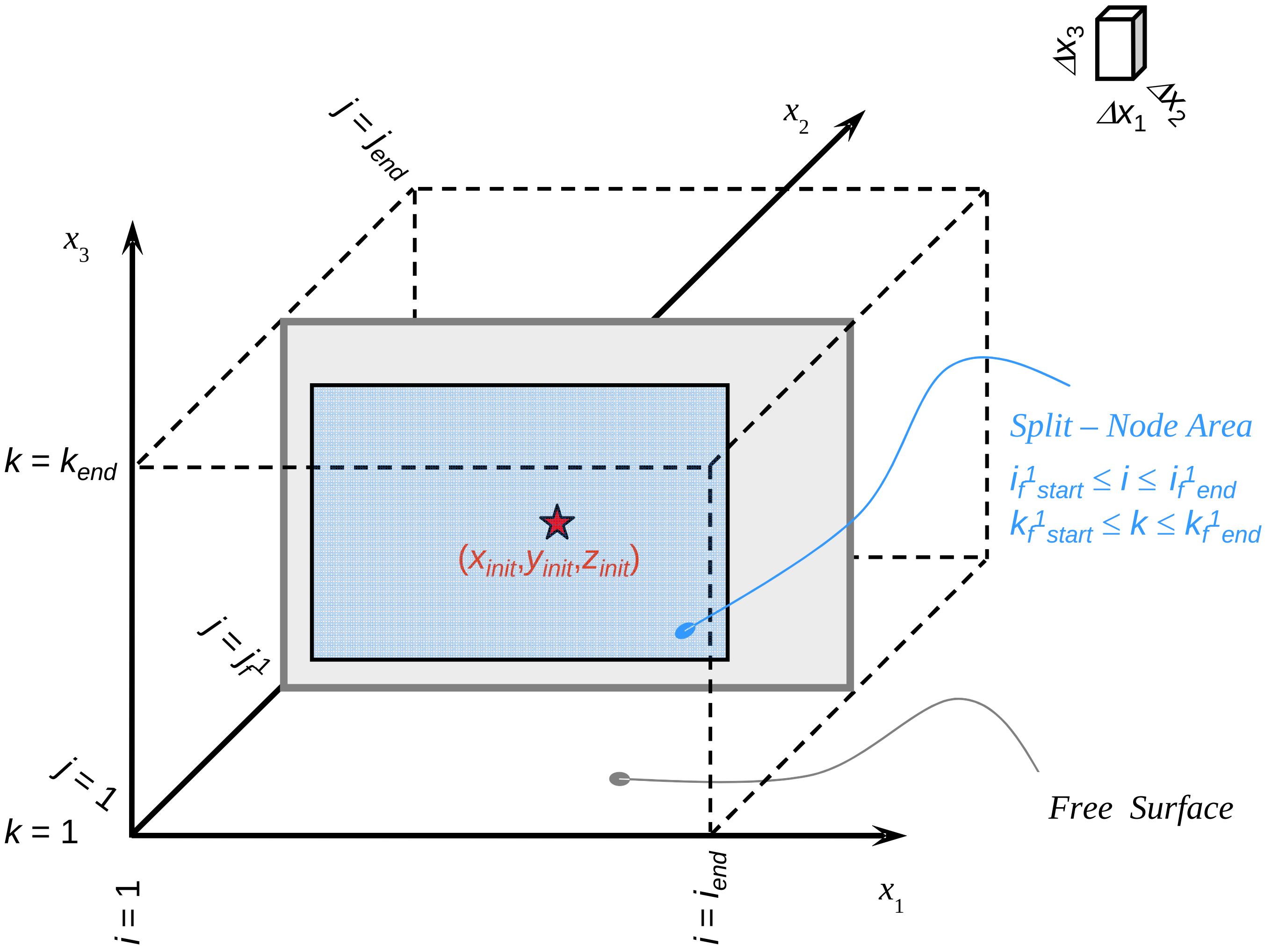
Rupture propagation in a *truly* 3 – D fault model

Numerical Method: FD 3 - D



$$\boldsymbol{u} = (\, u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t)\,)$$

In the assumed fault geometry, on a generic fault point (defined by the absolute coordinate (x_1, x_2^f, x_3)), at time *t*, the traction vector is:

$$\mathcal{T} = (\sigma_{21}, -\sigma_n^{eff}, \sigma_{23})$$

where:

$\sigma_n^{eff} = \sigma_n - p_{fluid}$	effective normal stress (normal stresses are negative for compression)
$\sigma_n = -\sigma_{22}$	is the regional normal stress (e. g. lithostatic stress: $\sigma_{22} = -p_0 \delta_{22} = -\rho\, g\, x_3$)
σ_{21}, σ_{23}	(shear stresses, associated to the adopted fault constitutive law)

3-D

In the assumed fault geometry, on a generic medium point (defined by the absolute coordinate (x_1,x_2,x_3)), at time *t,* the stress tensor matrix is:

$$\sigma_{ij}(x_1,x_2,x_3,t)=\lambda\, e_{kk}(x_1,x_2,x_3,t)\delta_{ij}+2\mu\, e_{ij}(x_1,x_2,x_3,t)$$

(i. e. the Hooke’ s law for a linealry homogeneous, isotropic medium, within the small displacement approximation)

where:

$$e_{ij} = ½\ (U_{i,j} + U_{j,i})$$

is calculated from the displacement field **U**, generated by the rupture propagation on the fault surface Σ.

We solve the fundamental elastodynamic equation, neglecting body forces **f**

$$\rho \ddot{U}_i = \sigma_{ij,j} + \cancel{f_i}$$

We discretize the volume in $x_1x_2x_3$ space by using cubic building blocks. The space is linearly elastic except that in **6 planes**, representing 4 dipping and 2 vertical faults

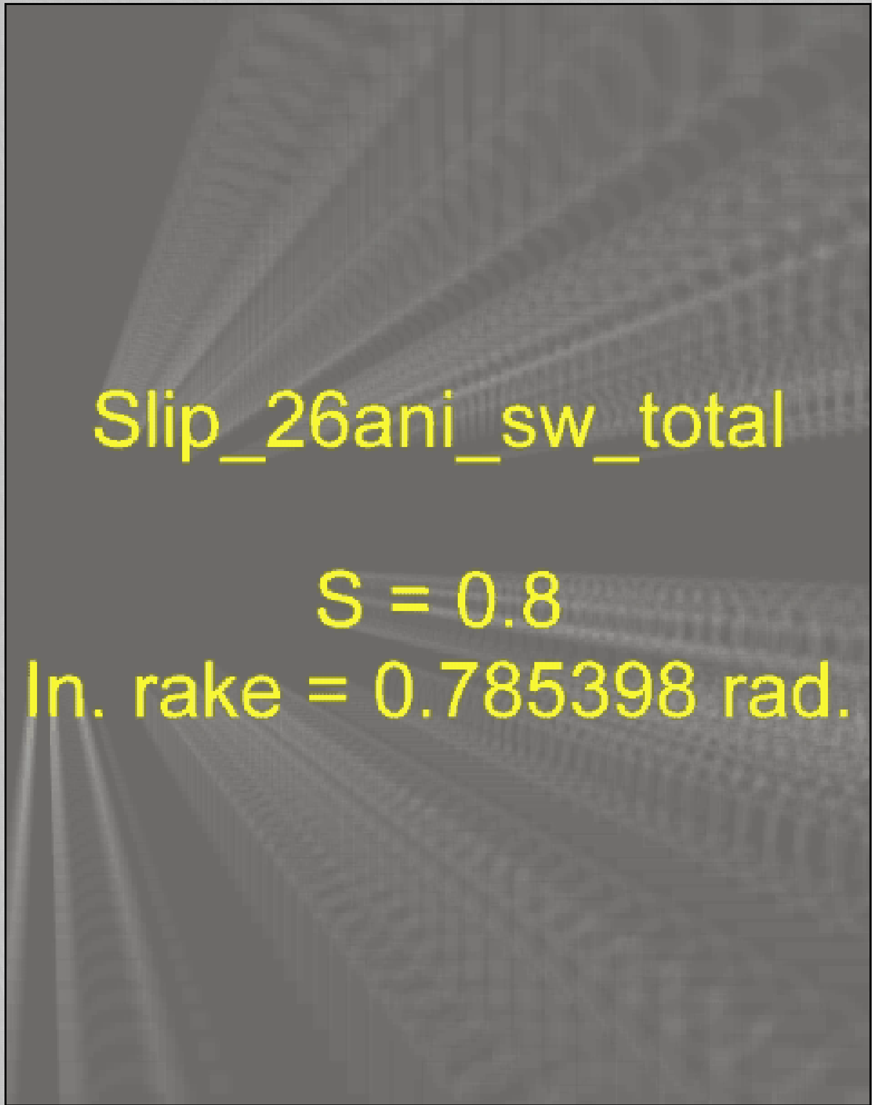
Displacements, forces and tractions are staggered in time with respect to the slip velocity components

An explicit displacement discontinuity is assumed between the two sides of faults: **Traction – at – Split – Node** scheme

We take into account the **rake rotation** during propagation: the rake direction is calculated from fault strength.

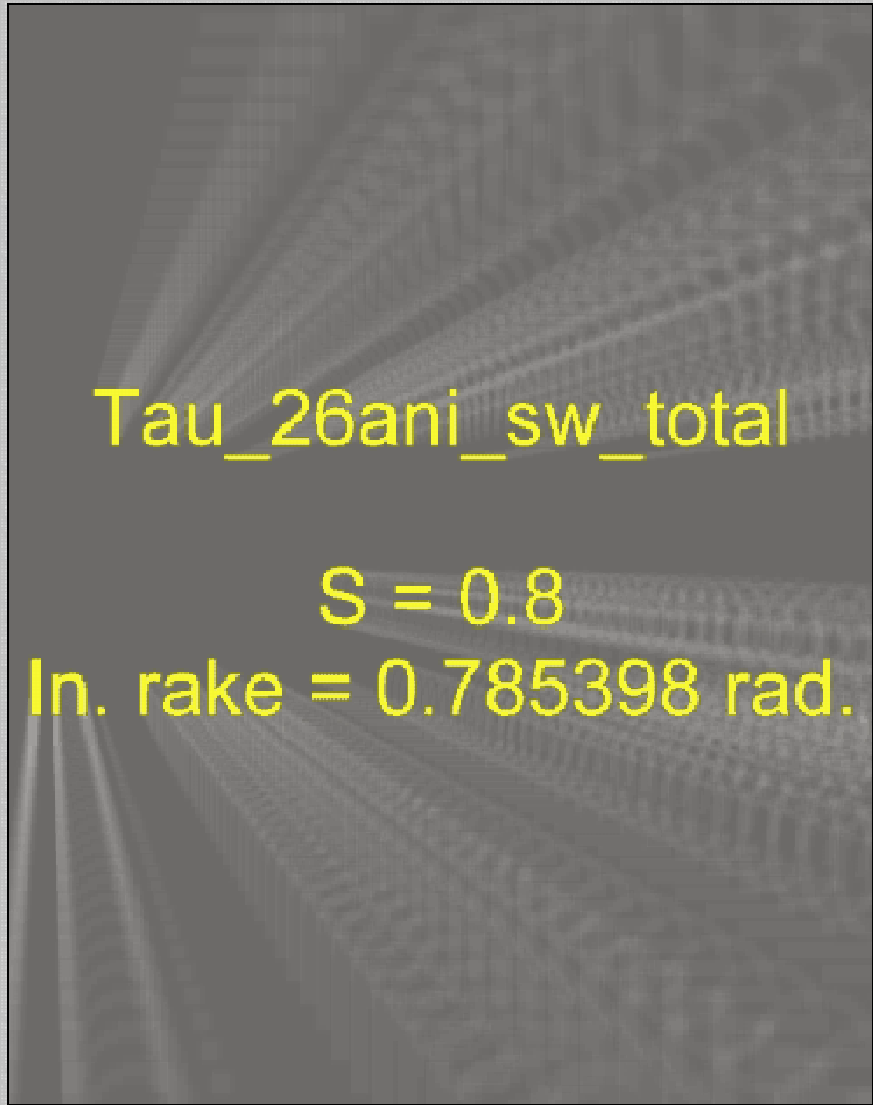
Reference Case

Slip



Anim_Slip_26ani_sw_total.avi

Traction



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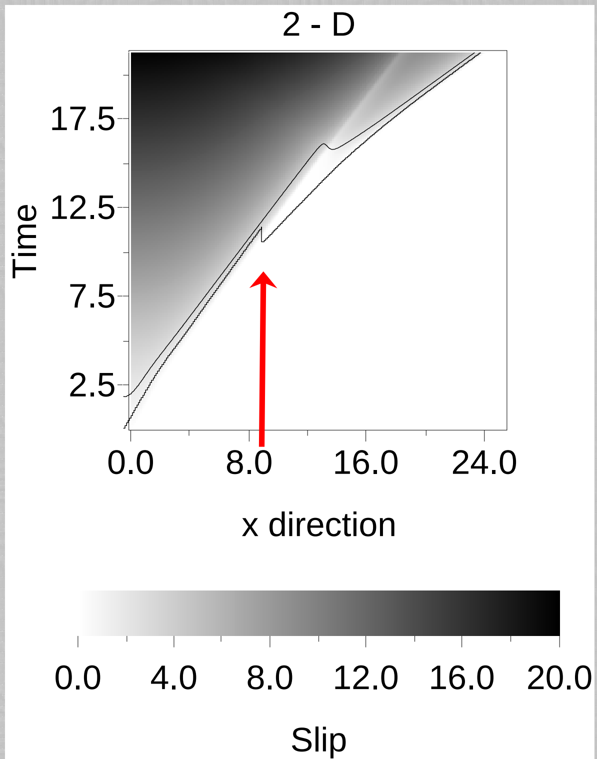
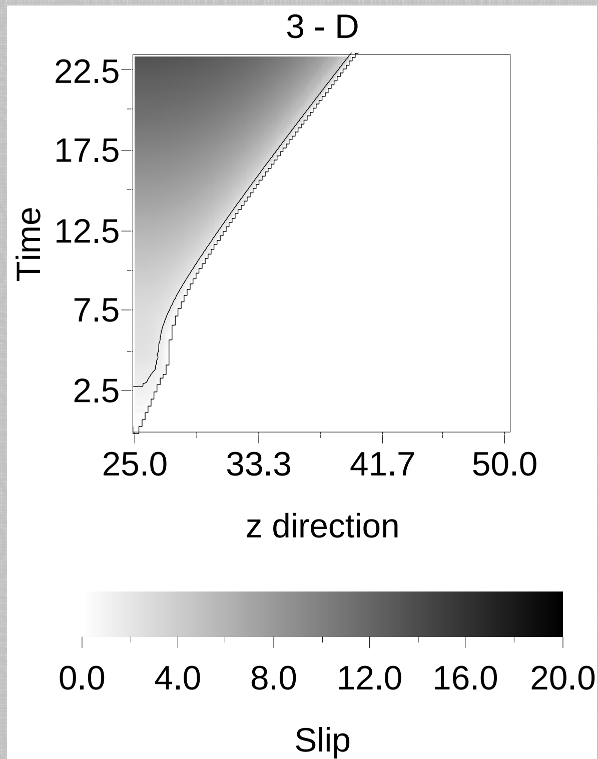
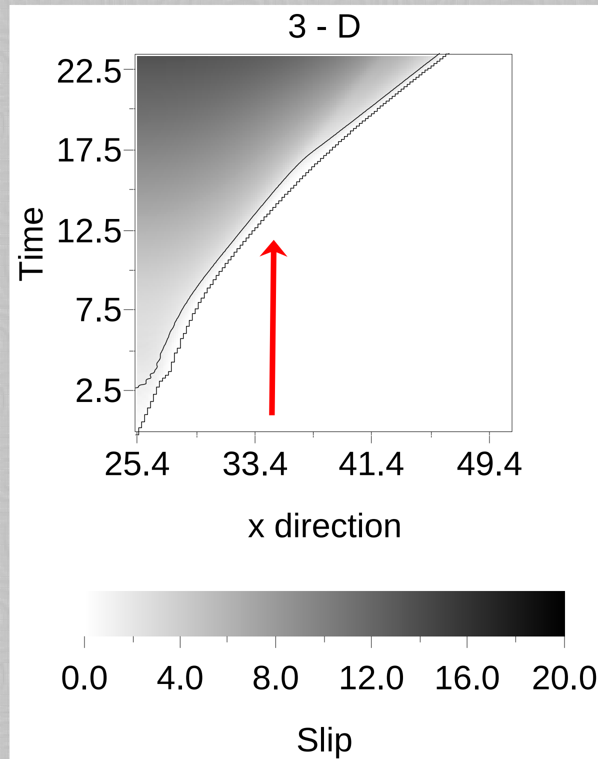
Comparision between 2 – D and 3 – D models #1



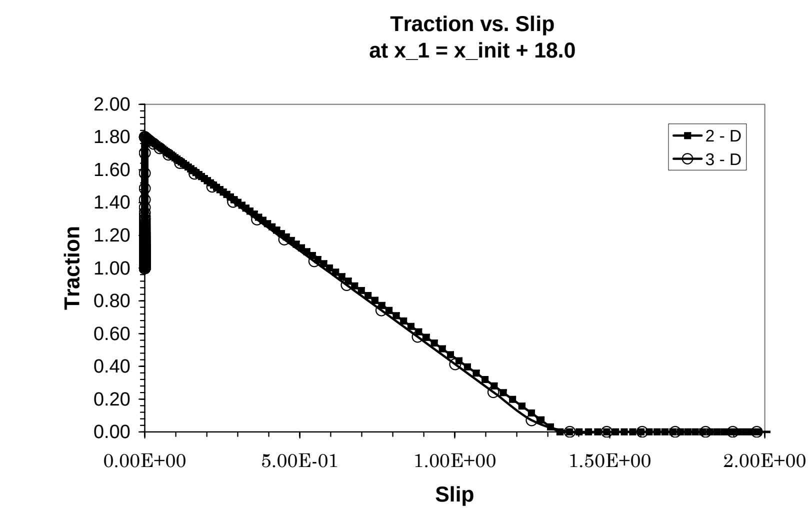
Fixed x_1 coordinate

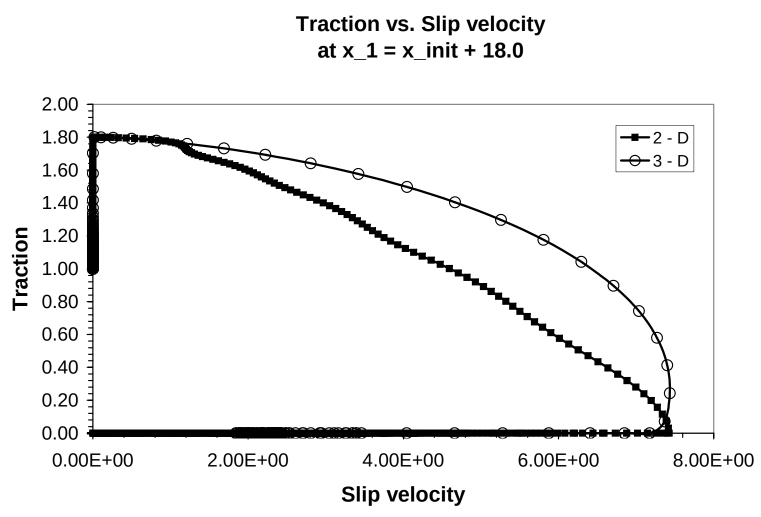


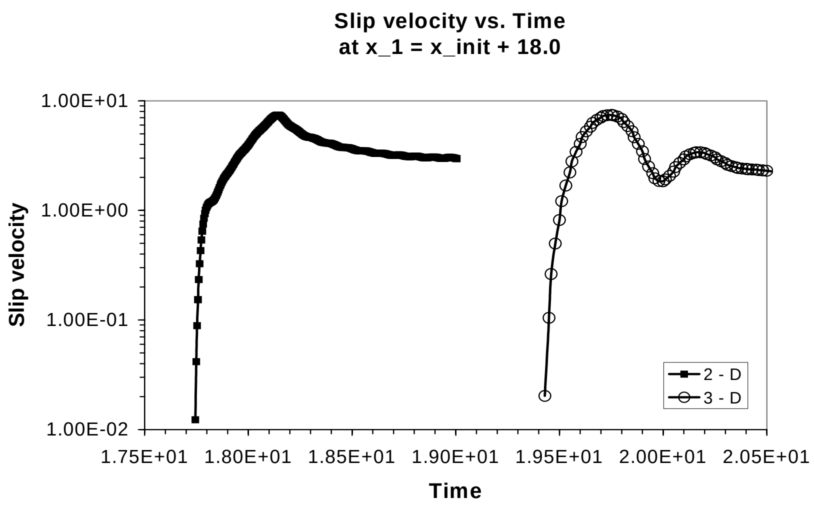
Fixed x_3 coordinate

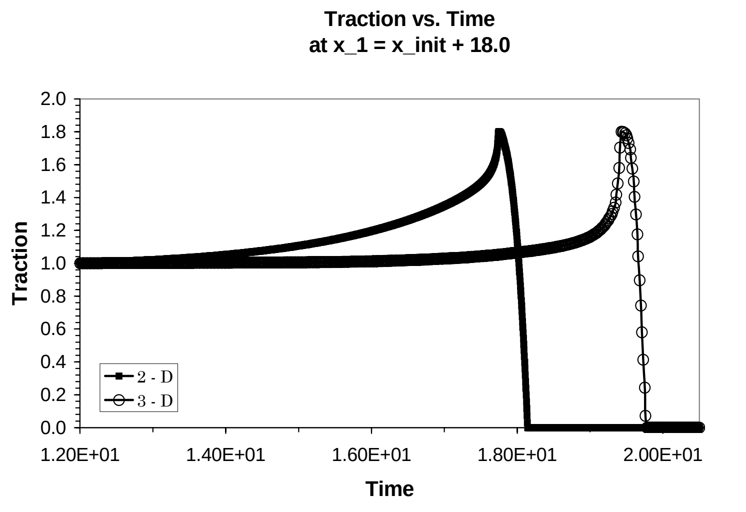


Comparision between 2 – D and 3 – D models #2

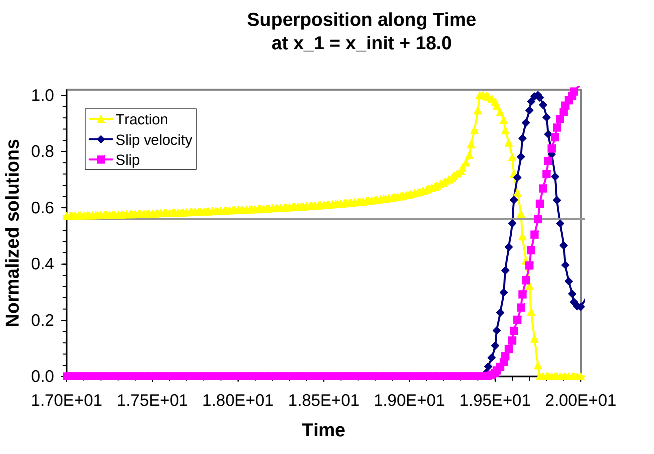


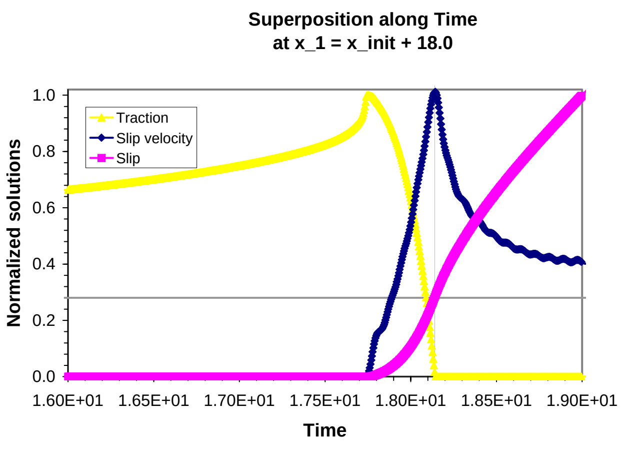


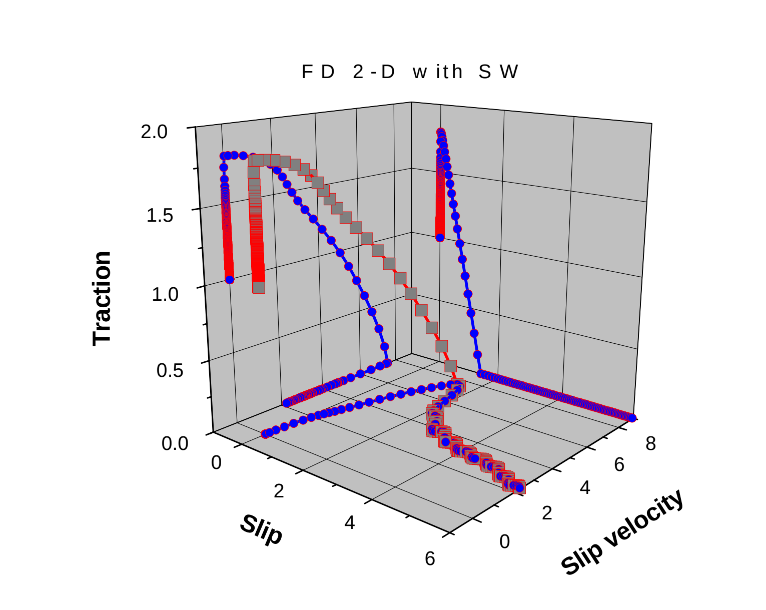


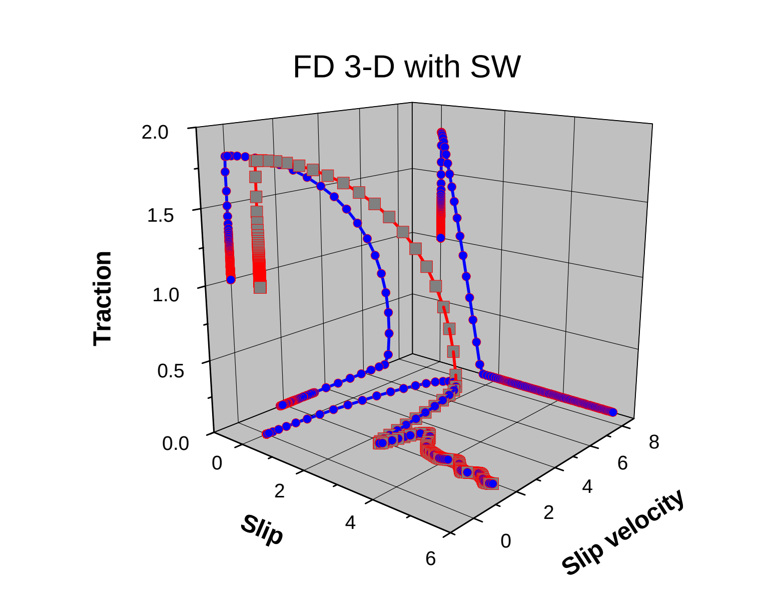


Comparision between 2 – D and 3 – D models #3

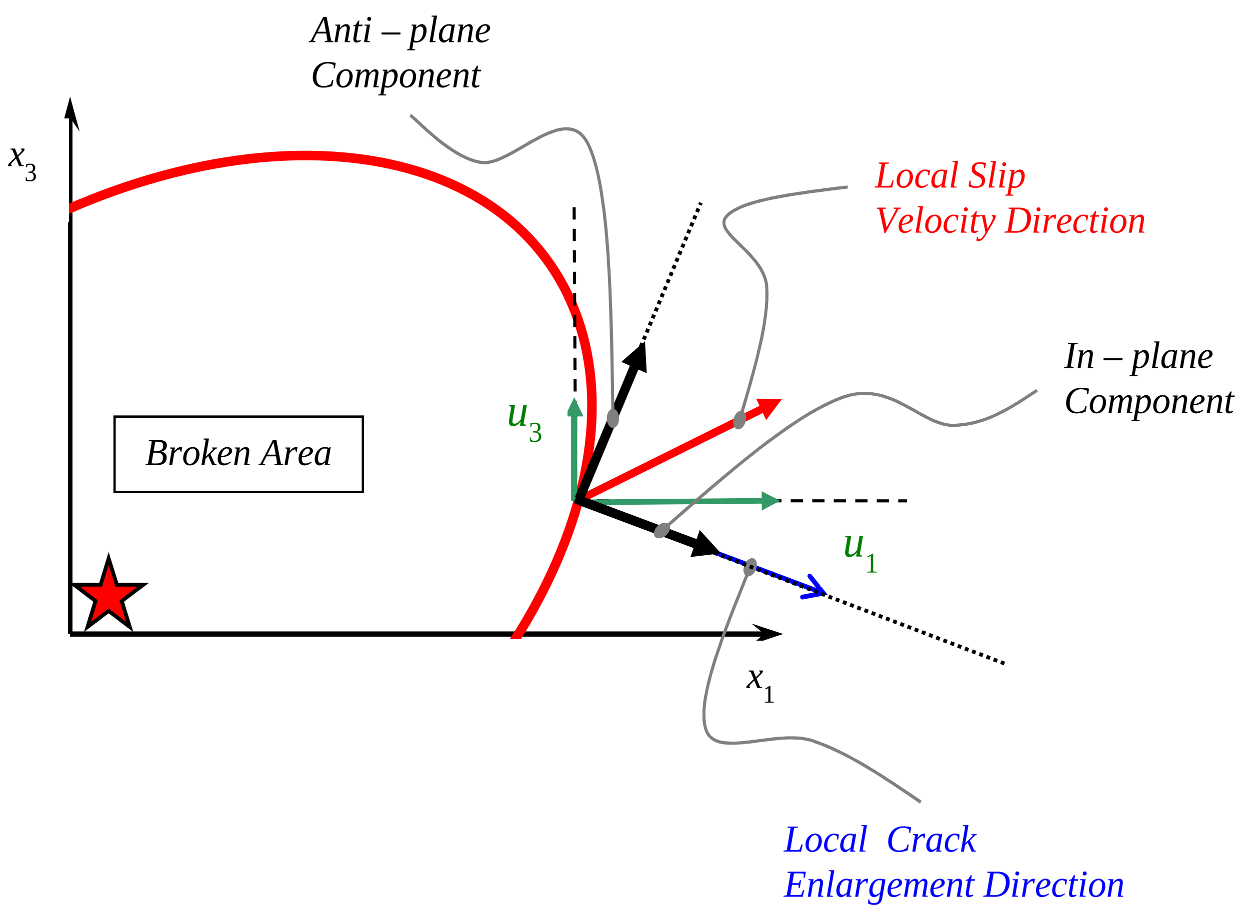








The rake rotation: the coupling of the two modes of propagation



Rake rotation #1: Theoretical background

- In the case of self – similar, expanding elliptical cracks the slip is everywhere parallel to the direction of pre – stress, even in the extreme situation of zero friction (*Burridge and Willis, 1969*).

- In the case of a finite circular crack *Madariaga (1976)* showed that rupture introduces a component perpendicular to the direction of pre – stress, which is quite small.

- The rake rotation is, by defintion, ***explicitely*** ***neglected*** in fault models where the pre – stress is assumed parallel to one coordinate axis and the slip is not allowed in the direction perpendicular to the pre – stress (*Aochi et al., 2000a, 2000b*; *Fukuyama and Madariaga, 2000*; *Madariaga et al., 1998*; *Nielsen and Olsen, 2000*) …

… as well as in models where the governing law is assumed in a vectorial form (i. e. independentely for each components of physical observables), but only one component is non null (*Fukuyama and Madariaga, 1998*; *Fukuyama et al., 2003*; *Olsen et al., 1997*).

Rake rotation #2: evidences

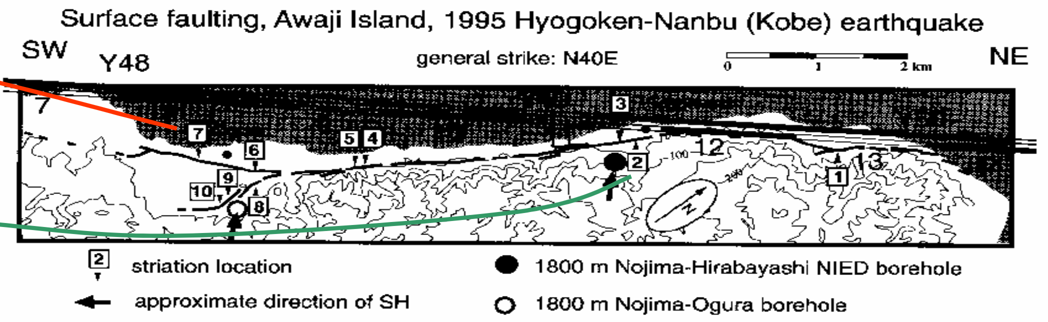


Figure 1. Map of Nojima fault on Awaji Island, showing elevation (m), surface faulting (heavy line), locations of fault striations (numbers in boxes), the Nojima-Hirabayashi NIED and the Nojima-Ogura boreholes, and the subfaults (numbered) in the original (Y48 to Y50) and interpolated (7 to 14) Yoshida slip models. Arrows show approximate direction of *SH* in boreholes.

From Spudich et al., (1998)

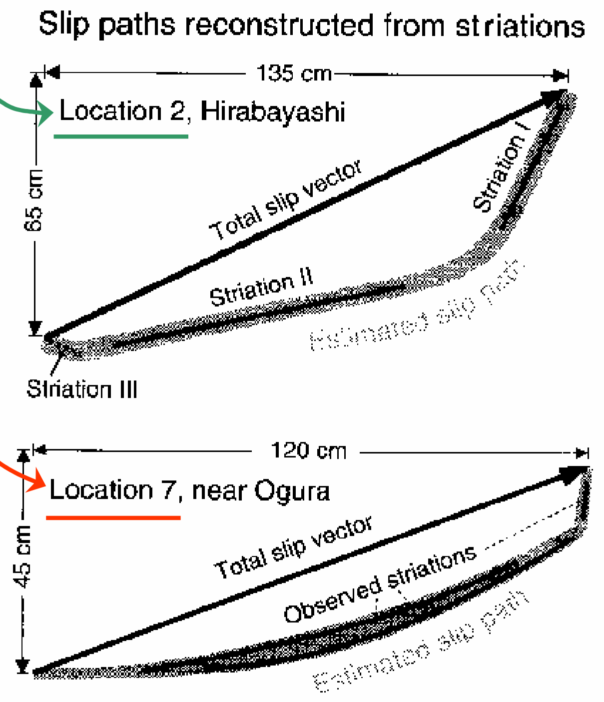
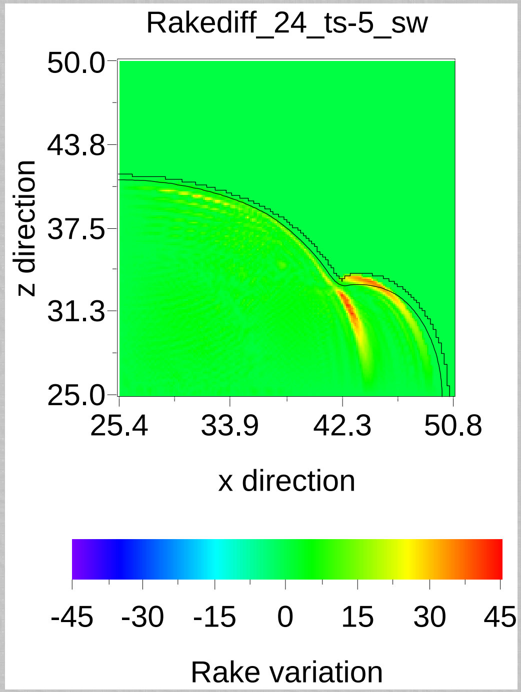


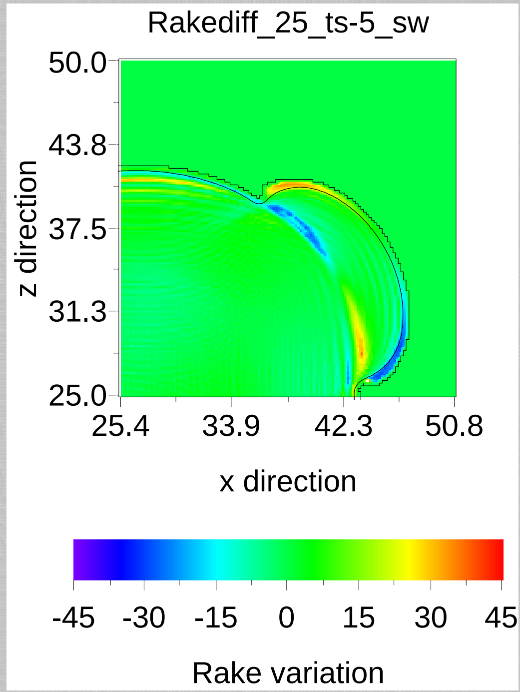
Figure 2. Black lines: striations observed at locations 2 and 7. Gray bands: slip paths inferred from striation locations 2 and 7. From Otsuki *et al.* (1997).

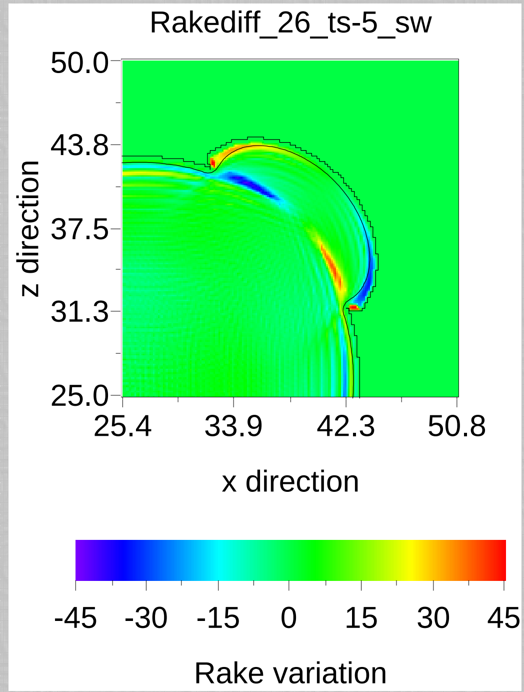
Etchecopar (1984), *Florensov and Solonenko (1965)*, *Kakimi et al. (1977)*, *Philip and Megard (1977)*.

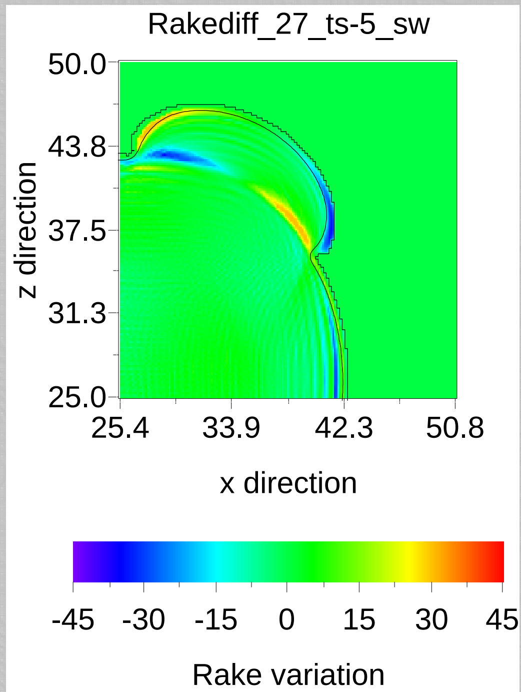
More recently curved striations (also called slickenlines) were seen in the Denali earthquake (*Haeussler et al. 2004*).

Curved striations were observed in the 1971 San Fernando; 1999 Hector Mine EQ; the 1992 Landers EQ; the 1980 El Asnam, Algeria EQ, and on the San Andreas in the Mecca Hills.


Rakediff_24_ts-5_sw
50.0
43.8
37.5
31.3
25.0
z direction
25.4
33.9
42.3
50.8
x direction
-45
-30
-15
0
15
30
45
Rake variation


Rakediff_25_ts-5_sw
50.0
43.8
37.5
31.3
25.0
z direction
25.4
33.9
42.3
50.8
x direction
-45
-30
-15
0
15
30
45
Rake variation


Rakediff_26_ts-5_sw
50.0
43.8
37.5
31.3
25.0
z direction
25.4
33.9
42.3
50.8
x direction
-45
-30
-15
0
15
30
45
Rake variation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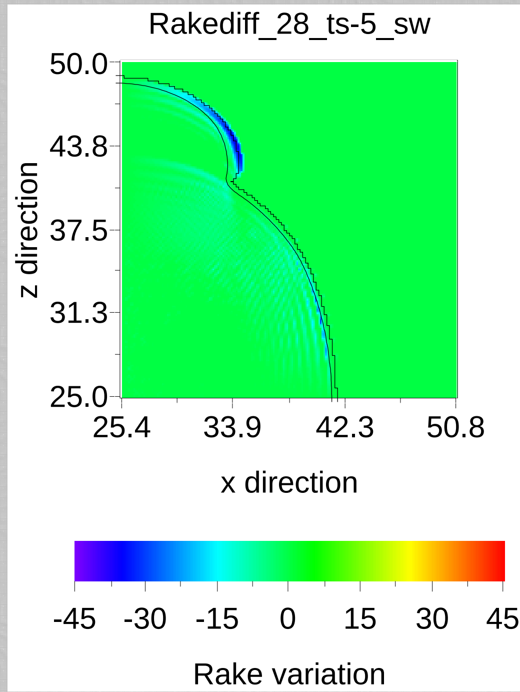
43.8
37.5
31.3
25.0
z direction
25.4
33.9
42.3
50.8
x direction
-45
-30
-15
0
15
30
45
Rake variation


Rakediff_28_ts-5_sw
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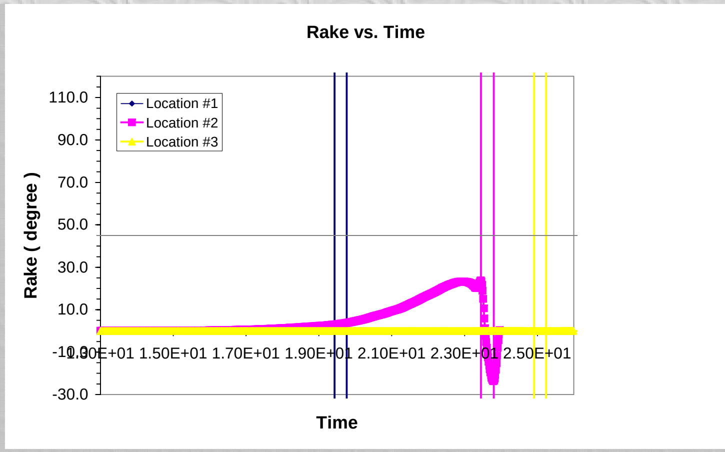
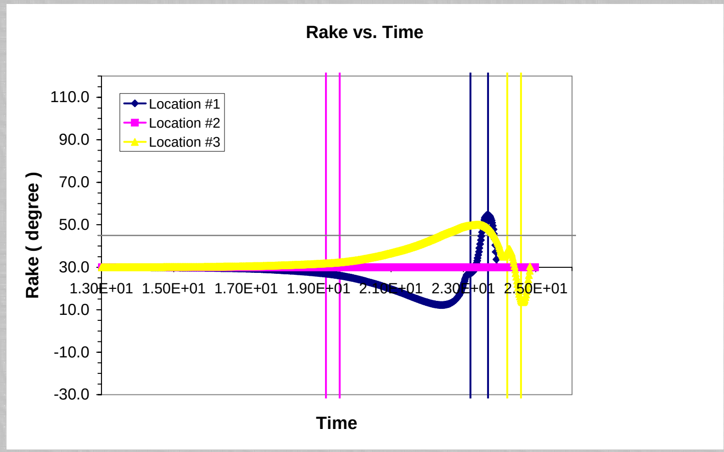
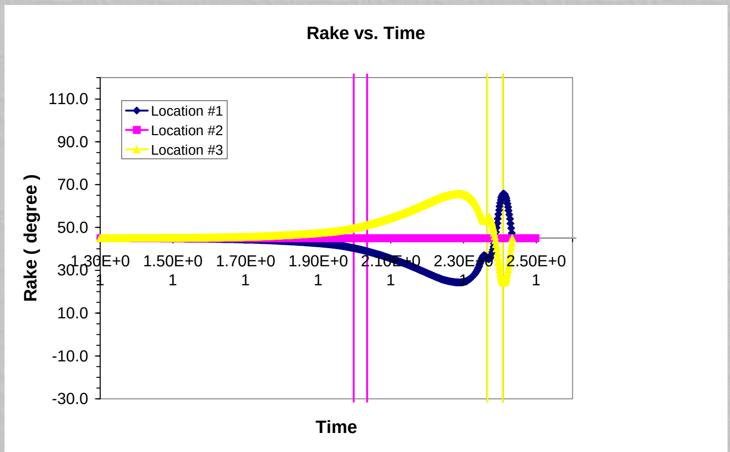
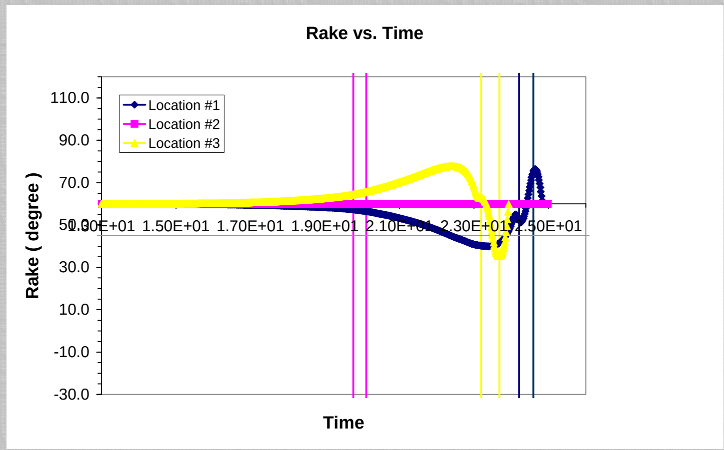
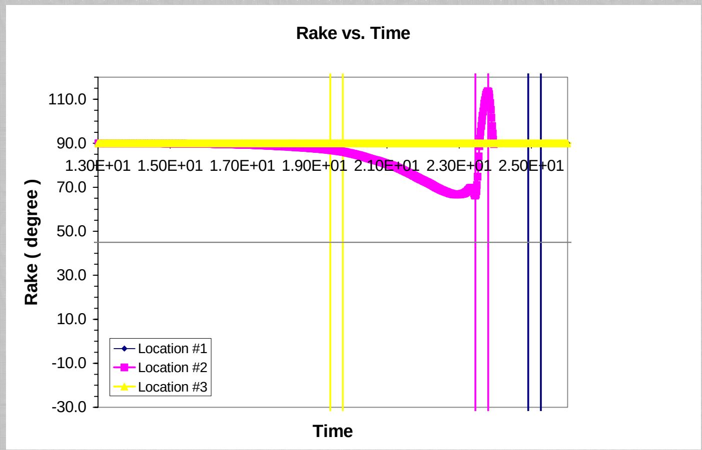
37.5
31.3
25.0
z direction
25.4
33.9
42.3
50.8
x direction
-45
-30
-15
0
15
30
45
Rake variation


Rake vs. Time
Location #1
Location #2
Location #3
110.0
90.0
70.0
50.0
30.0
10.0
-10.0
-30.0
1.30E+01 1.50E+01 1.70E+01 1.90E+01 2.10E+01 2.30E+01 2.50E+01
Rake (degree)
Time

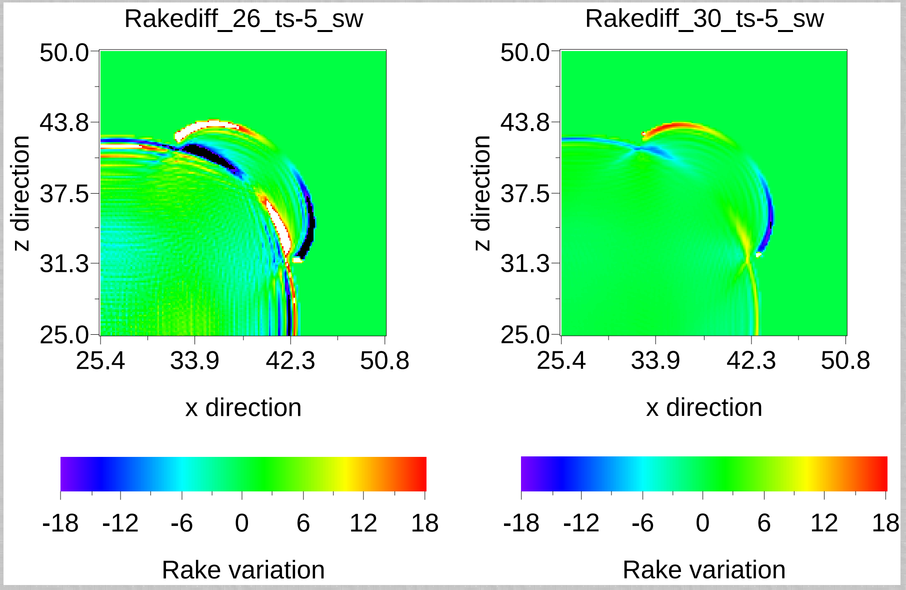

Rake vs. Time
Location #1
Location #2
Location #3
110.0
90.0
70.0
50.0
30.0
10.0
-10.0
-30.0
1.30E+01 1.50E+01 1.70E+01 1.90E+01 2.10E+01 2.30E+01 2.50E+01
Rake (degree)
Time

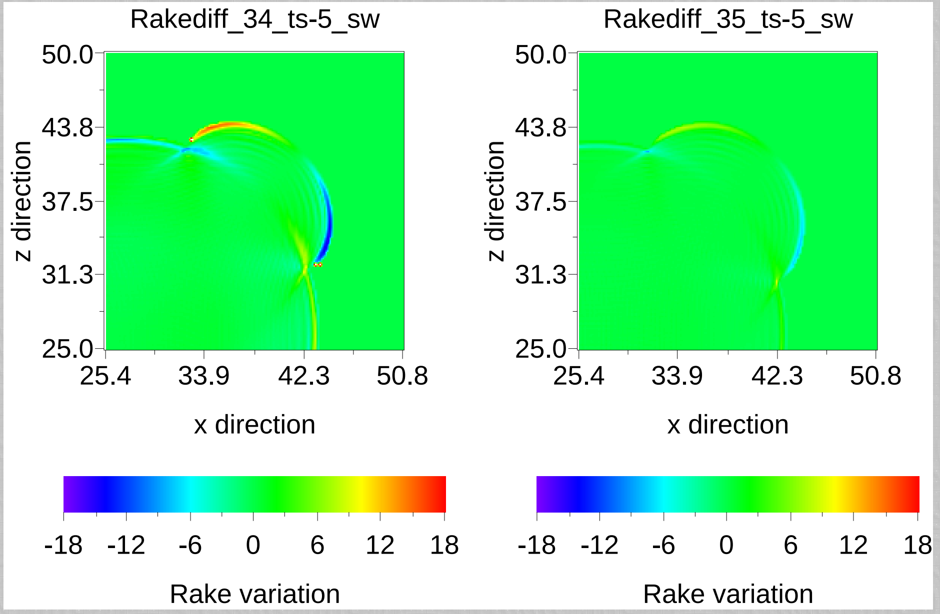

Rake vs. Time
Location #1
Location #2
Location #3
110.0
90.0
70.0
50.0
30.0
10.0
-10.0
-30.0
1.30E+01 1.50E+01 1.70E+01 1.90E+01 2.10E+01 2.30E+01 2.50E+01
Rake (degree)
Time

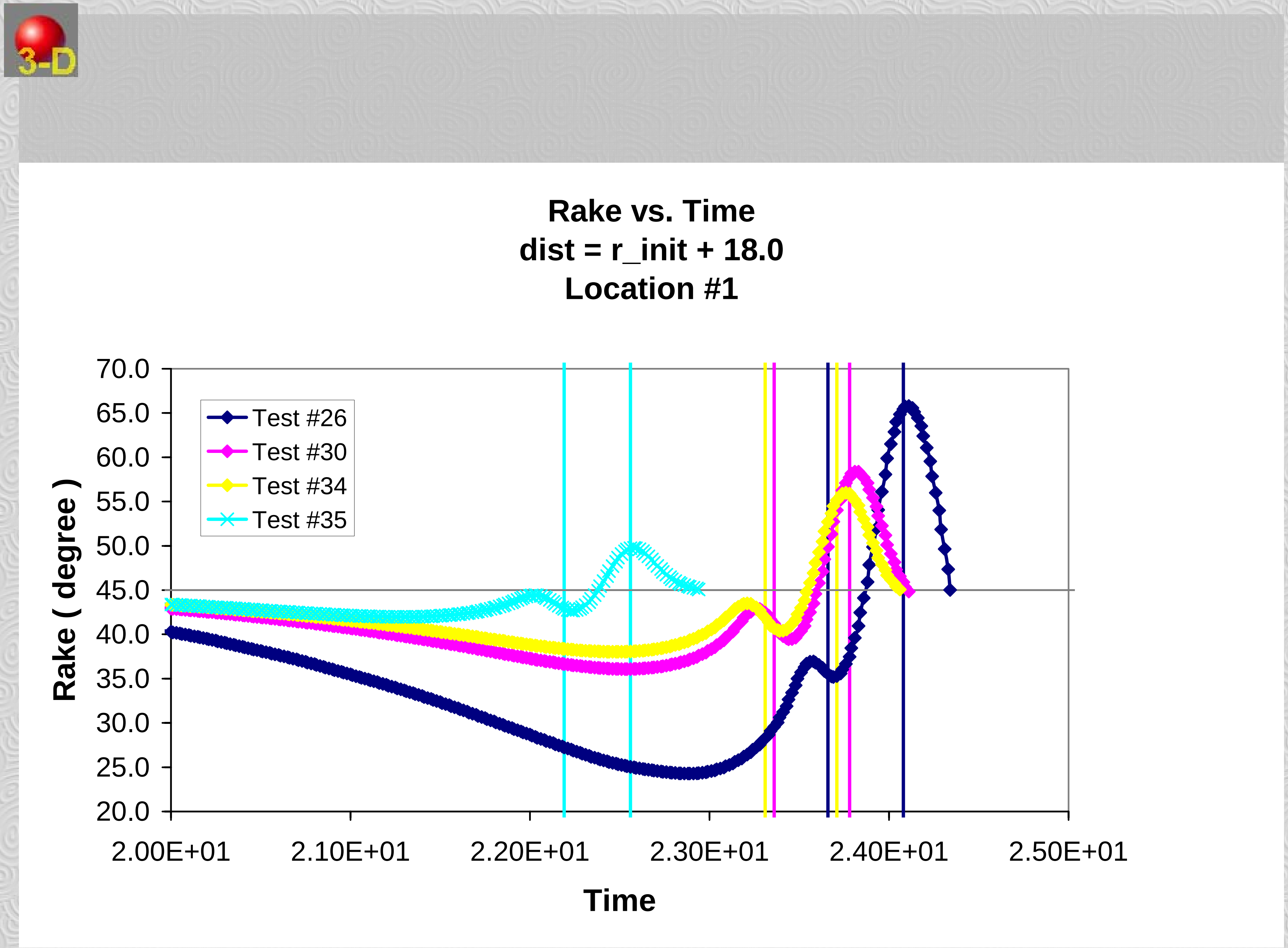

Rake vs. Time
Location #1
Location #2
Location #3
110.0
90.0
70.0
50.0
30.0
10.0
-10.0
-30.0
1.30E+01 1.50E+01 1.70E+01 1.90E+01 2.10E+01 2.30E+01 2.50E+01
Rake (degree)
Time


Rake vs. Time
110.0
90.0
70.0
50.0
30.0
10.0
-10.0
-30.0
1.30E+01 1.50E+01 1.70E+01 1.90E+01 2.10E+01 2.30E+01 2.50E+01
Location #1
Location #2
Location #3
Rake (degree)
Time

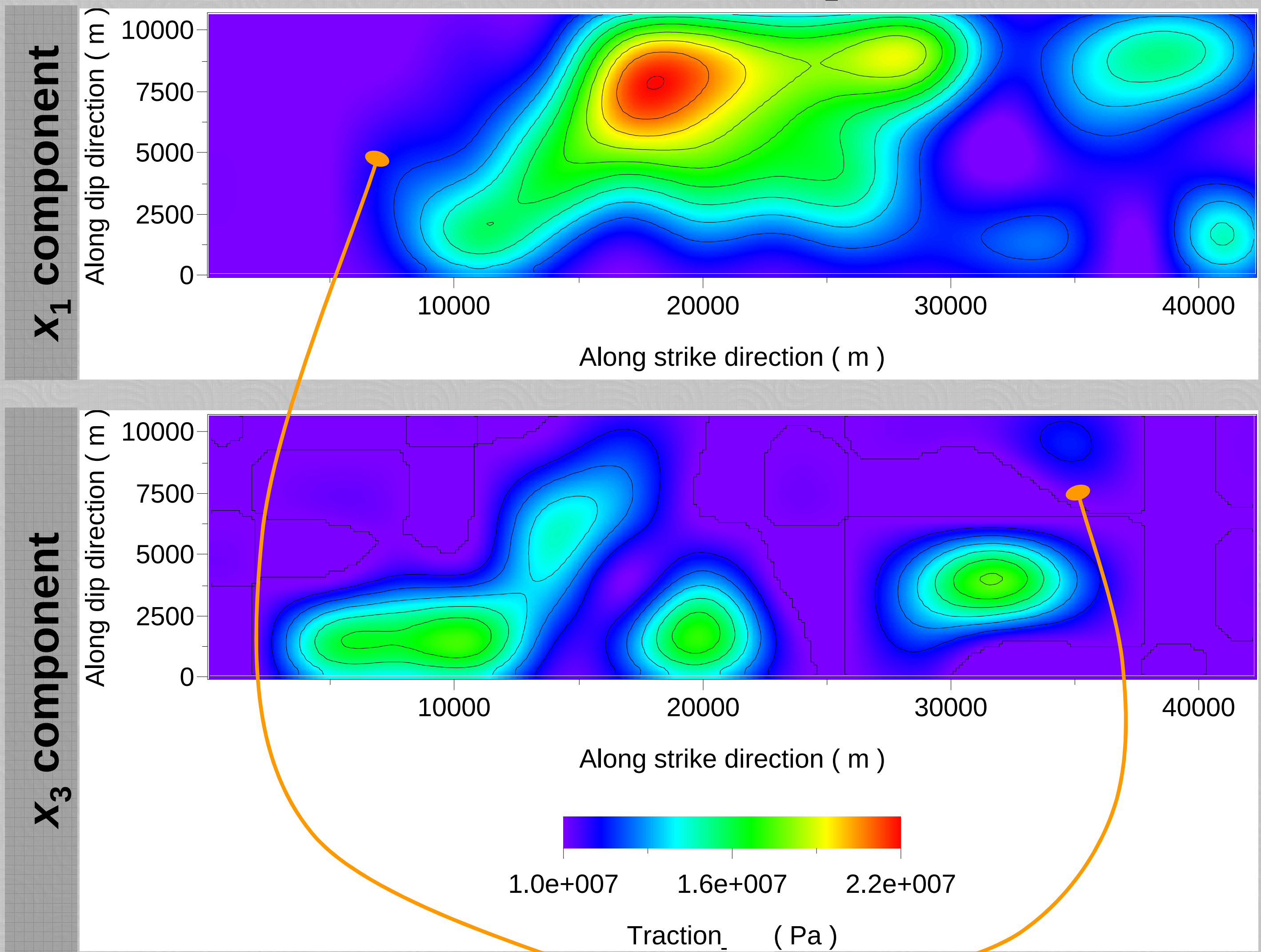
Rake rotation #4: dependence on the absolute stress level






3-D
Rake vs. Time
dist = r_init + 18.0
Location #1
Rake (degree)
70.0
65.0
60.0
55.0
50.0
45.0
40.0
35.0
30.0
25.0
20.0
2.00E+01
2.10E+01
2.20E+01
2.30E+01
2.40E+01
2.50E+01
Time
Test #26
Test #30
Test #34
Test #35

The rake rotation #5: path / modulus



$$\mathrm{T}_0(x_1,x_3) \equiv \mathrm{T}(x_1,x_3,0) = \left(T_1(x_1,x_3,0),0,T_3(x_1,x_3,0)\right)$$

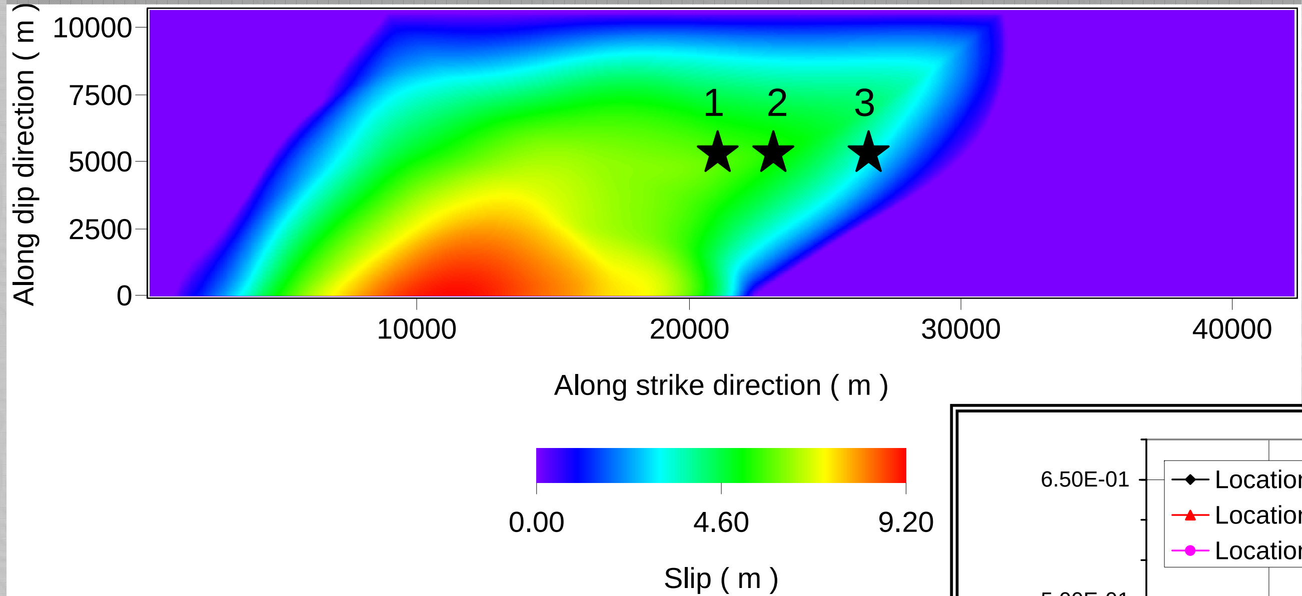
$$\mathcal{T}_{\mathbf{0}} = \mathrm{T}_0 + \Sigma_0$$

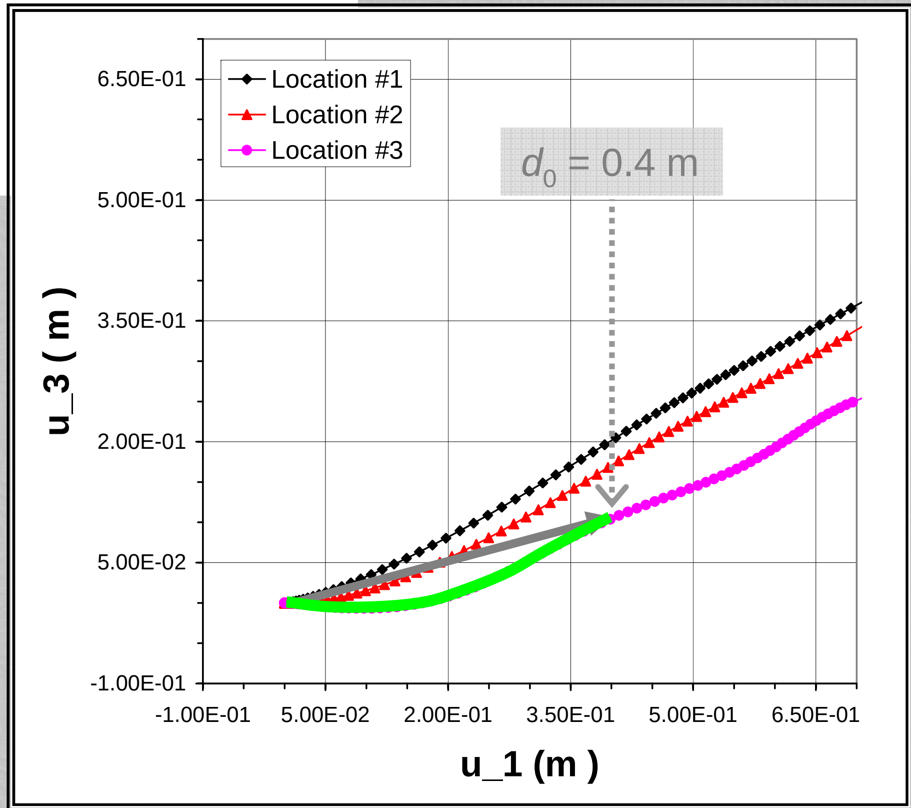
Normal Traction

$$\Sigma_0(x_1,x_3) \equiv \Sigma(x_1,x_3,0) = -\sigma_n^{\ eff}\,\hat{\mathbf{n}} = (0,-30\ \mathrm{MPa},\ 0)$$

Total Traction

Fault slip time snapshots – Linear SW assumed





Slip modulus:

$$u = u^{(mod)}\left(x_1, x_3, t\right) \equiv \left\| \mathbf{u}\left(x_1, x_3, t\right) \right\|$$

Slip path:

$$u = u^{(path)}\left(x_1, x_3, t\right) \equiv \int_0^t \left\| \mathbf{v}\left(x_1, x_3, t'\right) \right\| \mathrm{d}\,t'$$

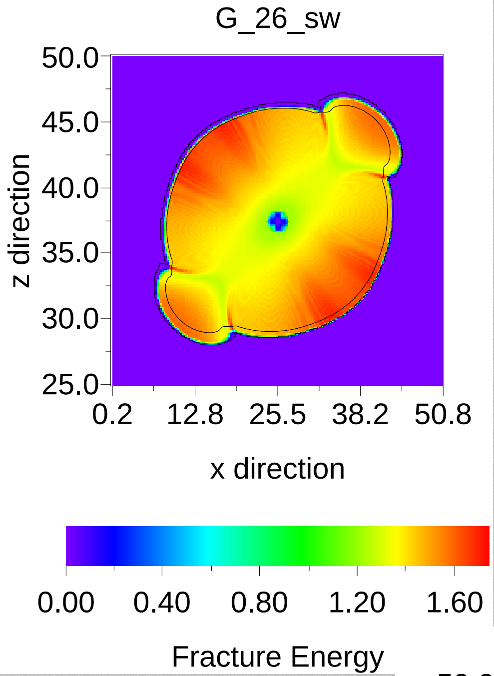
The ambiguity between modulus and path exists only for governing laws containing a dependence on *fault slip* (for instance in the case of rate – and state – dependent friction there is no other possibility than modulus of fault slip velocity).

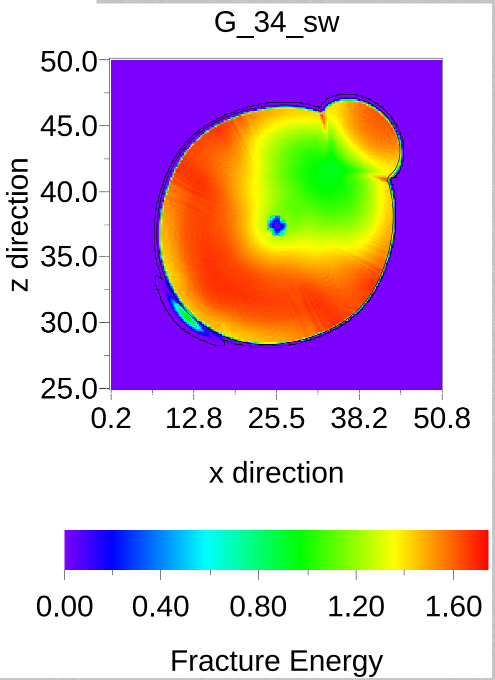
In the papers taking into account both components of fault slip (and fault slip velocity and fault traction)

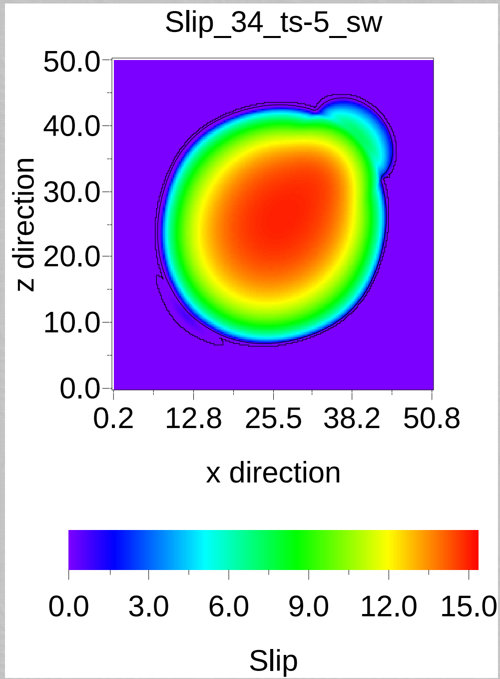
- *Bizzarri and Belardinelli (2007)*; *Bizzarri and Cocco (2005*, *2006a*, *2006b)*; *Bizzarri and Spudich (2007)*; *Olsen et al. (1997)* considered the dependence on slip modulus;

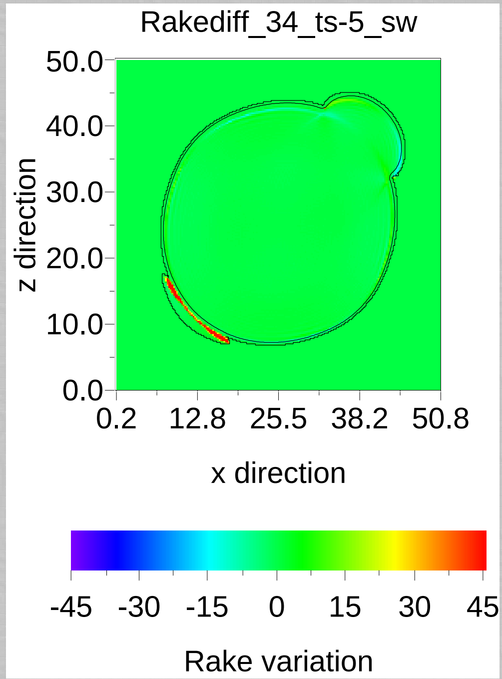
- *Dalguer and Day (2006)*; *Day et al., (1982a, 1982b)*; *Day et al. (2005)* considered the dependence on slip path.

Effect of the free surface









Slip complexity and heterogeneities

Direct evidences:

1) Shallow geometrical complexity observed at all scales (*Tchalenko and Ambrases, 1970*; *Aydin, 1978*; *Okubo and Aki, 1987*; *Aviles et al., 1987*; *Reches, 1988*; *Davy, 1993*; *Johnson et al., 1994*);

2) Profilemetry measurements along exumed fault surfaces (*Brown and Scholz, 1985*; *Power et al., 1988*; *Power and Tullis, 1991*; *Brown, 1995*);

3) Long – range property fluctuations in geophysical logs (*Hewett, 1986*; *Leary, 1991*).

Indirect evidences:

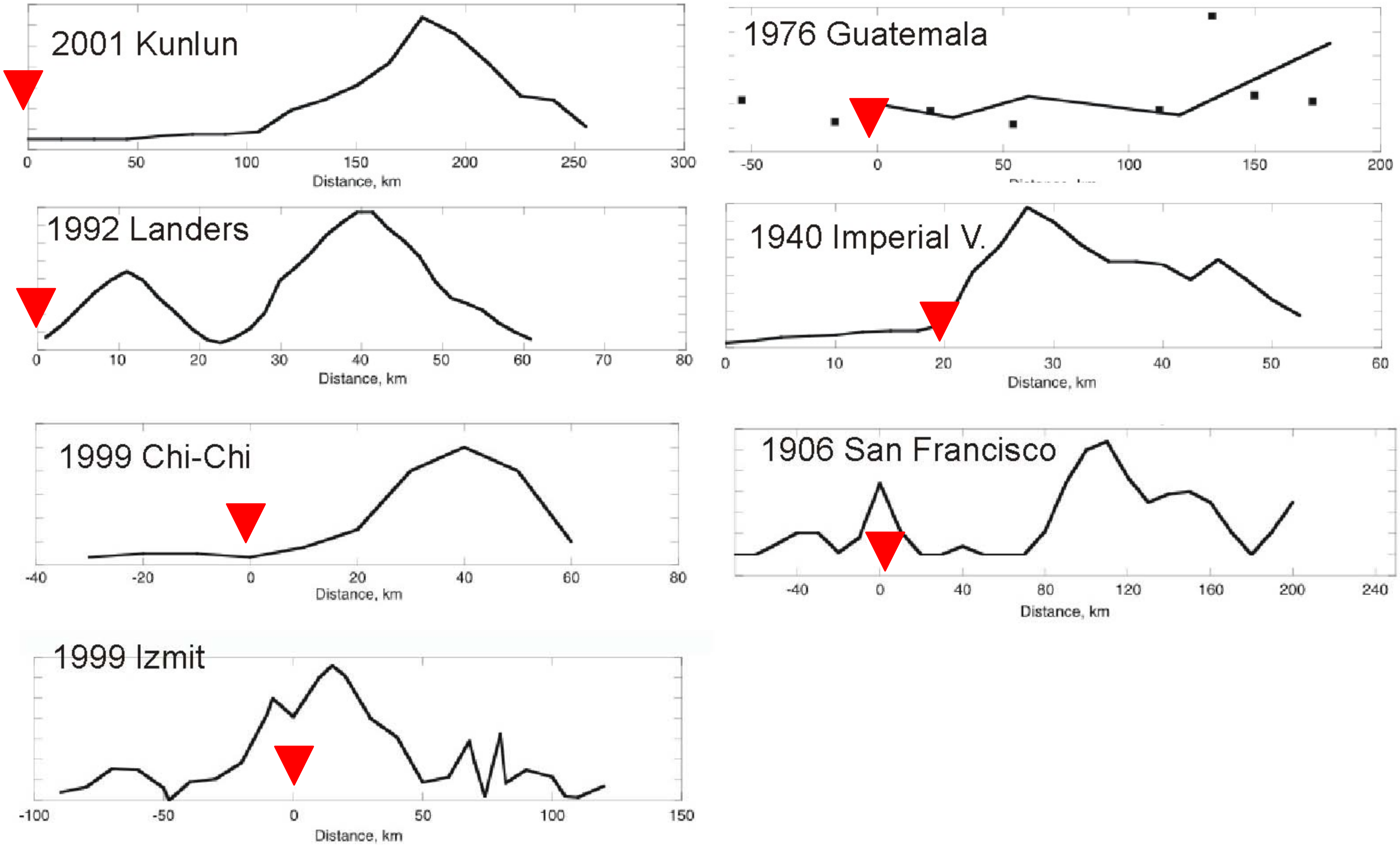
1) Complex distribution of earthquake hypocenters (*Kagan, 1994*) and of size and repeated time of earthquake occurrence;

2) Presence of abundance of incoherent high – frequency seismic radiation from earthquake rupture zones (*Hanks and McGuire, 1981*; *Papageorgiou and Aki, 1983*; *Joyner and Boore, 1988*; *Stevens and Day, 1994*);

3) Short risetimes in earthquake slip hystories (*Heaton, 1990*; *Wald, 1992*);

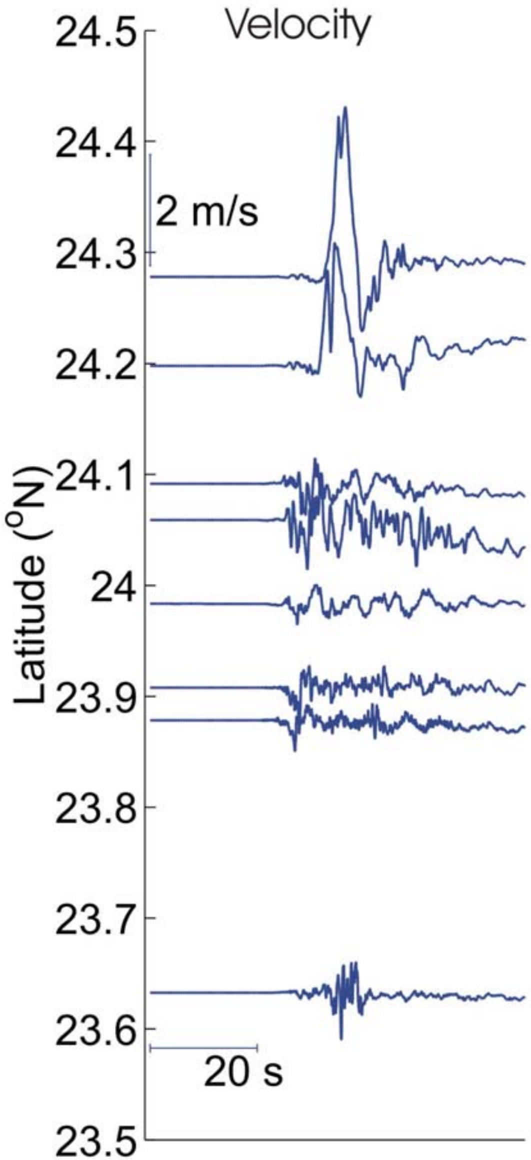
4) Stress drop fluctuations in small events (*Guo et al., 1992*; *Abercrombie and Leary, 1993*; *Hough and Dreger, 1995*).

Slip distribution of large earthquakes

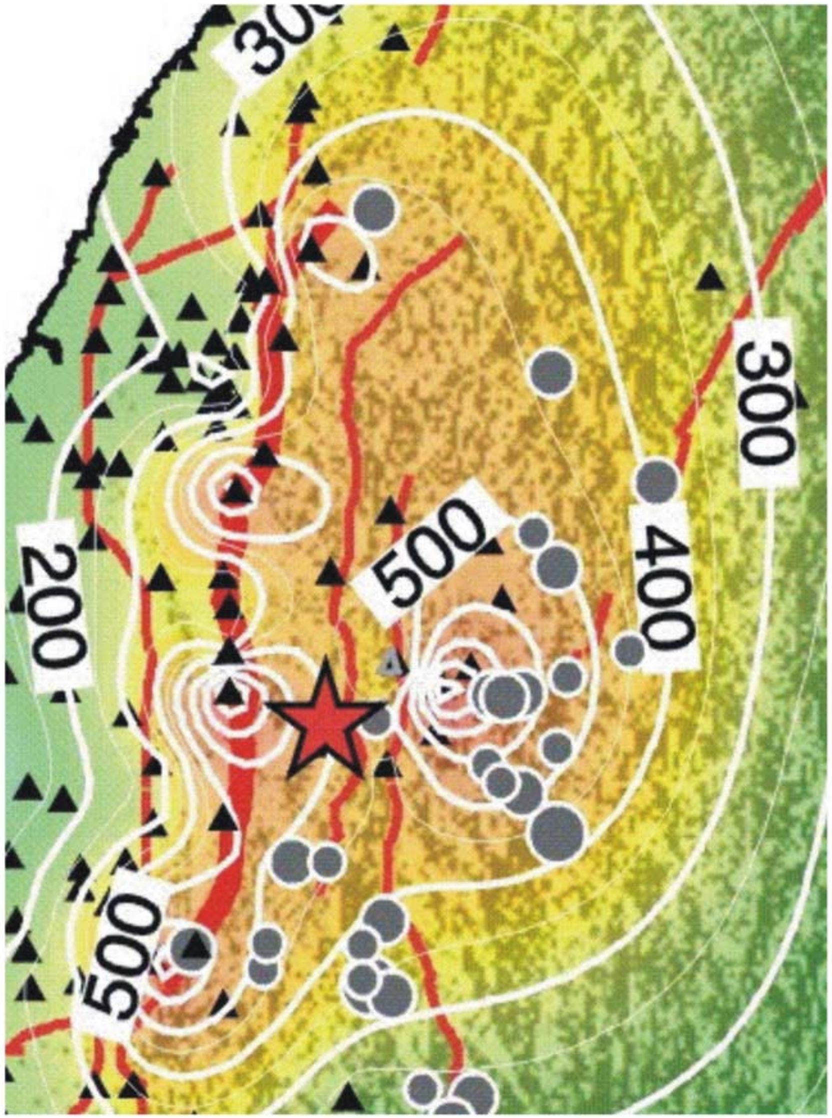


Ground motion from Chi – Chi, Taiwan, EQ

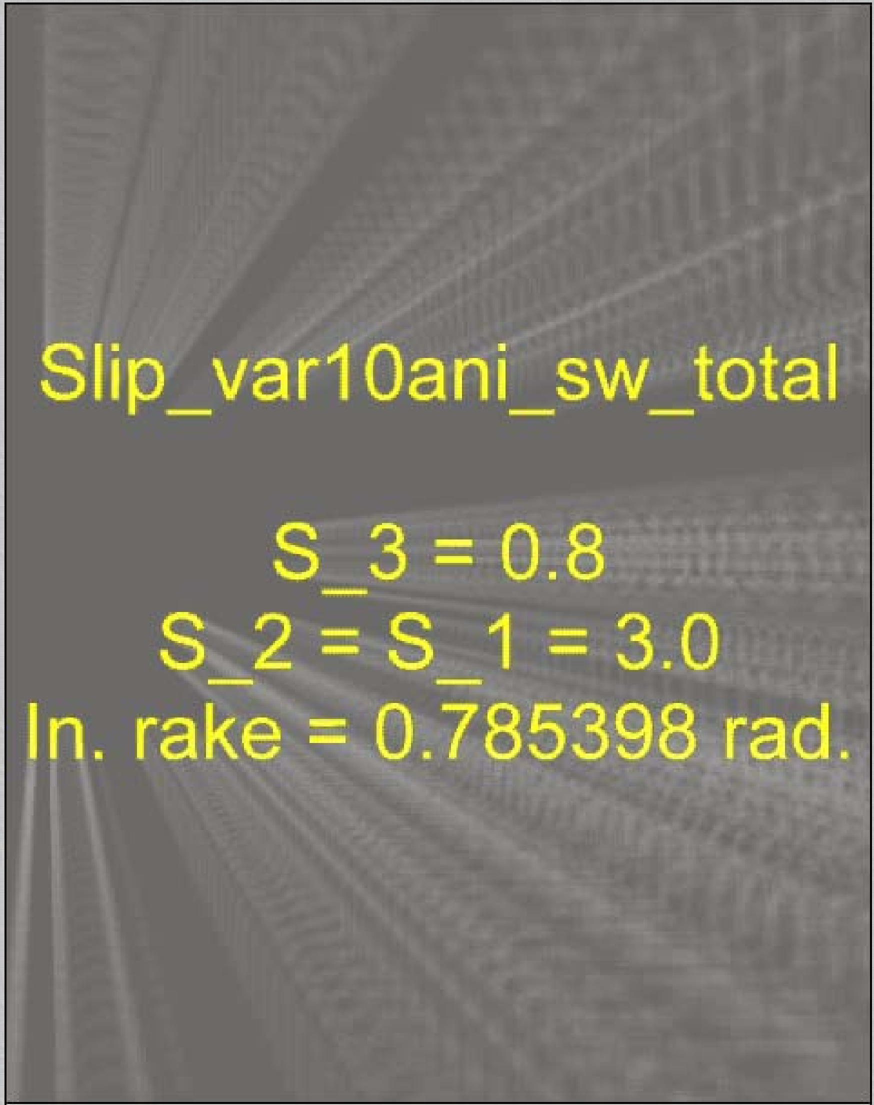
Brodsky and Kanamori (2001)



Ma et al. (1993)

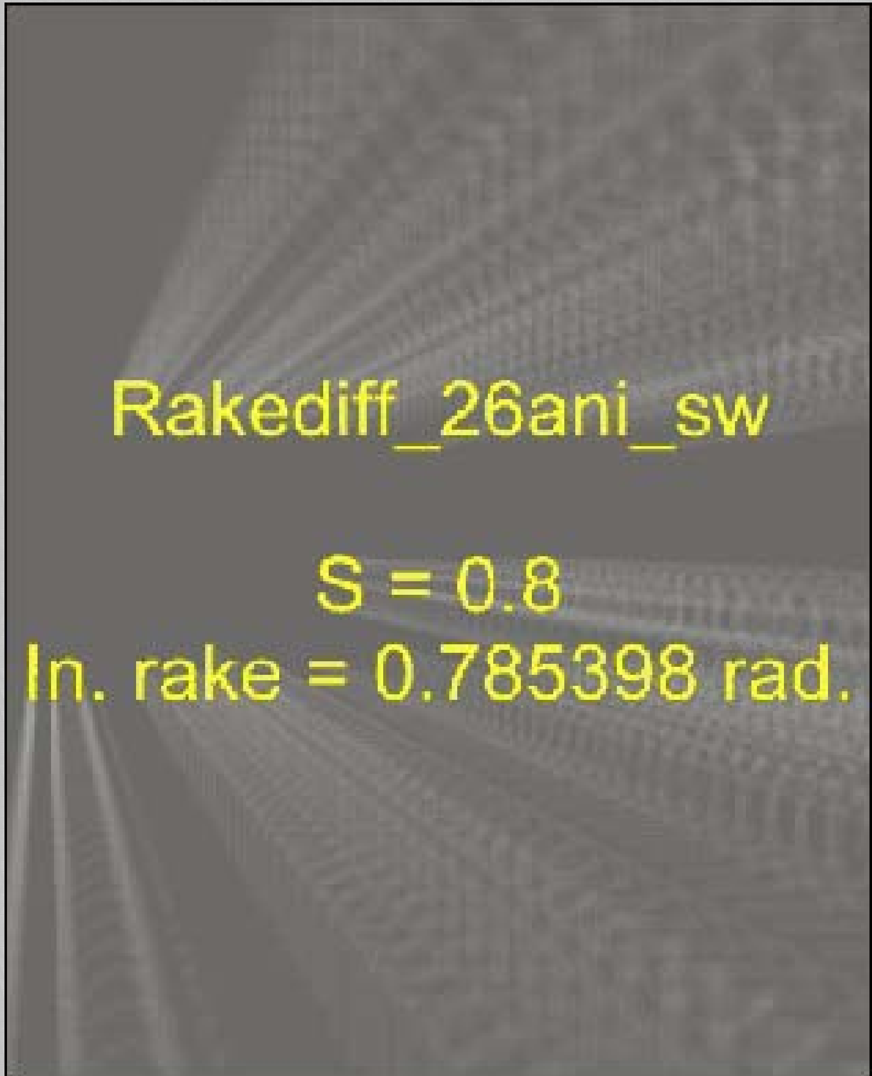


Effects of Strength Heterogeneity #1



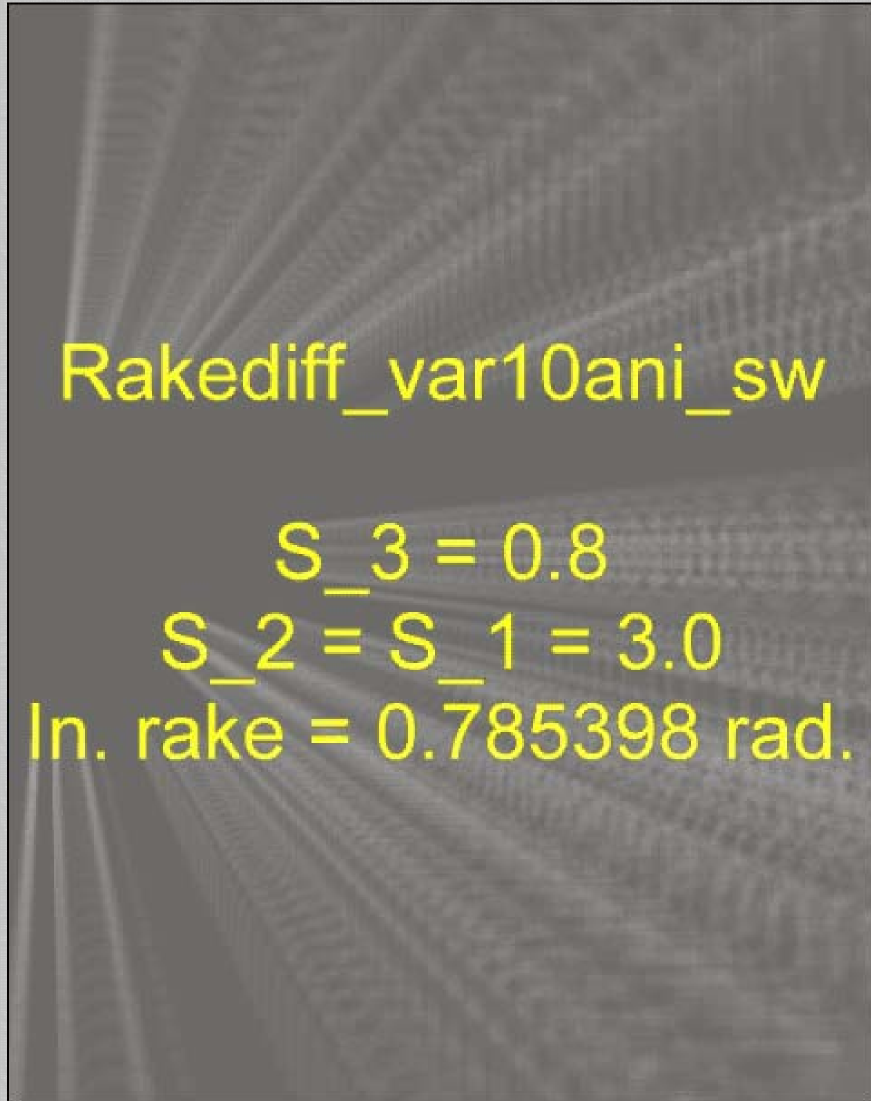
Anim_Slip_var10ani_sw_total.avi

Homogeneous



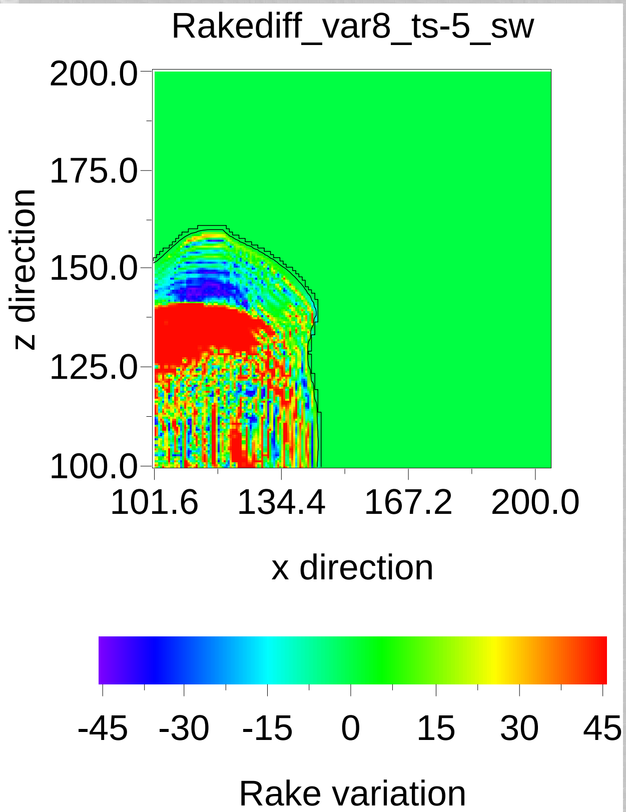
Anim_Rakediff_26ani_sw_total.avi

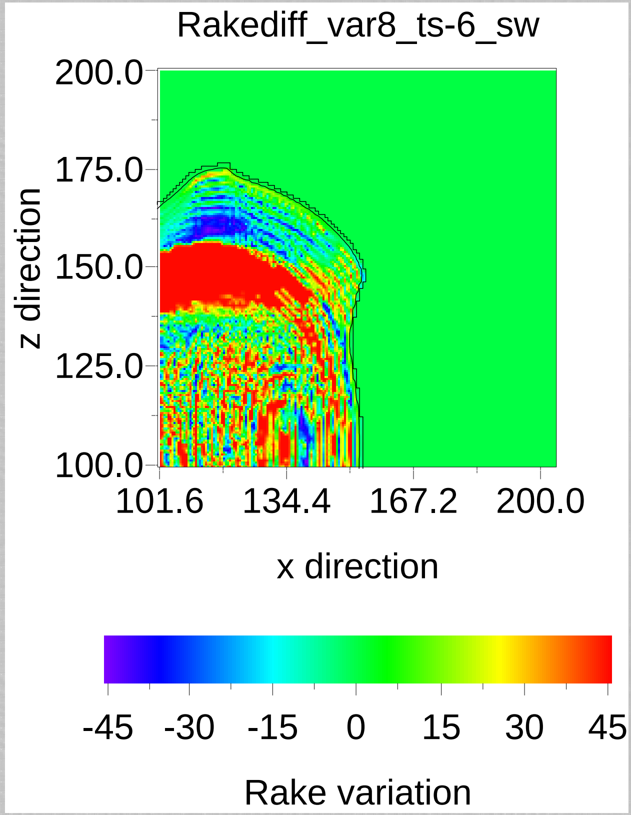
Heterogeneous

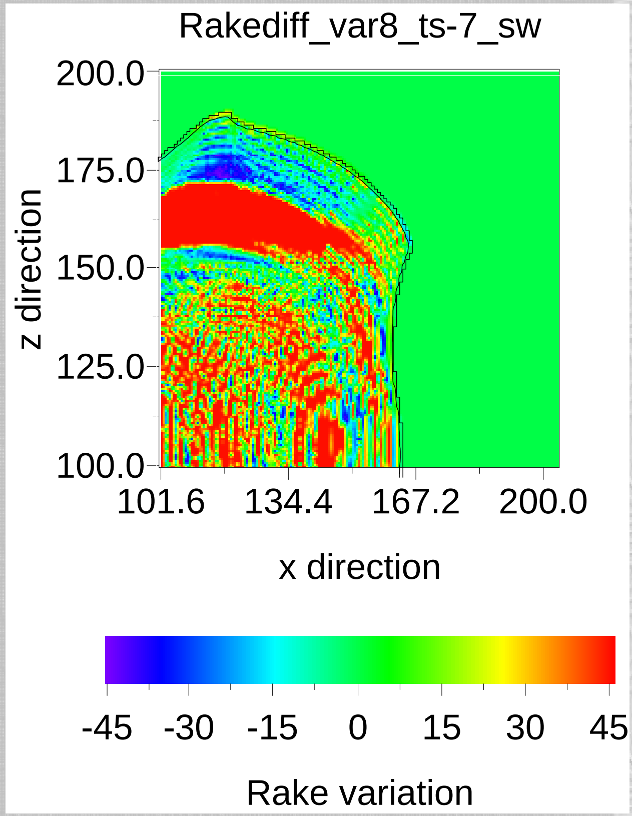


Anim_Rakediff_var10ani_sw_total.avi

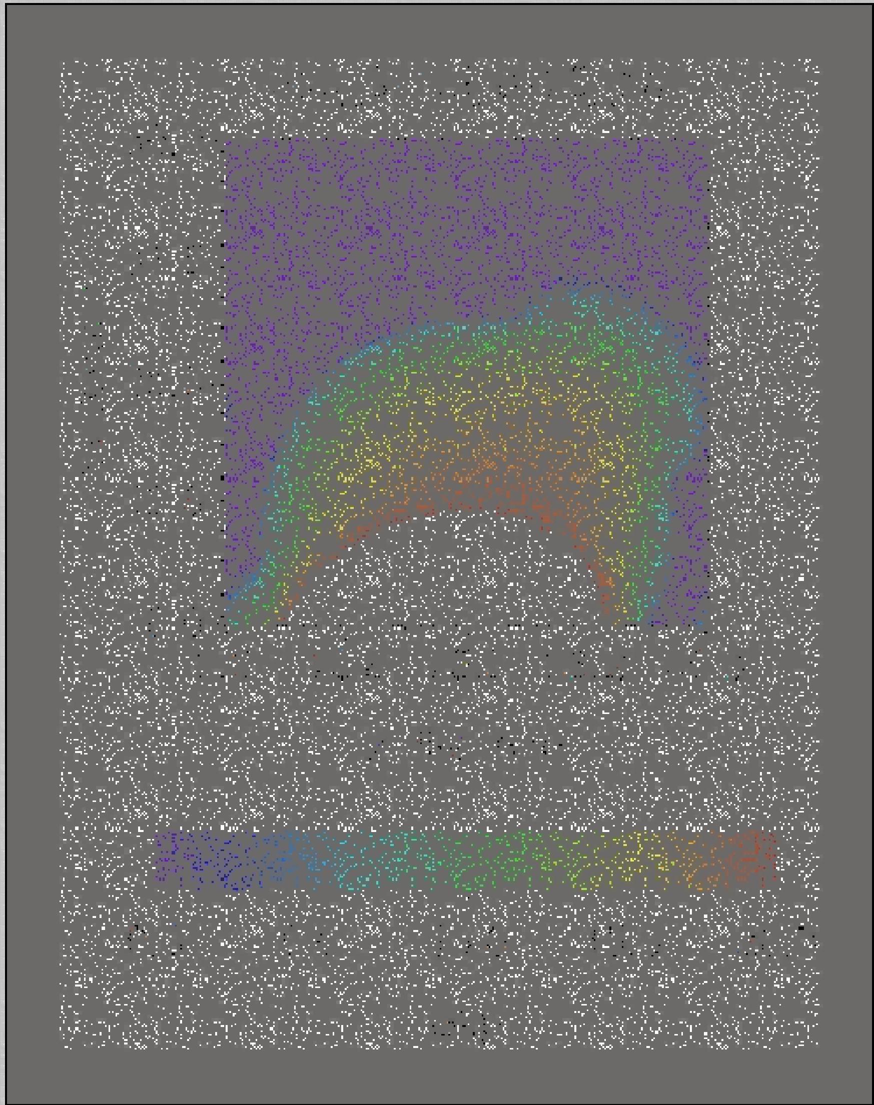
Effects of Strength Heterogeneity #2







Effects of Free Surface



Anim_Slip_37ani_sw_total.avi

This slide is empty intentionally.

Support Slides:
Parameters, Notes, etc.

To not be displayed directly. Referenced above.

Why " *truly* " 3 – D ?

Remembering the dimensionality d' of the problem:

2 – D Mode II (pure in – plane):	$\mathbf{u} = (u_1(x_1,t), 0, 0)$
2 – D Mode III (pure anti – plane):	$\mathbf{u} = (0, u_2(x_1,t), 0)$
3 – D Mixed mode:	$\mathbf{u} = (u_1(x_1,t), u_2(x_1,t), 0)$
3 – D having only one non null component:	$\mathbf{u} = (u_1(x_1,x_2,t), 0, 0)$
Truly 3 – D:	$\mathbf{u} = (u_1(x_1,x_2,t), u_2(x_1,x_2,t), 0)$

Test #	26ani_sw	3 – D	FD
Constitutive law	**Slip - weakening**		
Simulation Date	14-12-02		
System	*Mk*		
Categorized as	**Homogeneous**		
Input Set type	*Non - dimensional units*		
Δx , Δy , Δz	0.2	0.2	0.2
Arrays size	254	83	251
Iterations in time	350		
Mass density (ρ)	1.		
v_S , v_P	1.	1.732	
Initial stress (τ_0)	1.		
Yield stress (τ_u)	1.8		
Frictional level (τ_f)	0.		
Strength (S)	0.8		
Characteristic length (d_0)	1.3	1.3	1.3
Normal stress (σ_n)	1.		
Initial rake	0.785398 rad.		
Initial slip velocity	0.5		
Nucleation point	25.4	25.	
Fault type	*Vertical Strike - slip*		

Test #	**37ani_sw**	**3 – D**	**FD**
Constitutive law	**Slip - weakening**		
Simulation Date	15-10-02		
System	*Mk*		
Categorized as	**Homogeneous**		
Input Set type	*Non - dimensional units*		
Δx , Δy , Δz	0.2	0.2	0.2
Arrays size	254	83	251
Iterations in time	350		
Mass density (ρ)	1.		
v_S , v_P	1.	1.732	
Initial stress (τ_0)	1.		
Yield stress (τ_u)	1.8		
Frictional level (τ_f)	0.		
Strength (S)	0.8		
Characteristic length (d_0)	1.3	1.3	1.3
Normal stress (σ_n)	1.		
Initial rake	0.785398 rad.		
Initial slip velocity	0.5		
Nucleation point	25.4	25.	
Fault type	*Vertical Strike - slip*		

Test #	var10ani_sw	3 – D	FD
Constitutive law	**Slip - weakening**		
Simulation Date	19-12-02		
System	*Mk*		
Categorized as	**Heterogeneous**		
Input Set type	*Non - dimensional units*		
Δx , Δy , Δz	0.8	0.2	0.8
Arrays size	254	83	251
Iterations in time	700		
Mass density (ρ)	1.		
v_S , v_P	1.	1.732	
Initial stress (τ_0)	1.	1.	1.
Yield stress (τ_u)	1.8	4.	4.
Frictional level (τ_f)	0.	0.	0.
Strength (S)	0.8	3.	3.
Characteristic length (d_0)	1.3	1.3	1.3
Normal stress (σ_n)	1.		
Initial rake	0.785398 rad.		
Initial slip velocity	0.5		
Nucleation point	25.4	10.	
Fault type	*Vertical Strike - slip*		

Test #	var8_sw	3 – D	FD
Constitutive law	**Slip - weakening**		
Simulation Date	08-11-02		
System	*Mk*		
Categorized as	**Heterogeneous**		
Input Set type	*Non - dimensional units*		
Δx , Δy , Δz	0.8	0.2	0.8
Arrays size	254	83	251
Iterations in time	700		
Mass density (ρ)	1.		
v_S , v_P	1.	1.732	
Initial stress (τ_0)	1.	1.	1.
Yield stress (τ_u)	1.8	3.	3.
Frictional level (τ_f)	0.	0.	0.
Strength (S)	0.8	2.	2.
Characteristic length (d_0)	1.3	1.3	1.3
Normal stress (σ_n)	1.		
Initial rake	0.785398 rad.		
Initial slip velocity	0.5		
Nucleation point	25.4	10.	
Fault type	*Vertical Strike - slip*		