dot{U}$
Source Dimension: L
Critical Slip: D_c

Numerical Method: BIE 2 - D

We solve the fundamental elastodynamic equation, neglecting body forces ***f***

$$\rho \ddot{U}_i = \sigma_{ij,j} + \cancel{f_i}$$

Source integral rapresentation (*Betti*' s theorem, Integration in time limit in fault surface, Lamb' s problem):

$$u_n(\mathbf{x},t)=\int_{-\infty}^{+\infty} \mathrm{d}t' \int_{S(t')} \mathrm{d}\xi\, G_{n\alpha}(\mathbf{x}-\xi, t-t')\, \sigma_{\alpha 3}^{\,p}(\xi,t') \;;n=1,2,3;\alpha=1,2;\mathbf{x},\xi\in\mathbb{R}^3$$

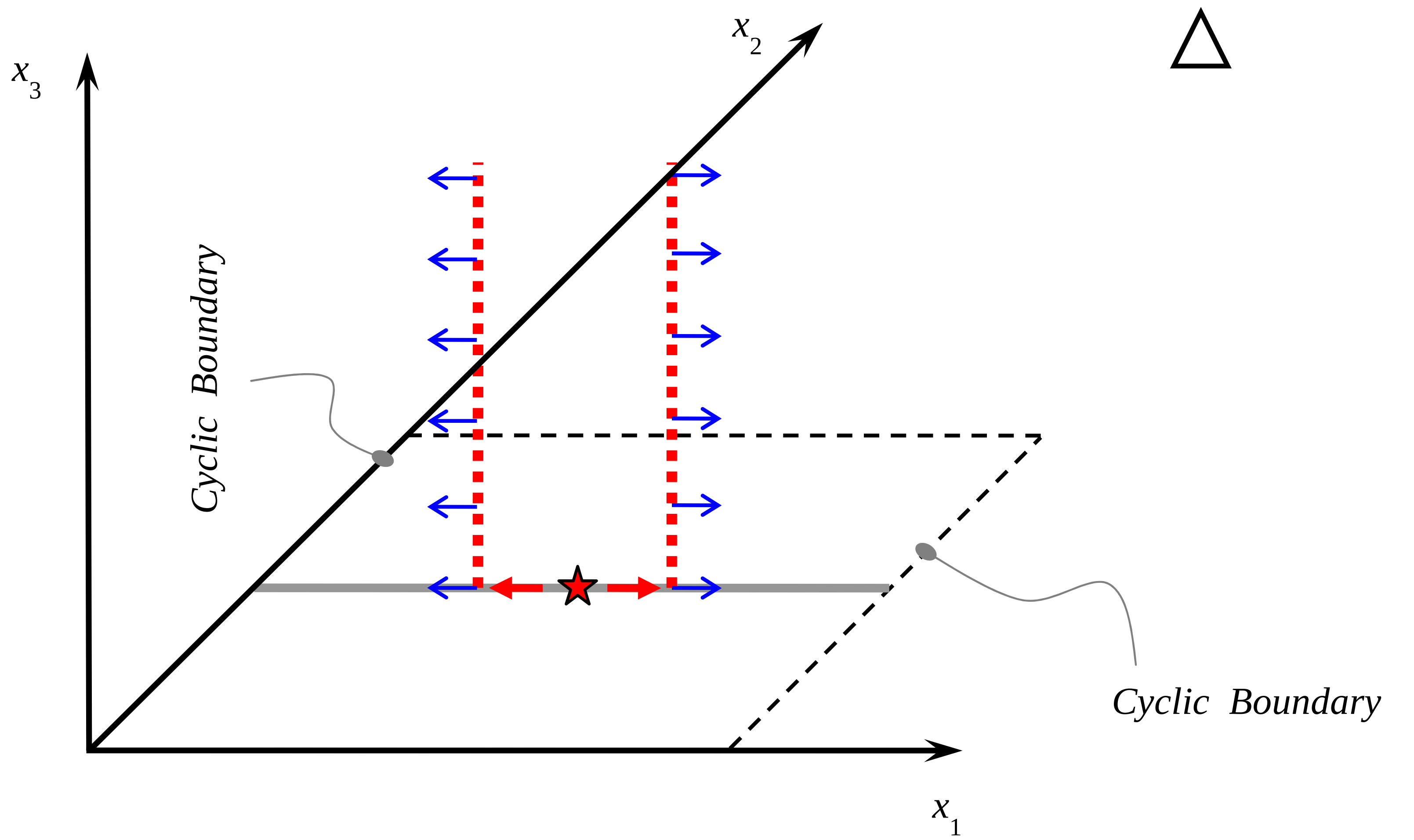
First neighbours decoupling (in the case of a 2 – D, pure in – plane rupture):

Traction

$$\begin{cases} u_1(x_1,t) + C\,\tau_1^{\,p}(x_1,t) = \mathcal{I}_1(x_1,t) \\ \tau_{0_1} + \tau_1^{\,p}(x_1,t) = \mu\sigma_n^{\,eff} \end{cases}$$

Friction

Numerical Method: FD 2 - D



$\boldsymbol{u} = (u_1(x_1, t), 0, 0)$

2-D

We solve the fundamental elastodynamic equation, neglecting body forces ***f***

$$\rho\ddot{u}_i = \sigma_{ij,j} + f_i$$

We discretize the x_1x_2 plane by using triangular cells (better performances)

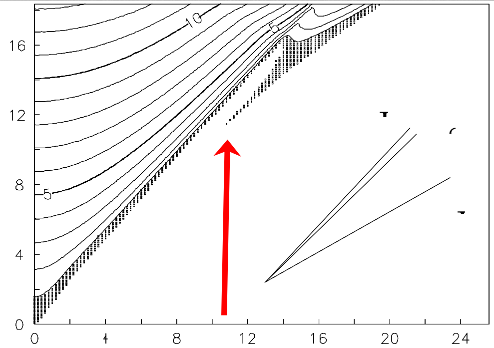
$$\rho \frac{\partial}{\partial t}\dot{u}_1 = \frac{\partial}{\partial x_1}\Sigma_{11} + \frac{\partial}{\partial x_2}\Sigma_{12}$$
$$\rho \frac{\partial}{\partial t}\dot{u}_2 = \frac{\partial}{\partial x_1}\Sigma_{12} + \frac{\partial}{\partial x_2}\Sigma_{22}$$

The plane is linear and elastic except in the fault intersection line, where a Fault Boundary Condition (TSN scheme) is adopted. In tihs line a constitutiva law is assumed to relate staggered stress with observables (slip, slip velocity, …)

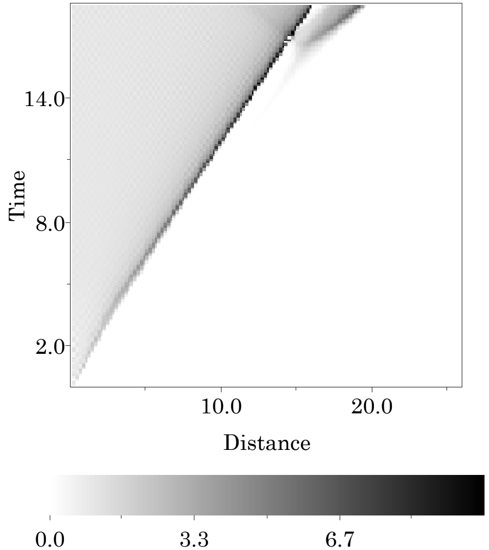
BIE vs. FD with SW #1

2-D

BIE



(a)



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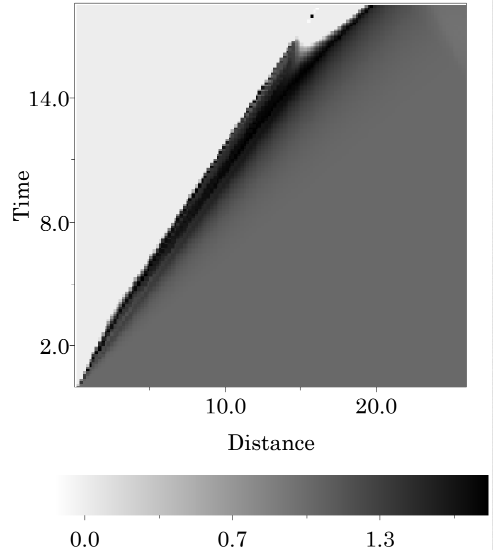

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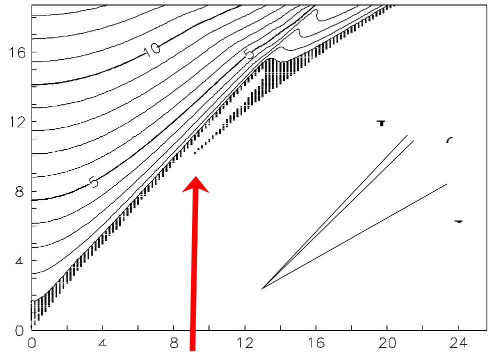
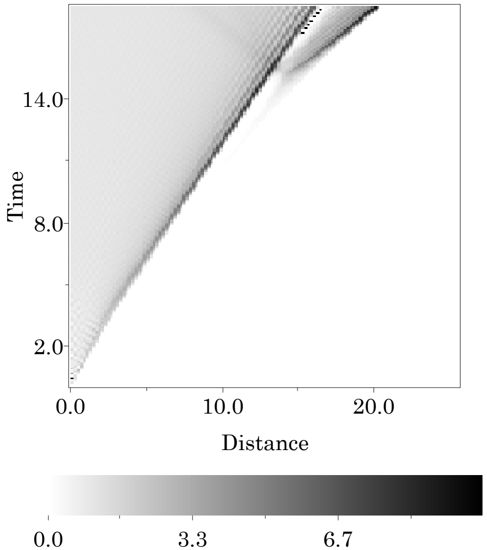
FD



(d)



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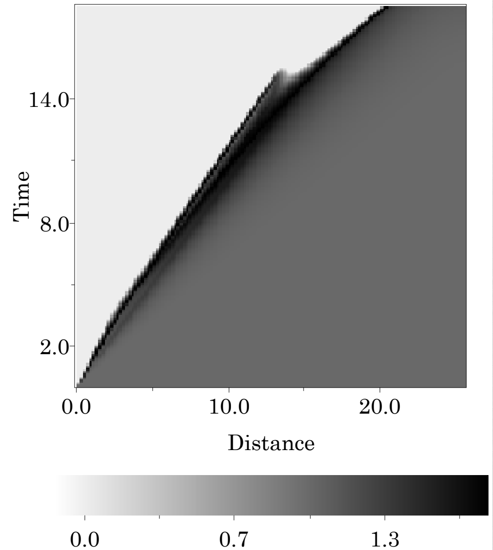
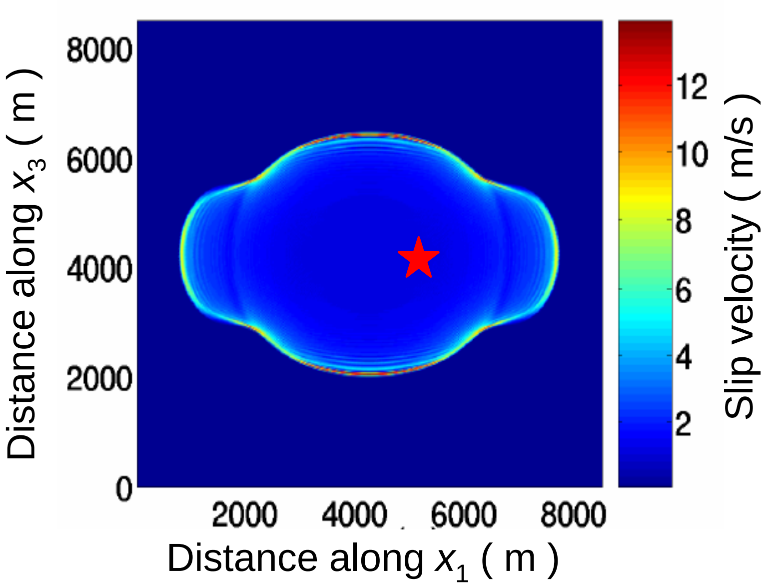


(f)

The cohesive zone

Time snapshot (t = 0.8 s) - SW law



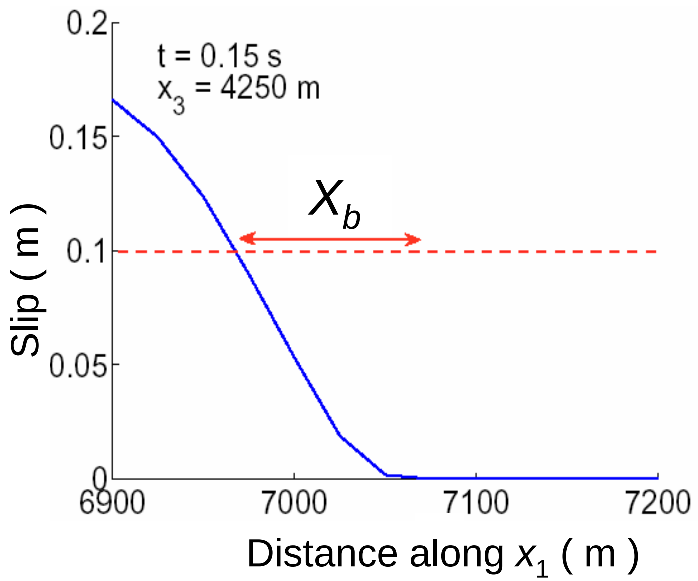
In the target location we can extimate:

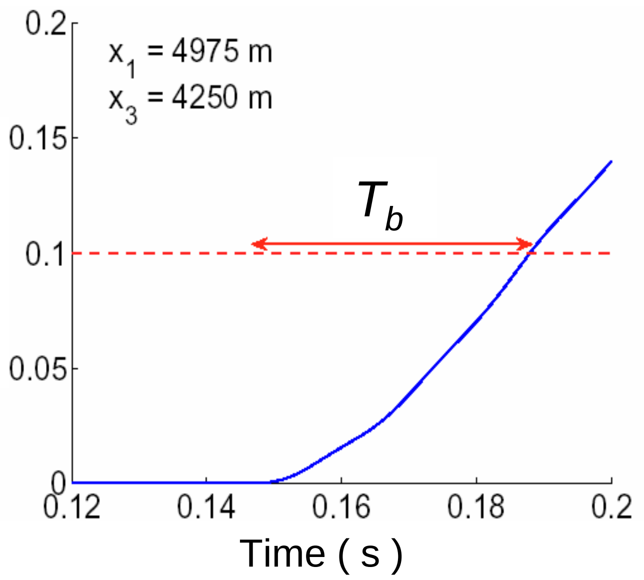
X_b = 105 m T_b = 0.04 s

From these quantities:

$v_{rupt} = X_b/T_b$ = 2625 m/s

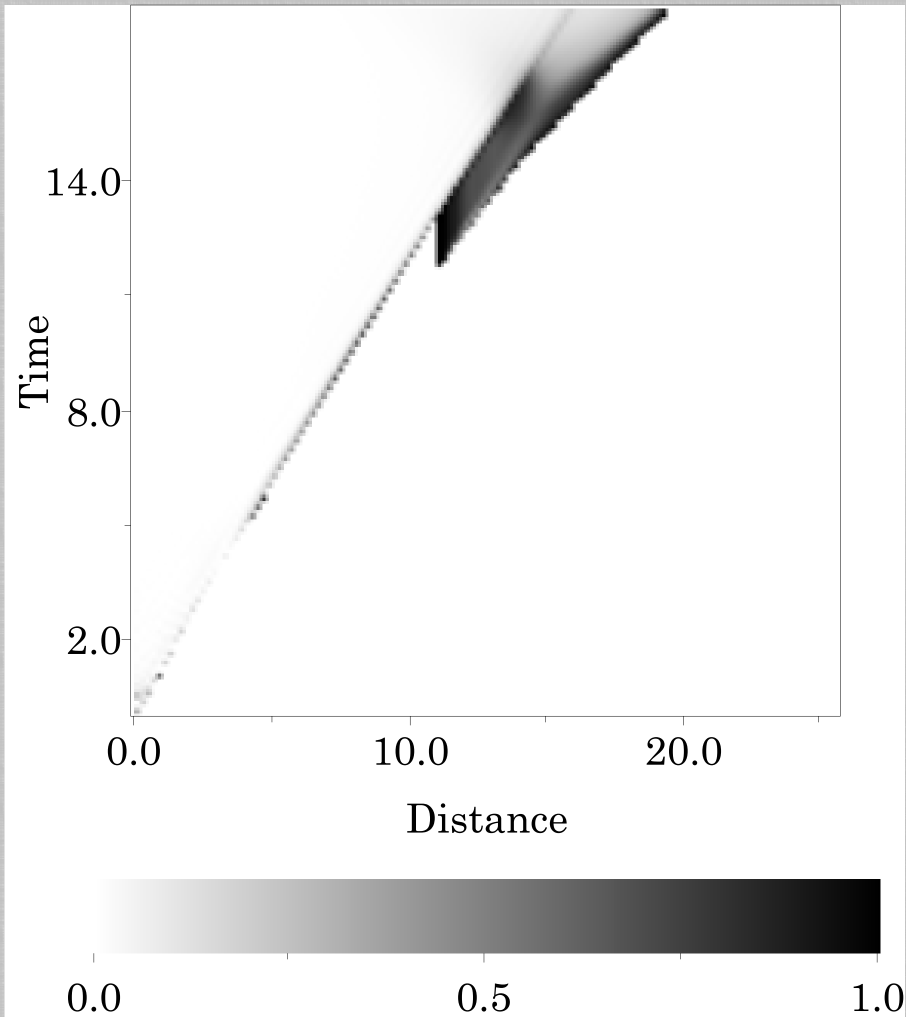
Local estimate



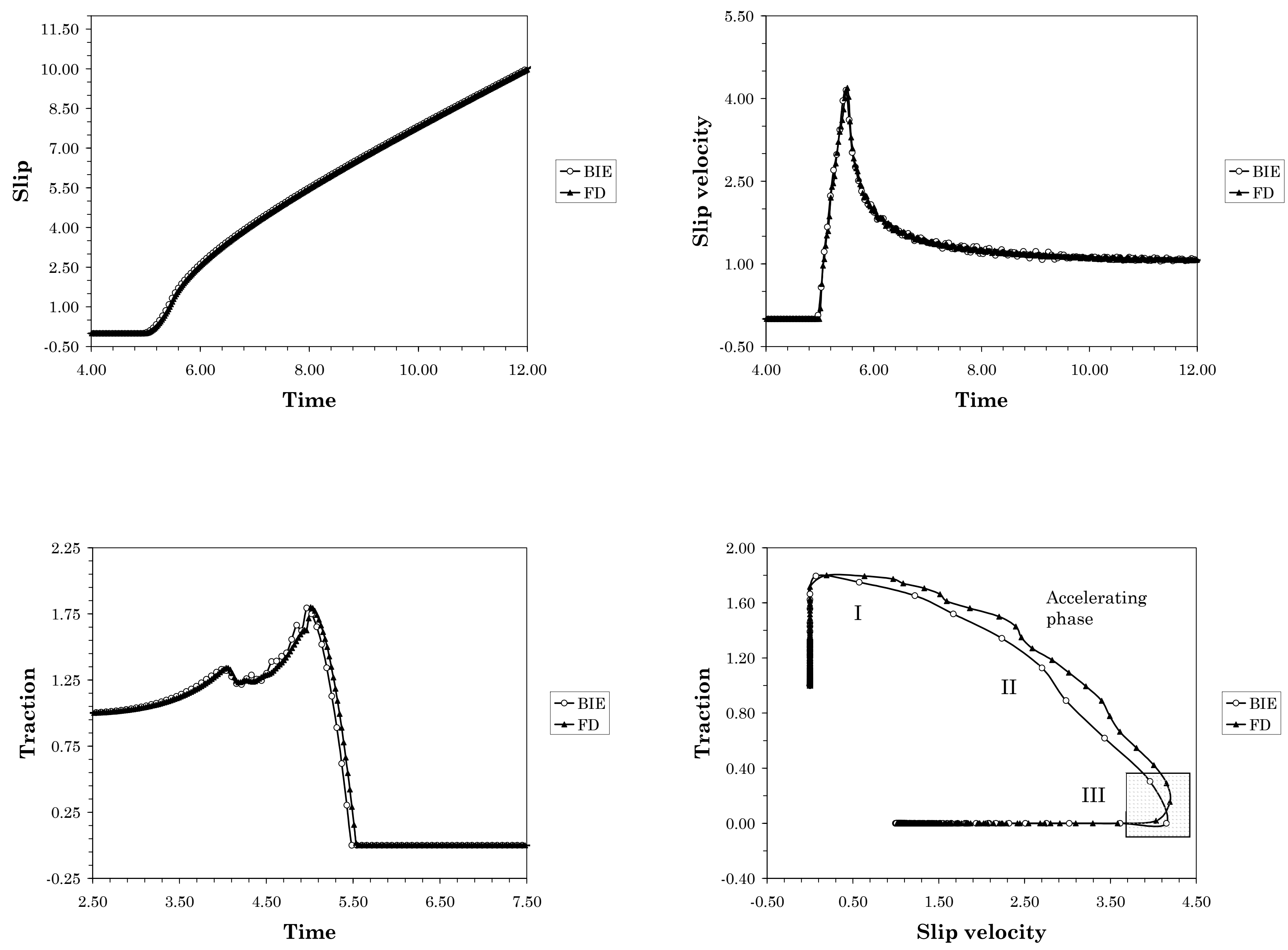


Misfit between slip modeled with BIE and FD

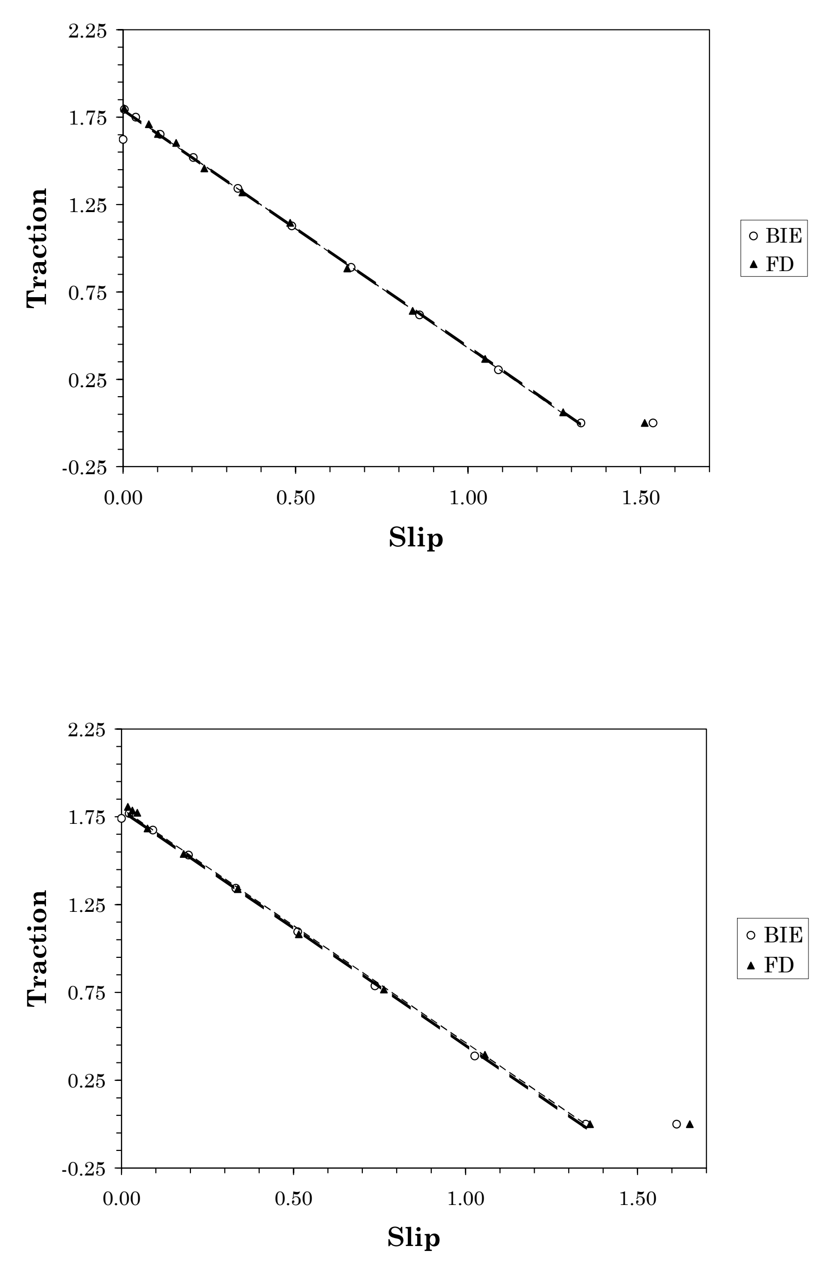
$$m(x_i, t_n) = \frac{\left| u^{(\mathrm{BIE})}(x_i, t_n) - \tilde{u}^{(\mathrm{FD})}(x_i, t_n) \right|}{\left| u^{(\mathrm{BIE})}(x_i, t_n) + \tilde{u}^{(\mathrm{FD})}(x_i, t_n) \right|}$$



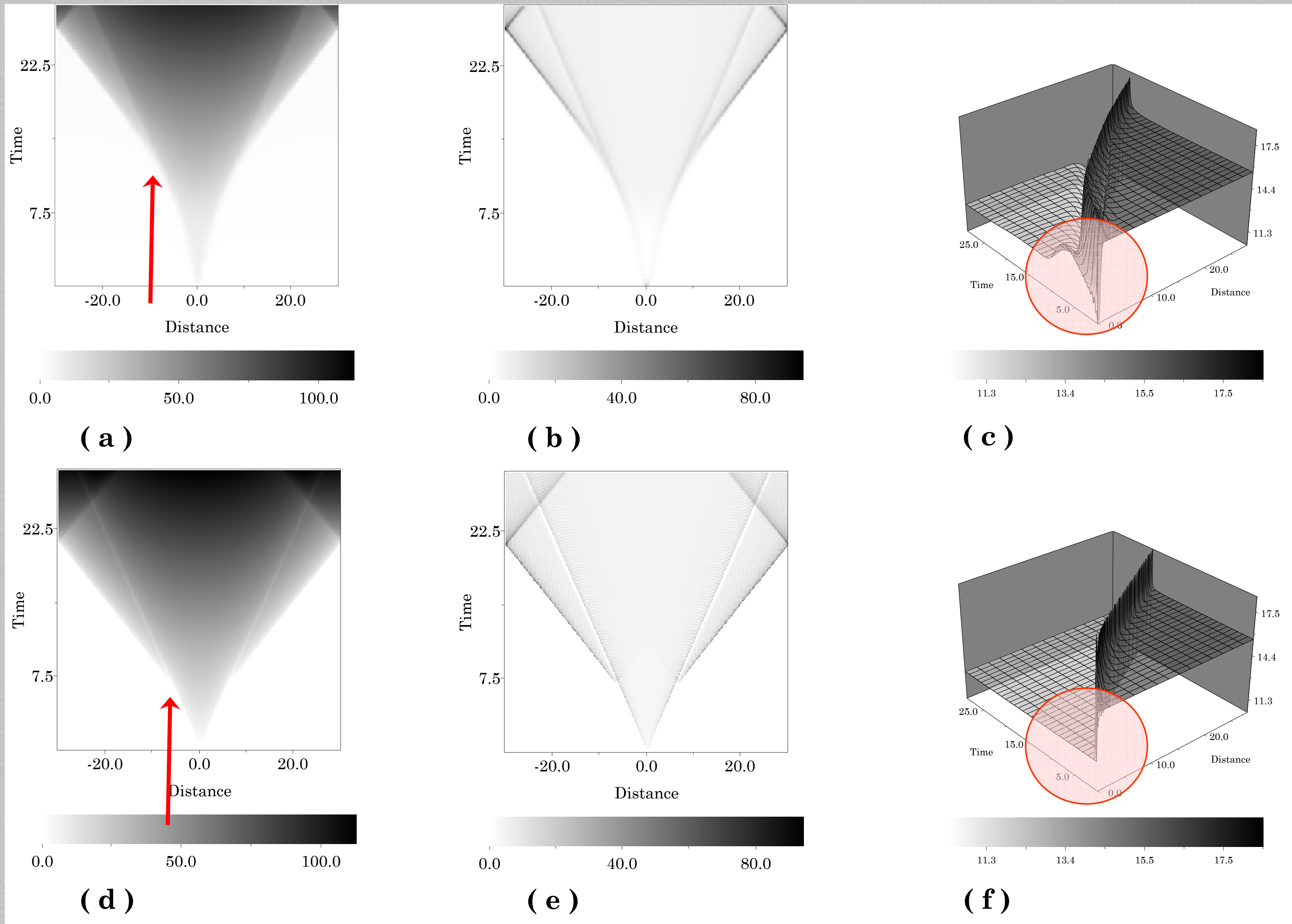
BIE vs. FD with SW #2



BIE vs. FD with SW #3

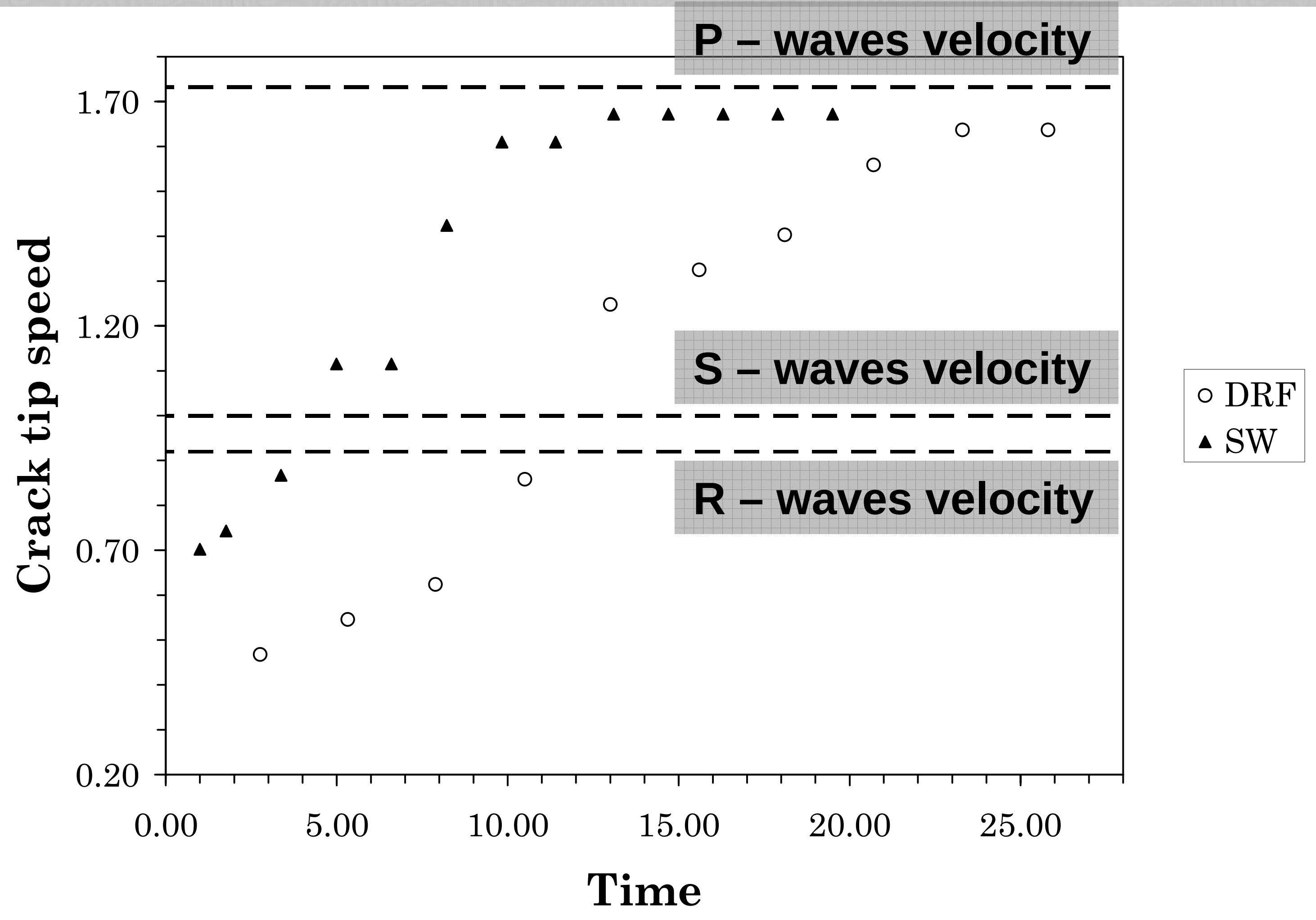


SW vs. DR law #1



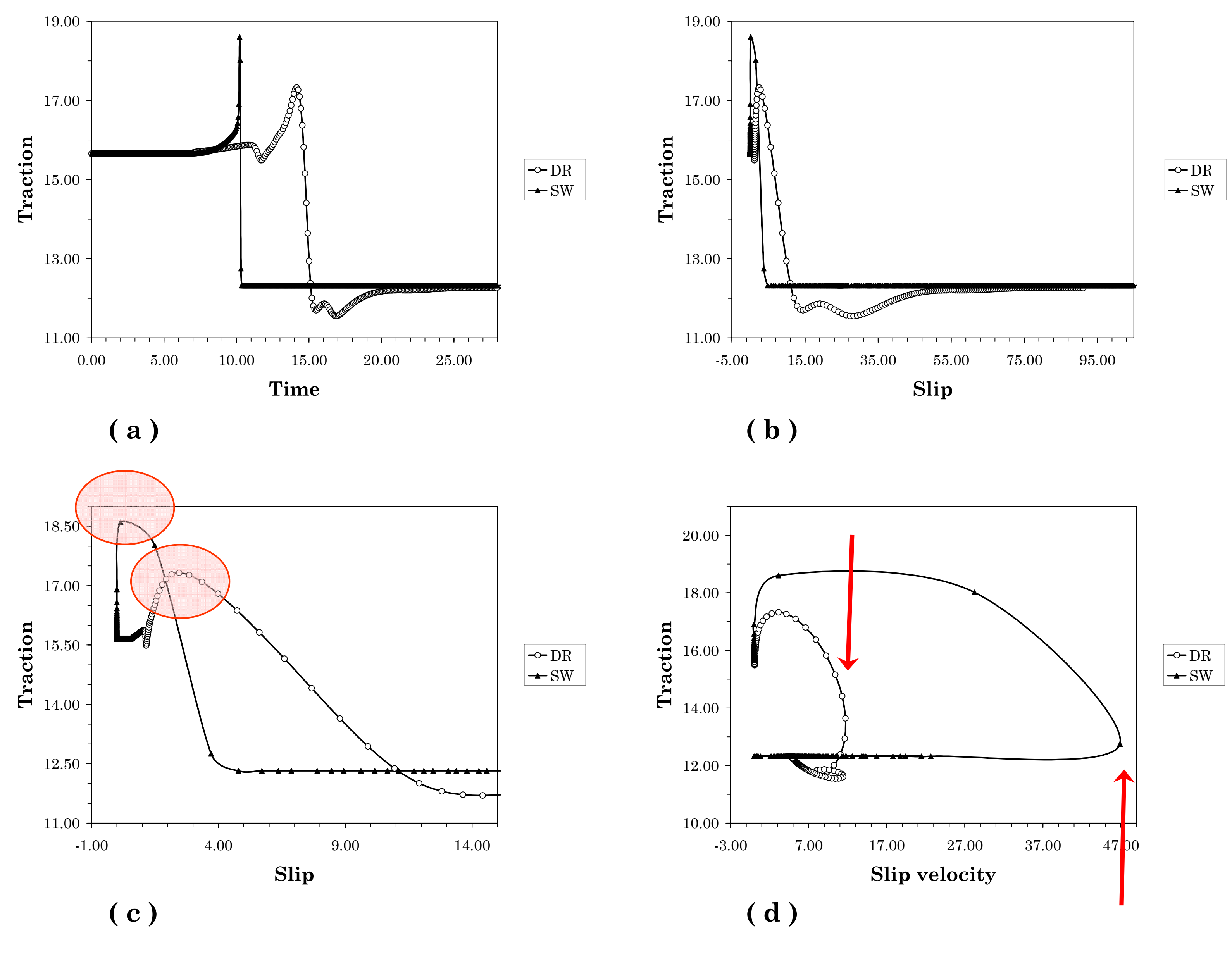
22.5
7.5
Time
-20.0
0.0
20.0
Distance
0.0
50.0
100.0
(a)
0.0
40.0
80.0
(b)
25.0
15.0
5.0
0.0
10.0
20.0
17.5
14.4
11.3
11.3
13.4
15.5
17.5
(c)
(d)
(e)
(f)

SW vs. DR law #2



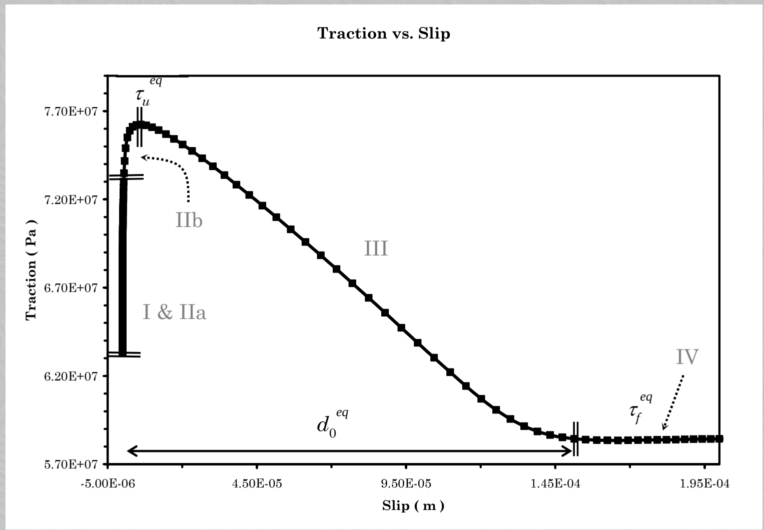
P – waves velocity
S – waves velocity
R – waves velocity
Crack tip speed
1.70
1.20
0.70
0.20
0.00
5.00
10.00
15.00
20.00
25.00
Time
DRF
SW

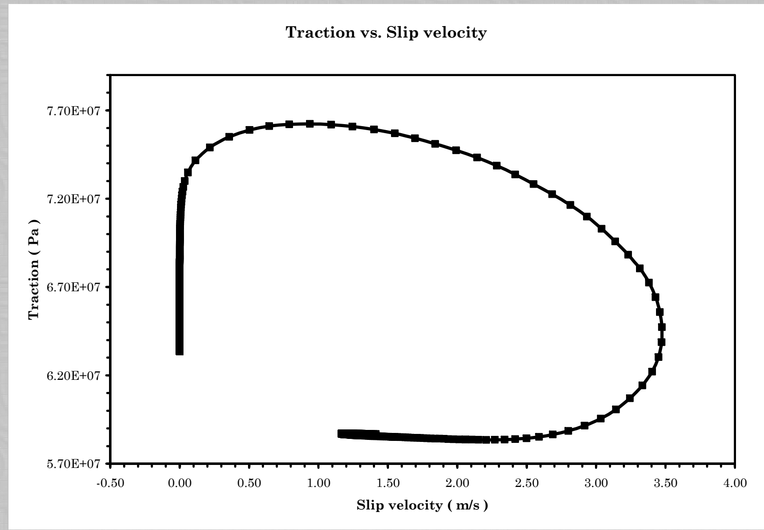
SW vs. DR law #3

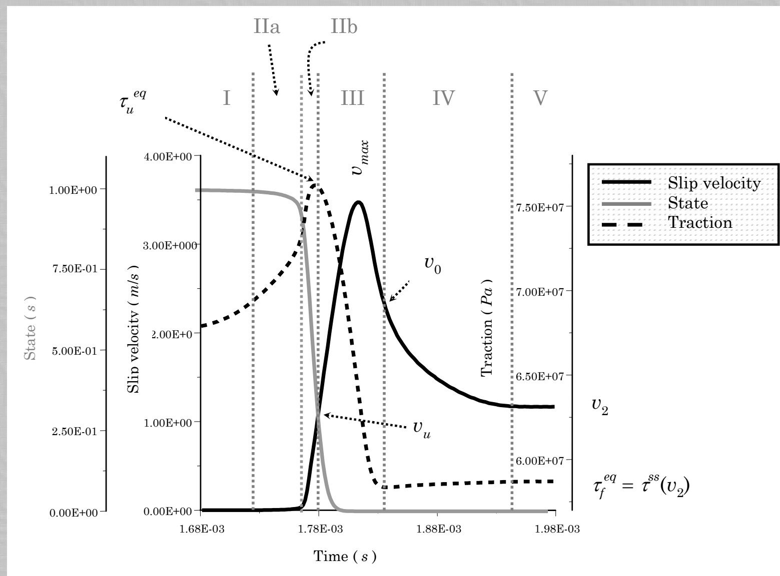


Traction
Time
DR
SW
(a)
Traction
Slip
DR
SW
(b)
Traction
Slip
DR
SW
(c)
Traction
Slip velocity
DR
SW
(d)

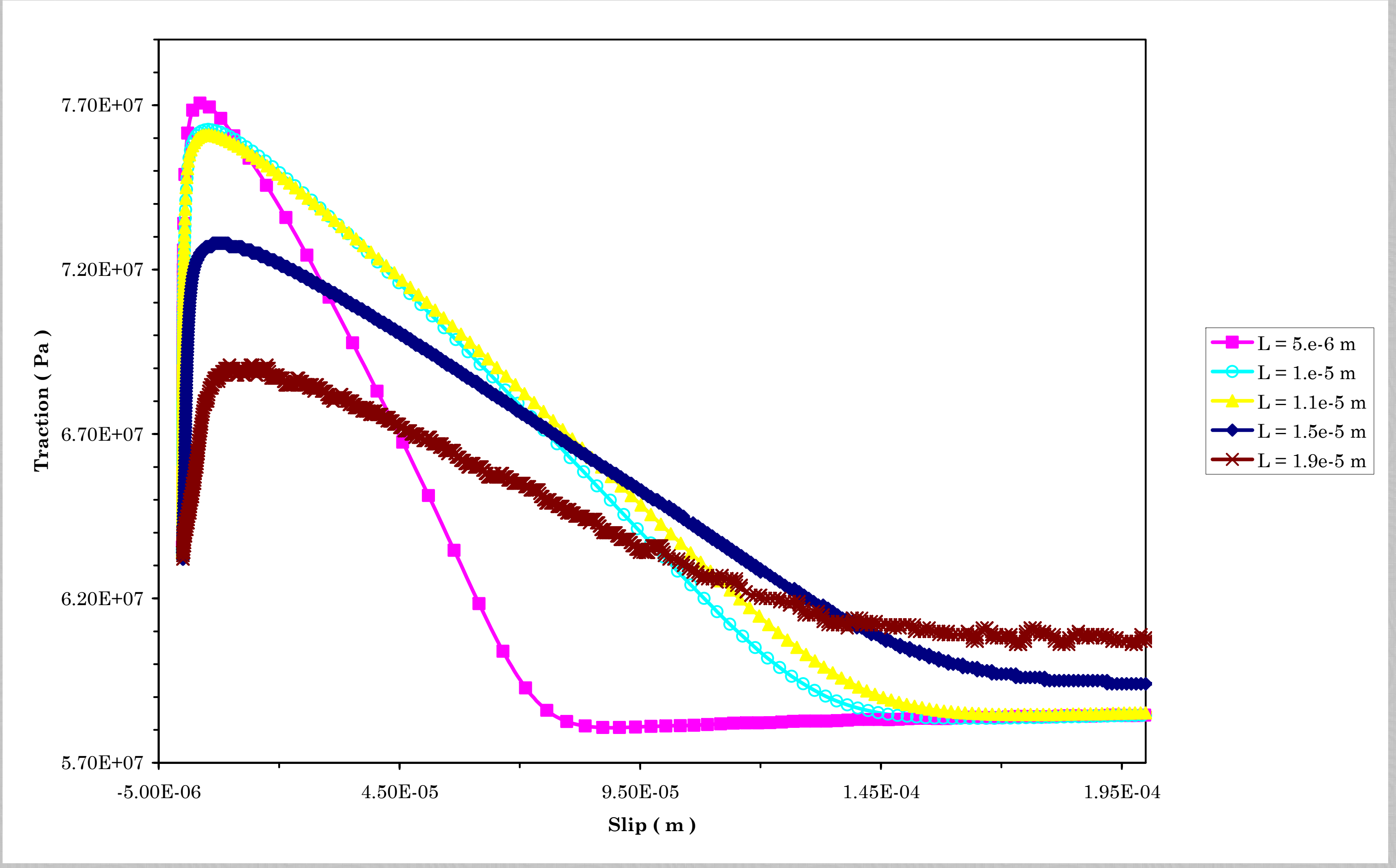
The dynamic propagation. The cohesive zone and the breakdown





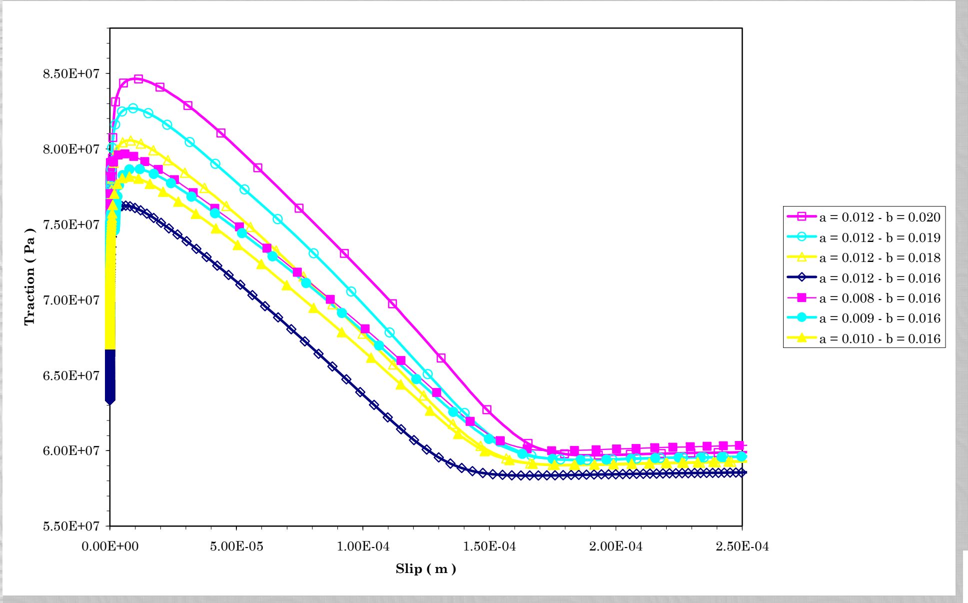


Dependence on *L* parameter

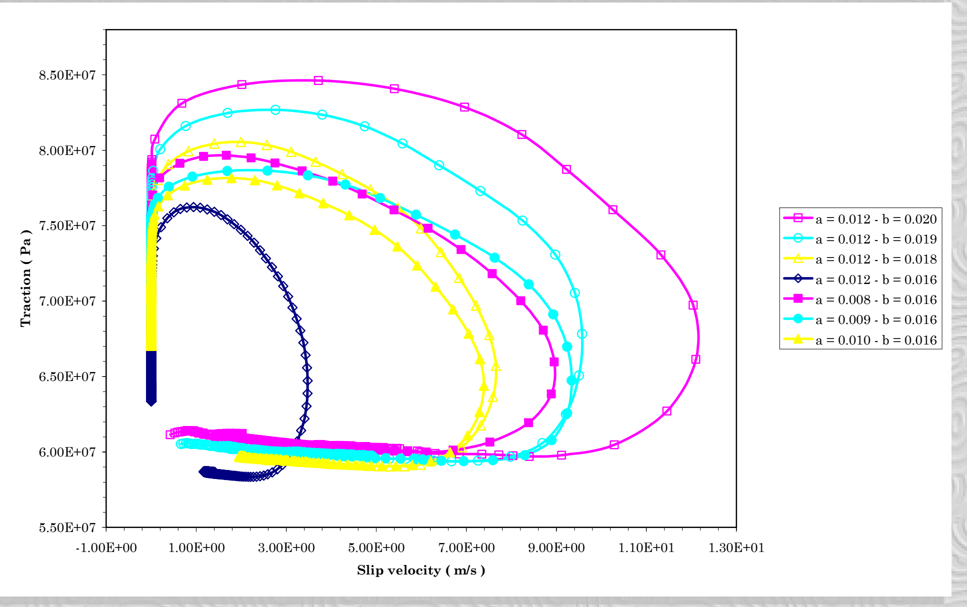


Dependence on *a* and *b*

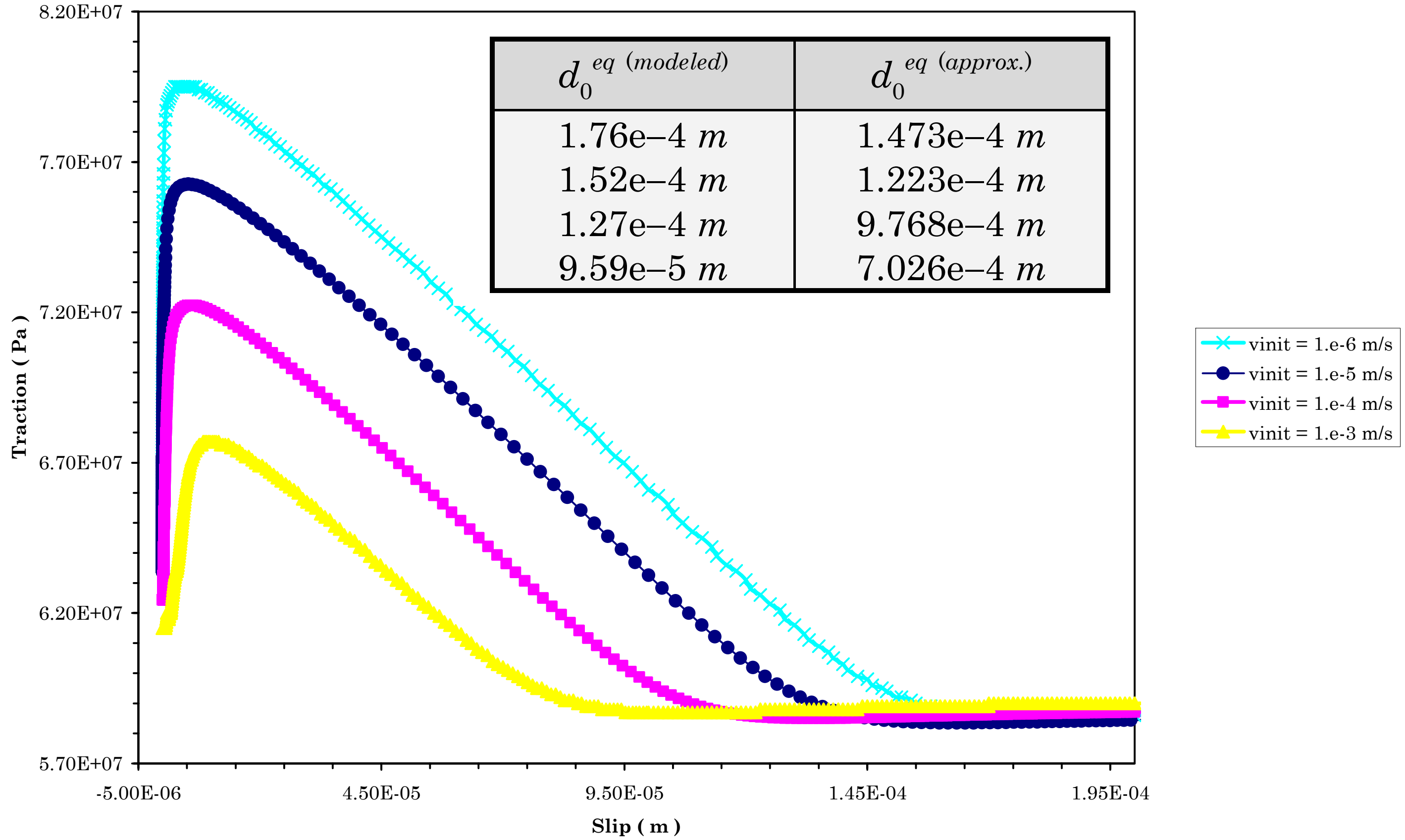
Slip – weakening curves



Phase portraits



Dependence on the initial velocity



d_0^{eq} (modeled)	d_0^{eq} (approx.)
1.76e–4 m	1.473e–4 m
1.52e–4 m	1.223e–4 m
1.27e–4 m	9.768e–4 m
9.59e–5 m	7.026e–4 m

Limitation for modeling dynamic Rupture and seismic vawe genration

- Because the initial slip velocity is totally arbitrary, it is difficult in the framework of R&S formulation to **prescribe** the traction evolution and the **SW** behavior within the cohesive zone.

- We can only infer an approximated value of the equivalent slip – weakening distance from the proposed scaling law. Moreover, the difference between d_0^{eq} and L depends on the adoption of a slowness (ageing) evolution equation.

Theoretical interpretations

➢ We derived analytical expressions that relate the yield and the kinetic frictional stresses to the constitutive parameters and to slip:

$$\tau = \left[\mu_* + a \ln\left(\frac{v}{v_*}\right) + b \ln\left(\frac{v_*}{v_{init}}\right) - b\frac{u}{L} \right] \sigma_n^{eff}$$

$$\tau_f^{eq} = \left[\mu_* + (b - a) \ln\left(\frac{v_*}{v_2}\right) \right] \sigma_n^{eff}$$

$$D_0^{eq} = L \, ln\left(\frac{V_0}{V_i}\right) \approx \frac{\tau_u^{eq} - \tau_f^{eq}}{b \, \sigma_n} L$$

➢ These relations hold under the assumptions that and that slip velocity is large enough to neglect the term 1 / *v*. This yields

$$\phi(u) = \frac{L}{v_{init}} e^{-\frac{u}{L}}$$

Numerical estimates of characteristic length

- Laboratory experiments:

Laboratory scale ⟺ **fault dimension ~ 20 *m*   *Δx* ~ 0.01 m   L ~ 10 $^{-5}$ *m***

$L \sim 10^{-5}$ *m*   **$d_o^{eq} \sim 10^{-4} - 10^{-3}$ *m***

- Extending our calculations to real faults

  ✓ **Estimates of D_o from ground motions or kinematic source models range within $0.5 \leq d_o \leq 1$ *m*.**

  [*Ide and Takeo*, 1997; *Olsen et al.*, 1997; *Guatteri and Spudich*, 2000]

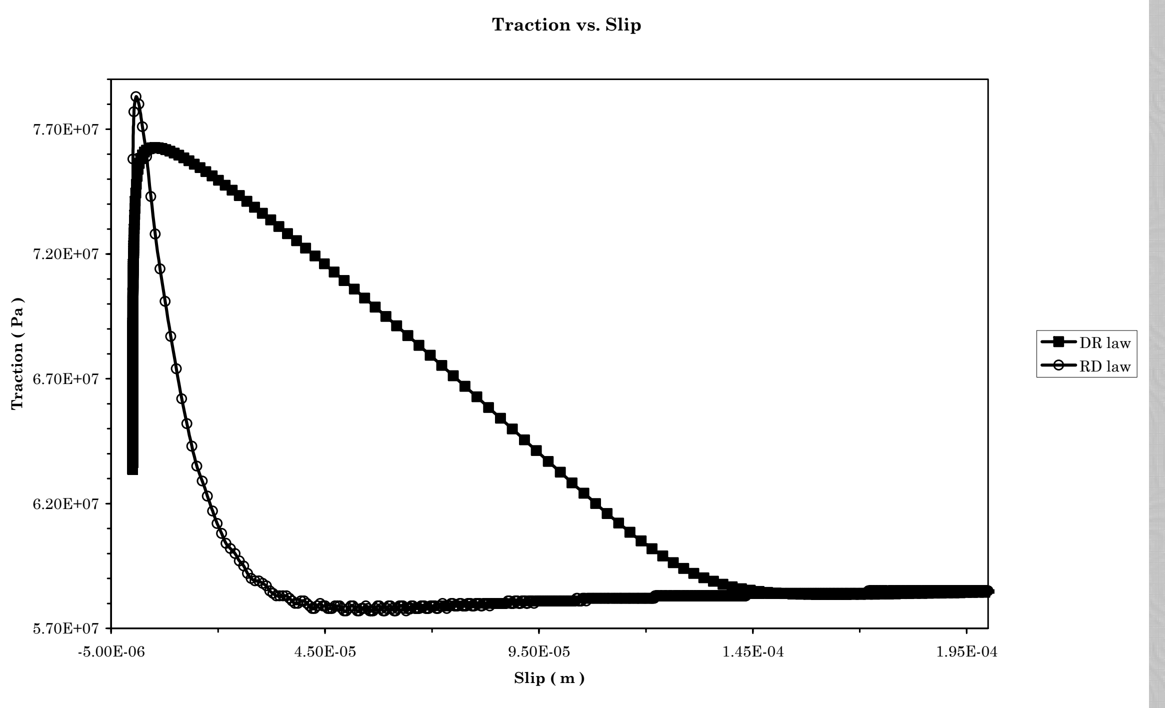
  ✓ **There is a trade - off between strength - excess and the slip weakening distance d_o [$d_o < 0.3$ *m* are not resolved]**

  ✓ **Estimate of d_o inferred from kinematic inversion models are biased due to smoothing constraints used in the inverse - problems formulation [*Guatteri and Spudich*, 2000]**

Fault scale ⟺ **fault dimension ~ 20 *km*   *Δx* ~ 10 *m*   *L* ~ 10 $^{-2}$ *m***

$L \sim 10^{-2}$ *m* = 1 *cm*   **$d_o^{eq} \sim 10^{-1}$ *m***

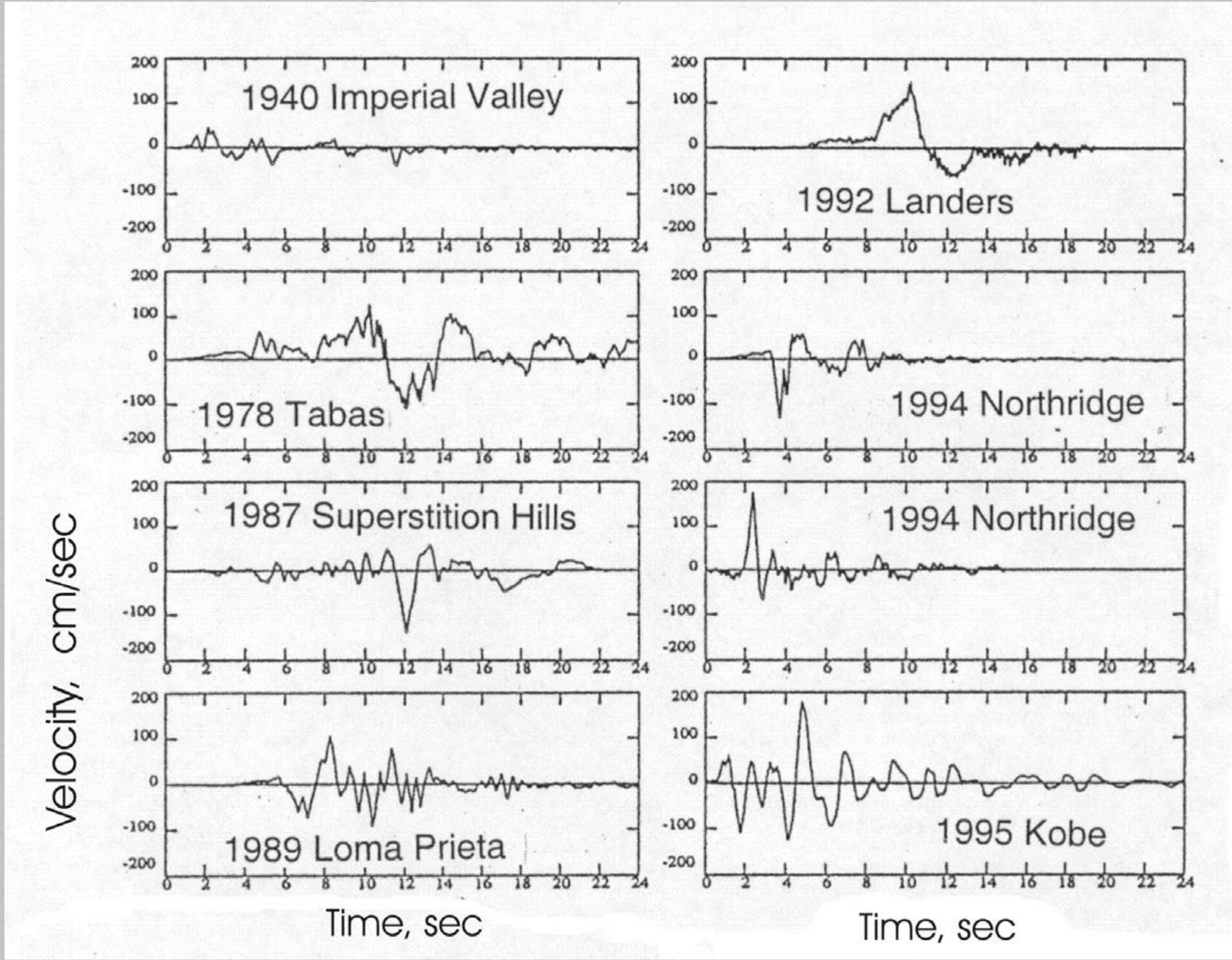
Differences between DR and RD



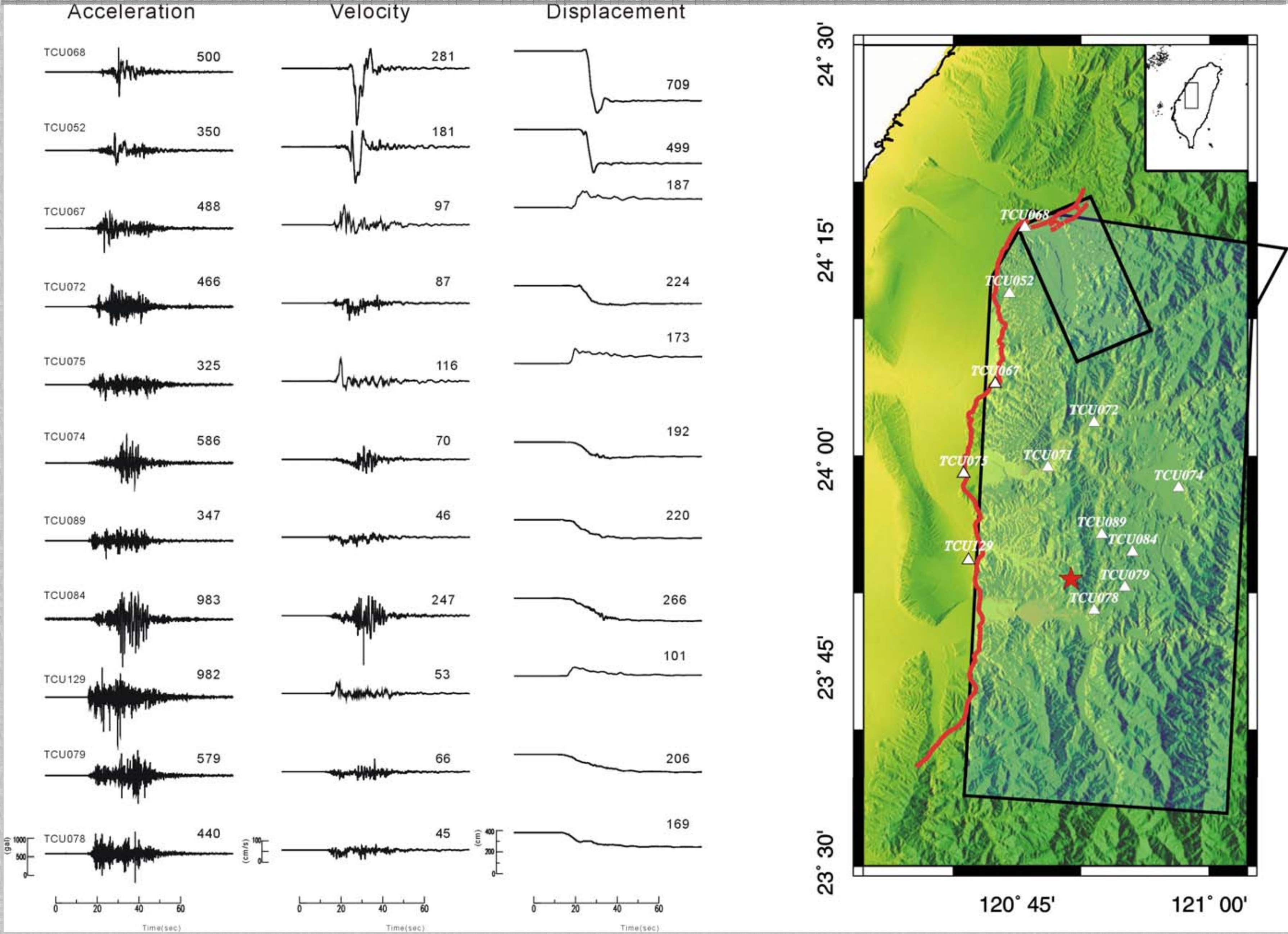
$d_0^{eq\,(RD)}$

$d_0^{eq\,(DR)}$

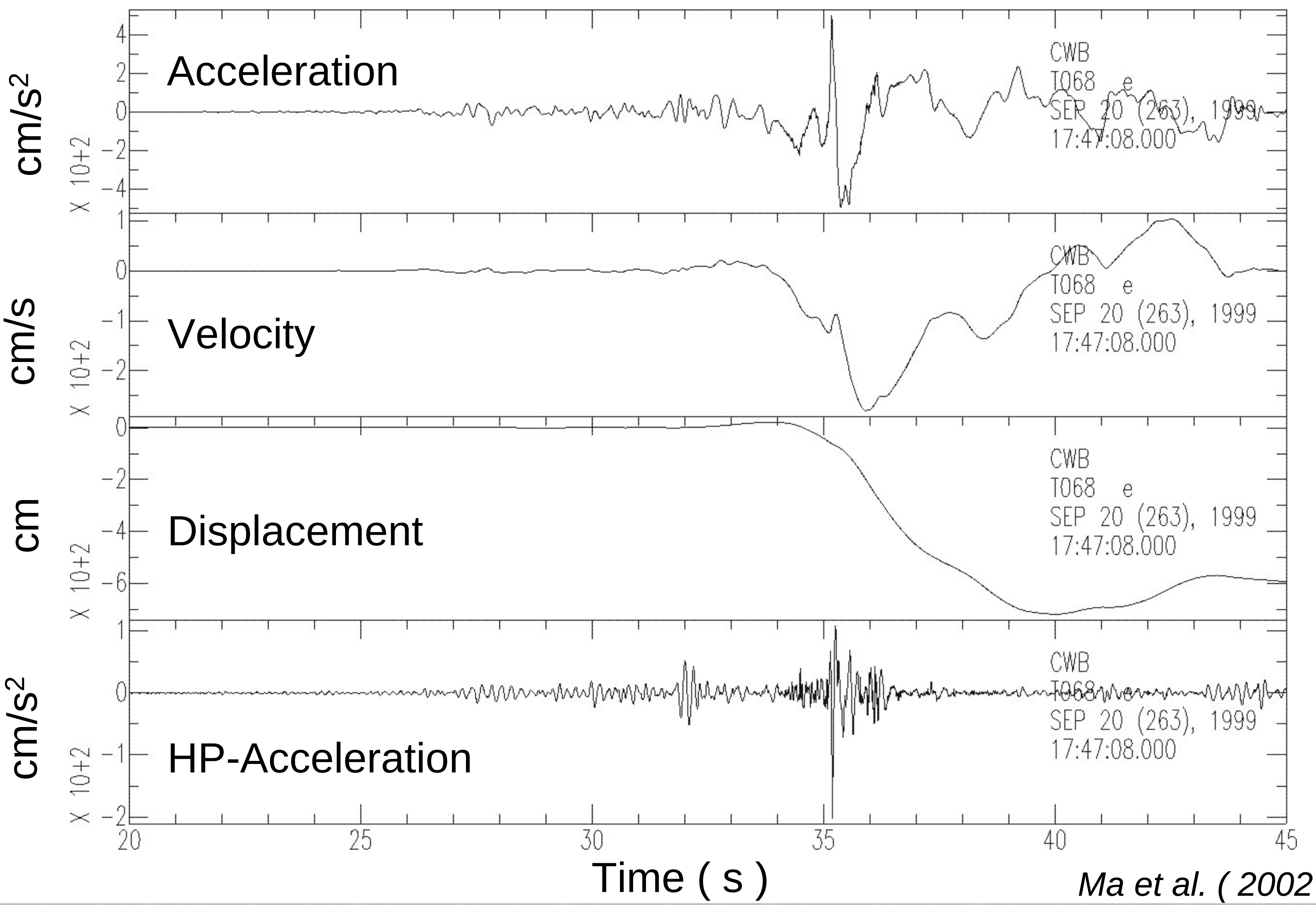
How to relate model results to physical observables



Ground motion from Chi – Chi, Taiwan, EQ



Chi – Chi, Taiwan, EQ; CWBT068



Ma et al. (2002)

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Support Slides:
Parameters, Notes, etc.

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Remembering constitutive law...

LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

Remembering constitutive law...

DIETERICH IN REDUCED FORMULATION

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} + 1 \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi\, v}{L} \end{cases}$$

Remembering constitutive law...

RUINA – DIETERICH

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n{}^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) \end{cases}$$