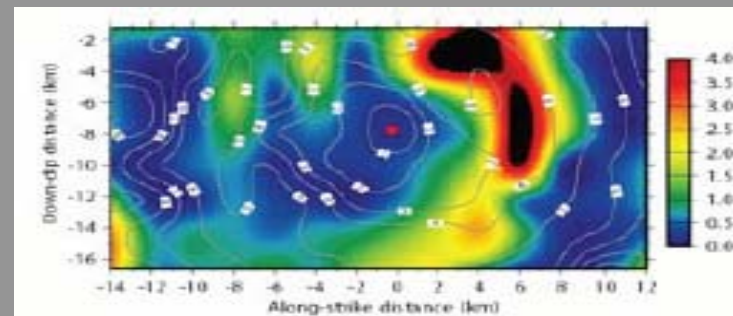




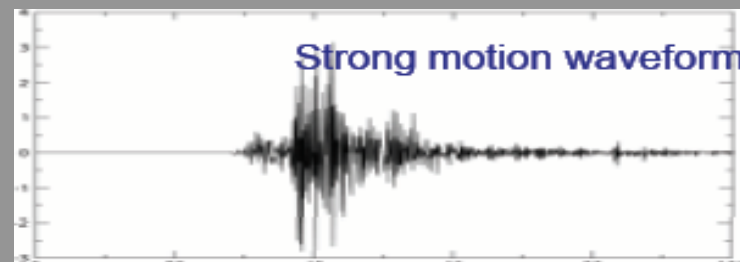
**Fault governing laws  
( constitutive equations )**

# Seismologists need traction

- ✓ To apply fracture mechanics on mathematical planes representing the fault surfaces;
- ✓ To numerically simulate the spontaneous rupture nucleation, propagation, healing and arrest in dynamic earthquake models;

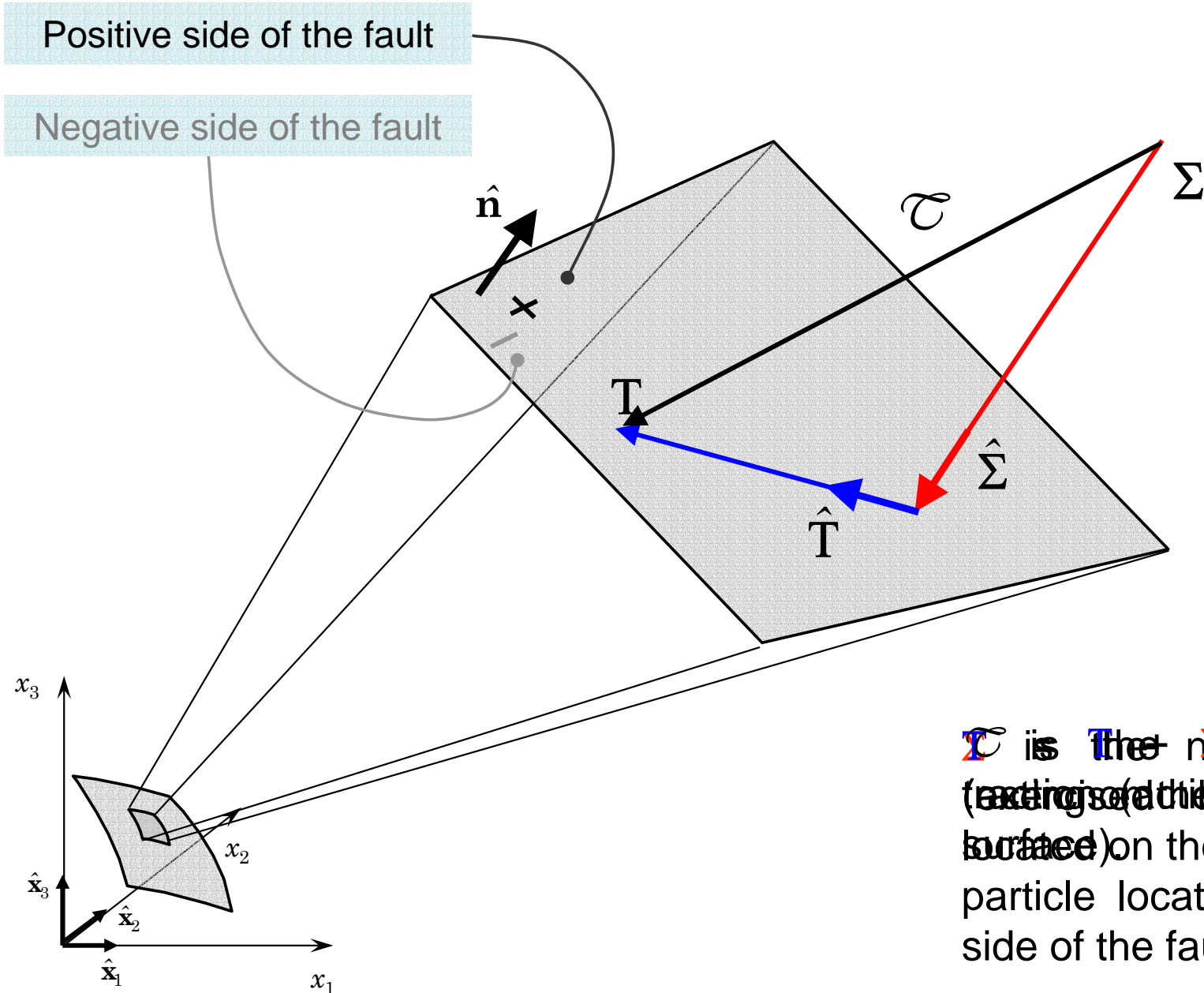


- ✓ To model seismic wave propagation in the surrounding medium;



- ✓ To predict ground shaking.

# Notations and symbols



$$\mathcal{T}^{(\hat{n})} = \mathbf{T}^{(\hat{n})} + \Sigma^{(\hat{n})} \quad \text{total traction (acting on the fault surface).}$$

$$\mathcal{T}_j^{(\hat{n})} = n_i \sigma_{ij}^{eff} \quad \text{Cauchy's formula, where } \mathcal{T}^{(\hat{n})} = (\mathcal{T}_1^{(\hat{n})}, \mathcal{T}_2^{(\hat{n})}, \mathcal{T}_3^{(\hat{n})}),$$

$$\mathbf{n} = (n_1, n_2, n_3) \text{ and}$$

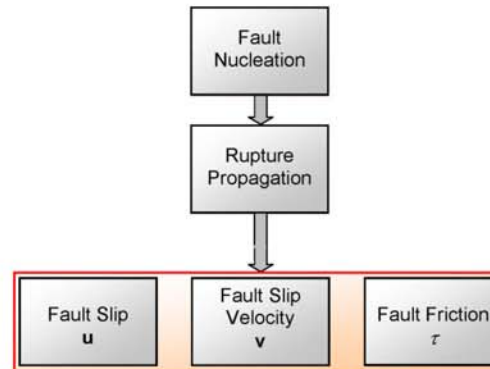
$$\sigma_{ij}^{eff} = \sigma_{ij} + p_{fluid} \delta_{ij} = \begin{bmatrix} -\sigma_{n_1}^{eff} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & -\sigma_{n_2}^{eff} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & -\sigma_{n_3}^{eff} \end{bmatrix}$$

where:  $\sigma_{n_i}^{eff} = \sigma_{n_i} - p_{fluid} = -\sigma_{ii} - p_{fluid}$  and stresses are assumed to be negative for compression

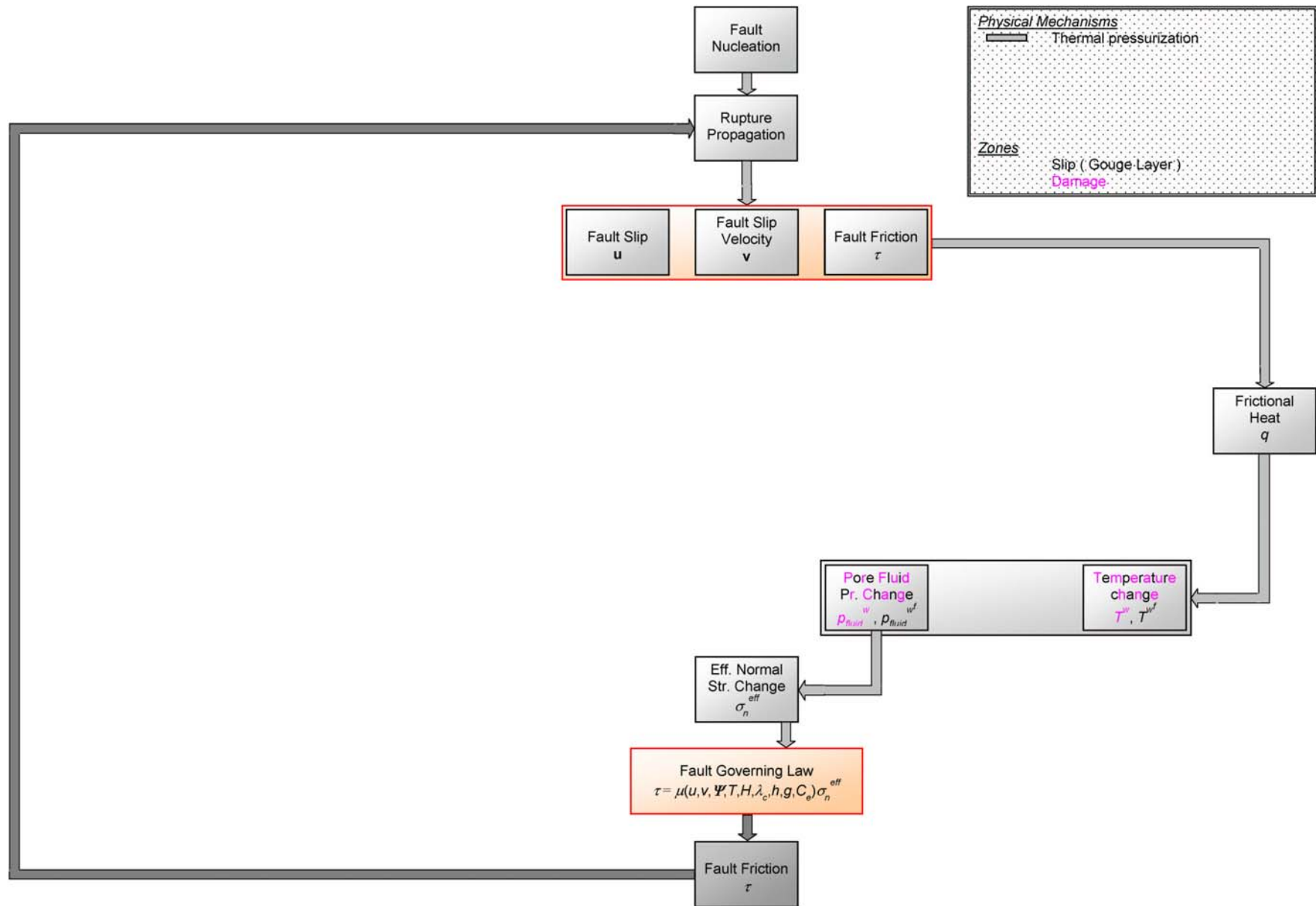
$$T_j^{(\hat{n})} = n_i \sigma_{ij}^{eff} - n_j (n_i \sigma_{ik}^{eff} n_k) \quad \text{shear traction}$$

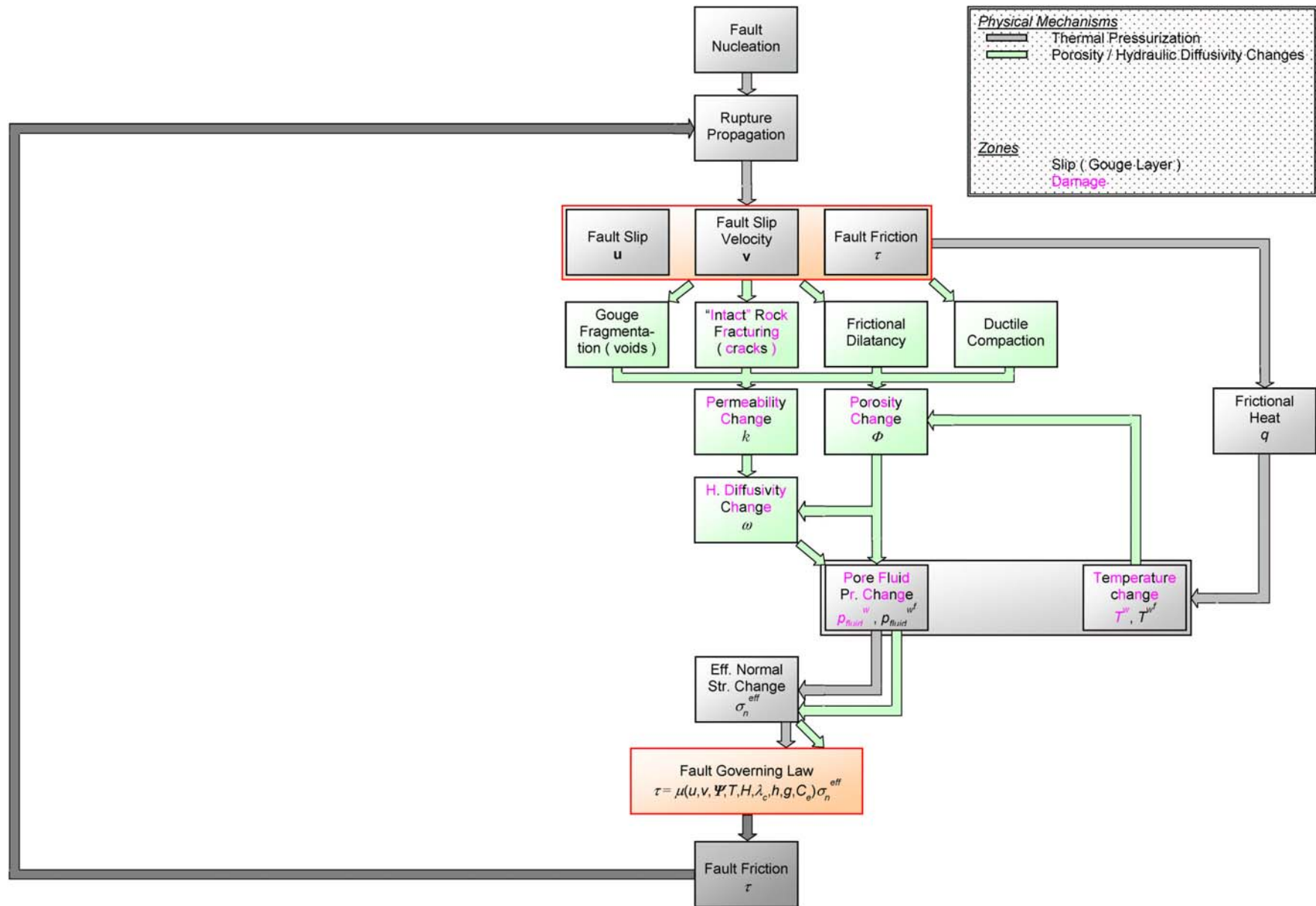
$$\Sigma_j^{(\hat{n})} = n_j (n_i \sigma_{ik}^{eff} n_k) \quad \text{normal traction}$$

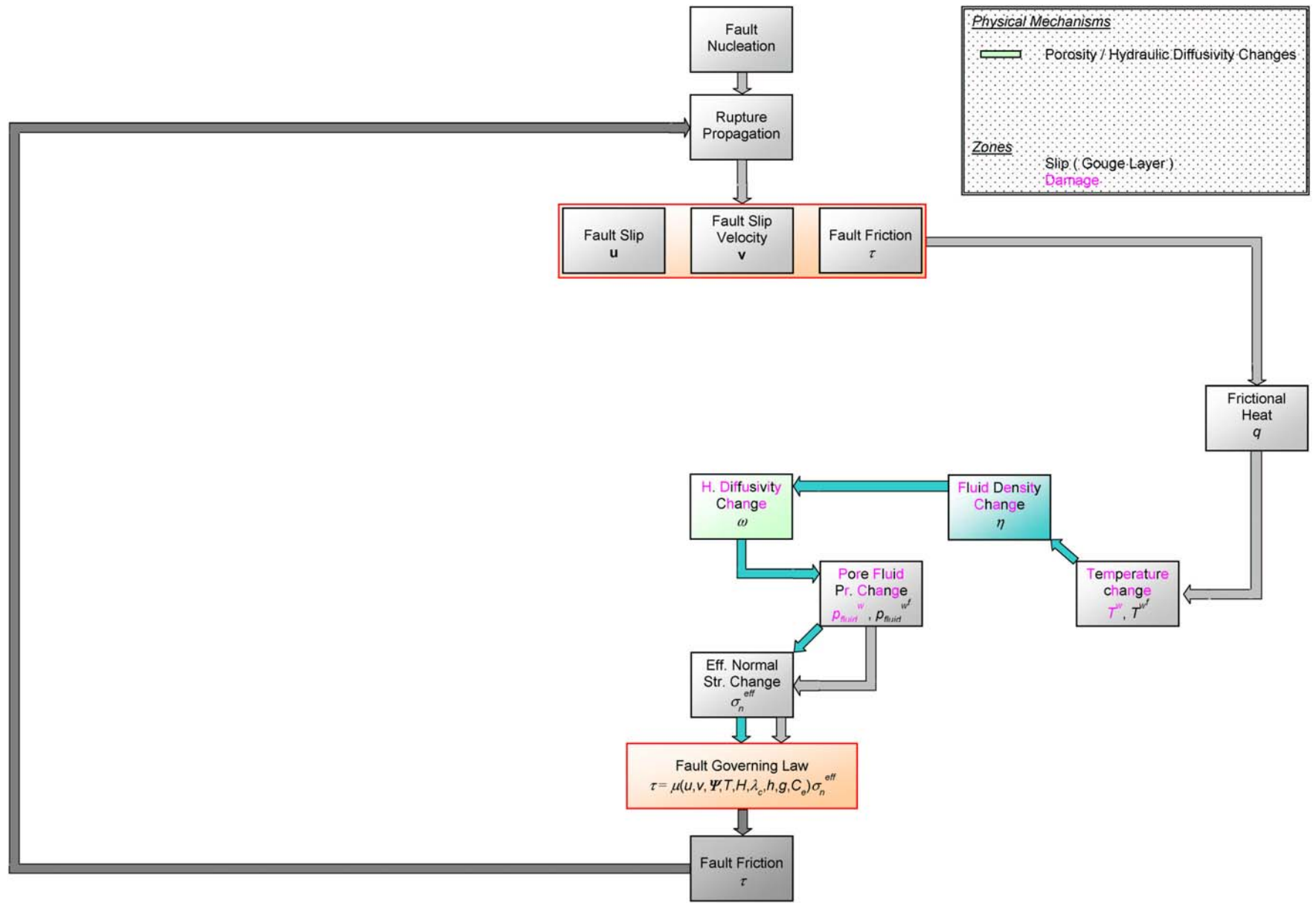
# Physical Phenomena in Faulting



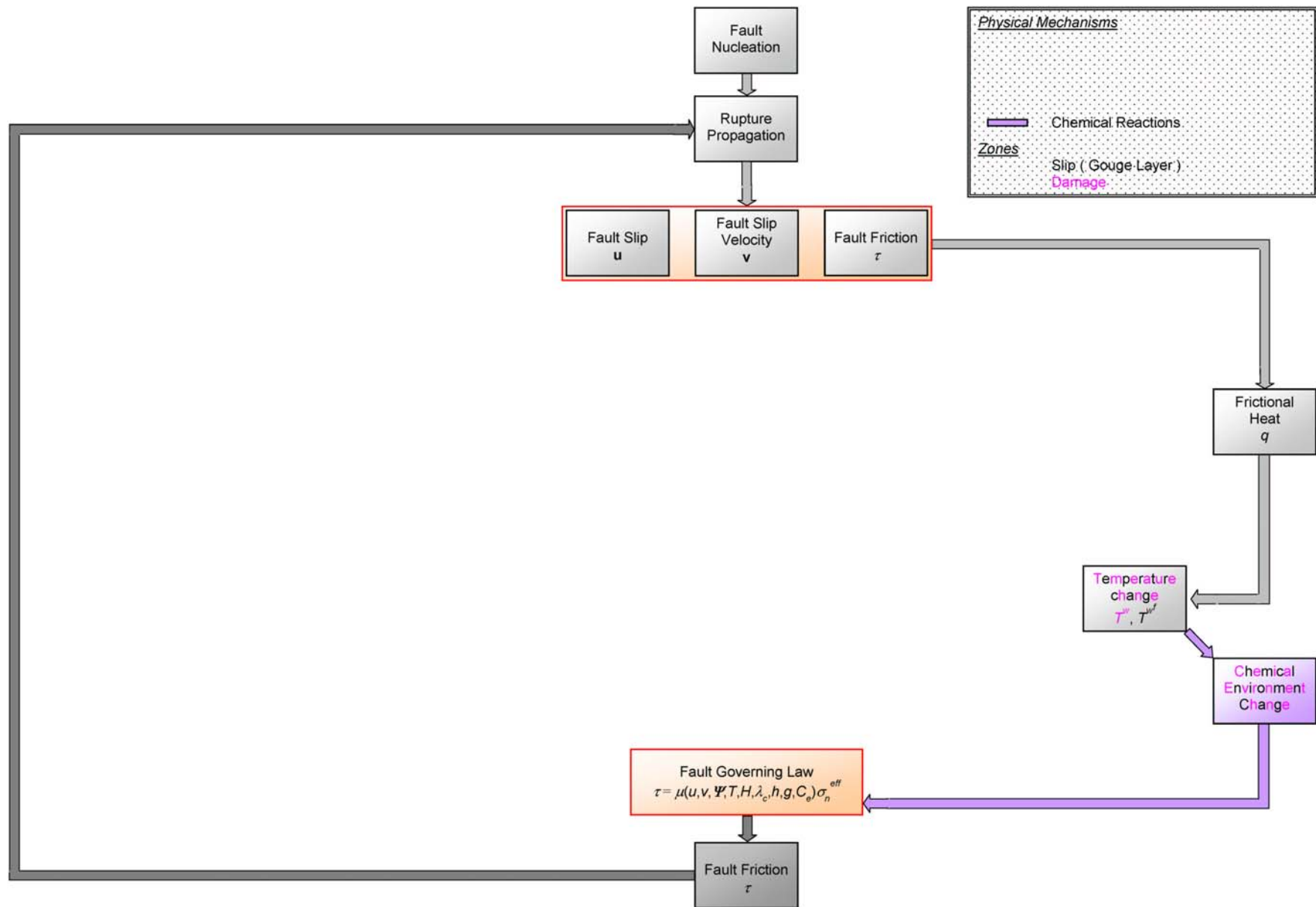


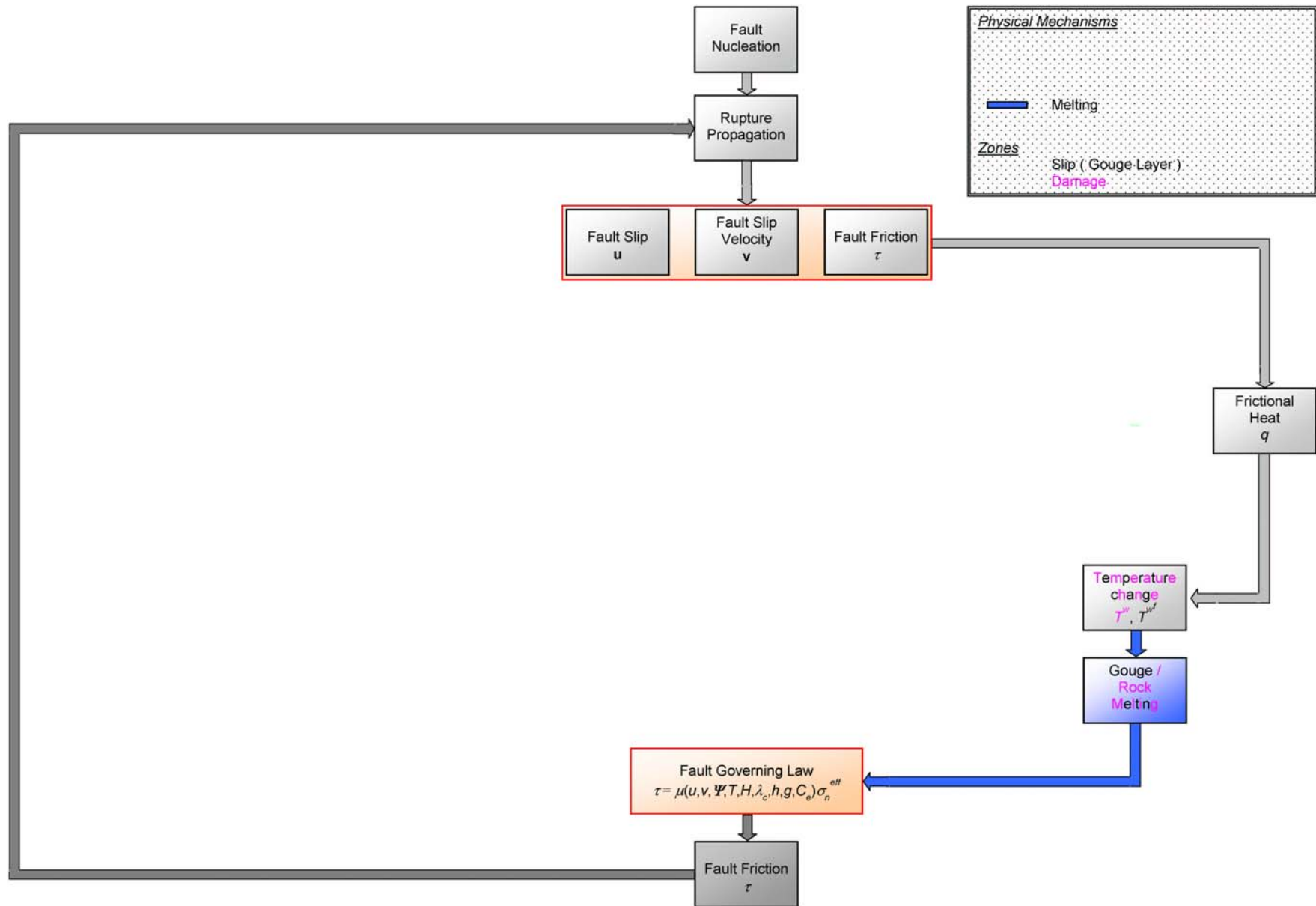


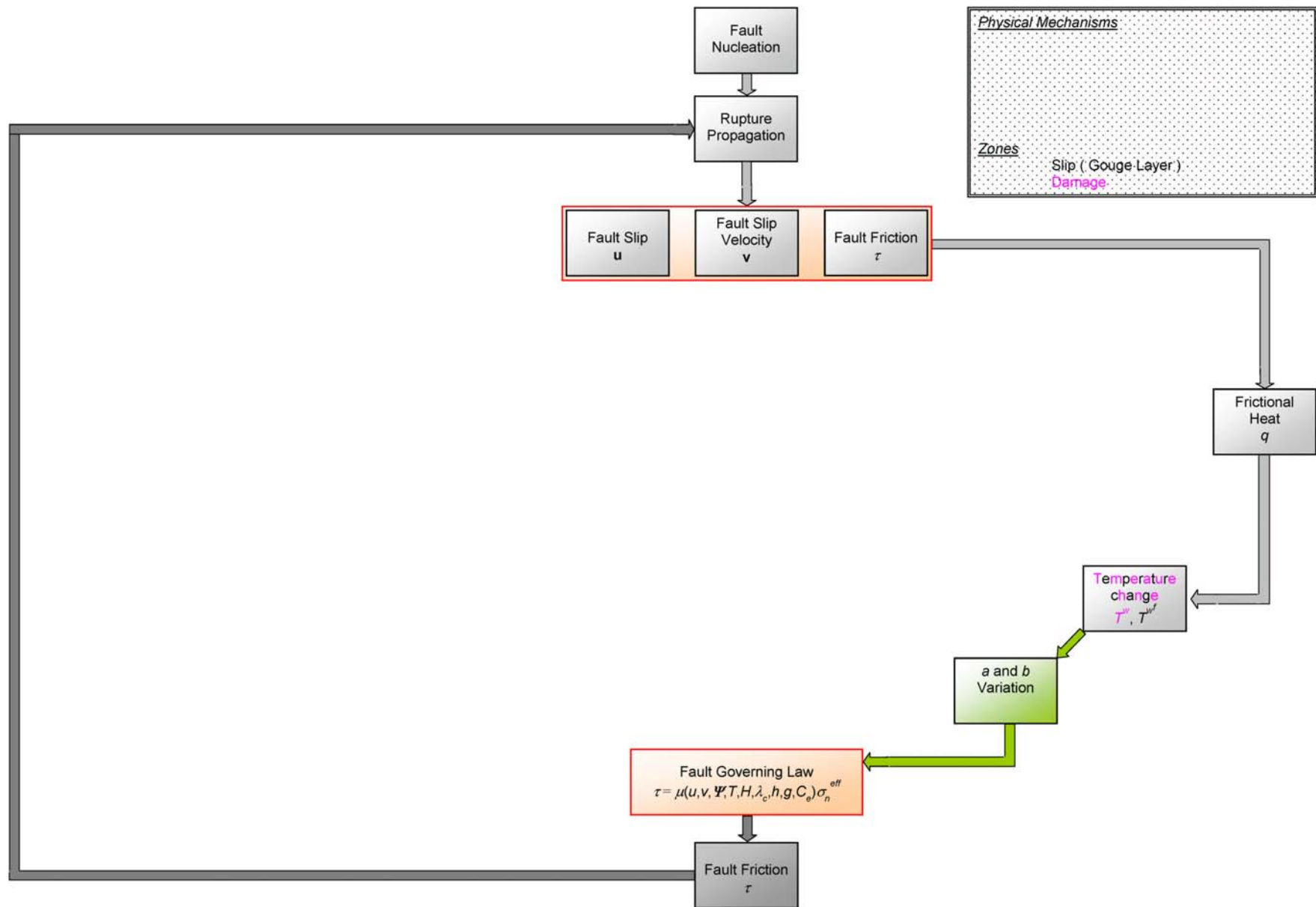


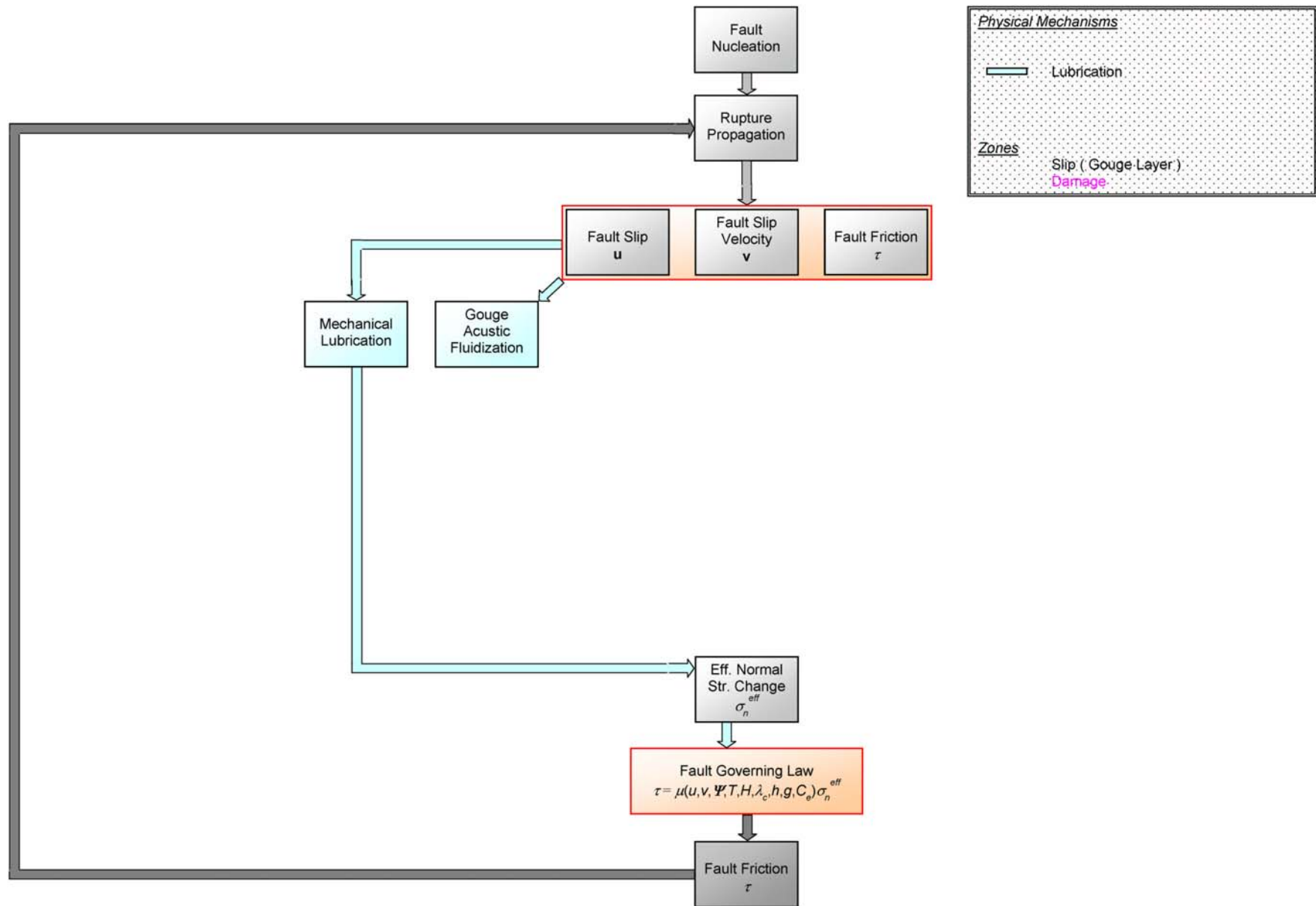


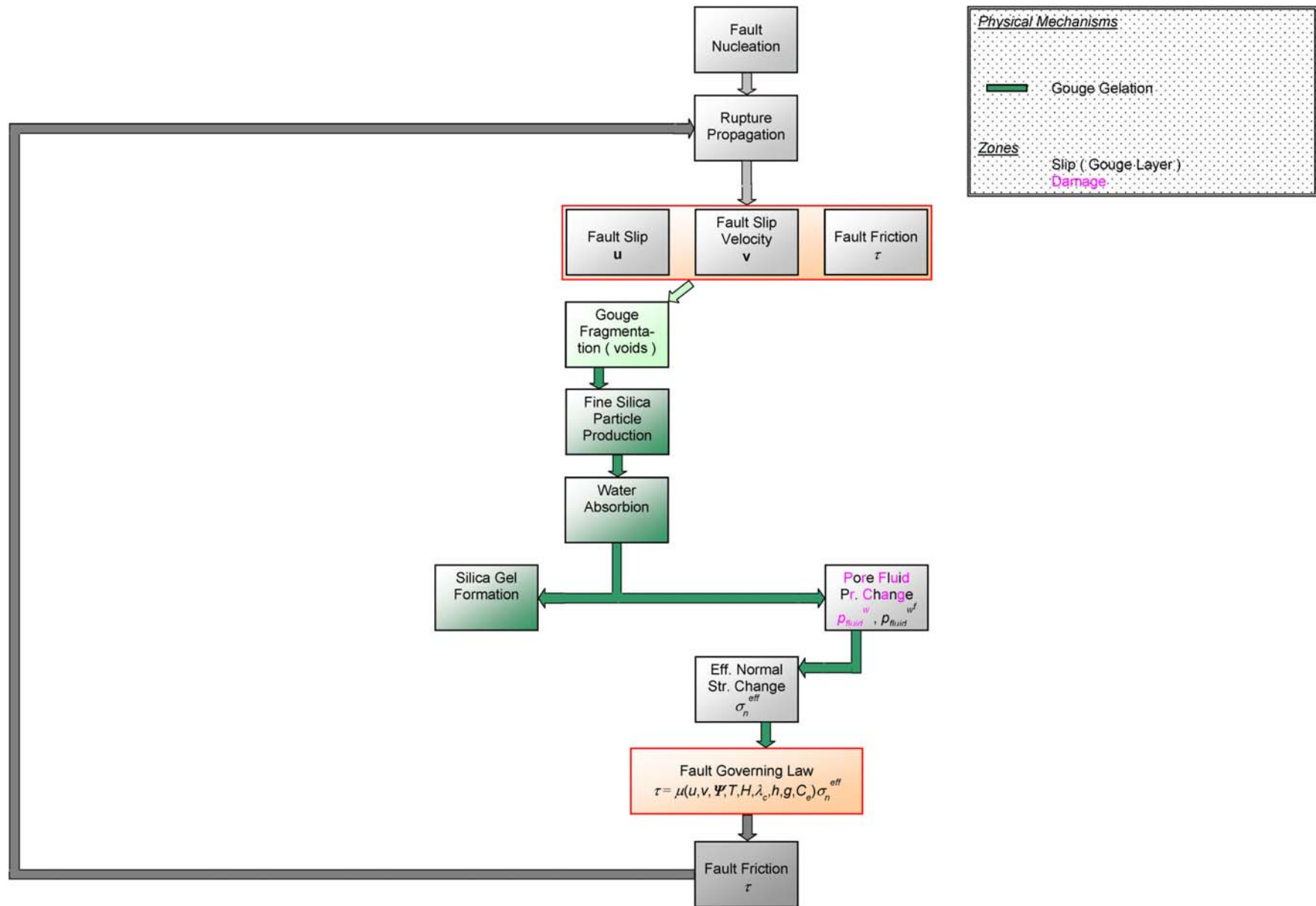


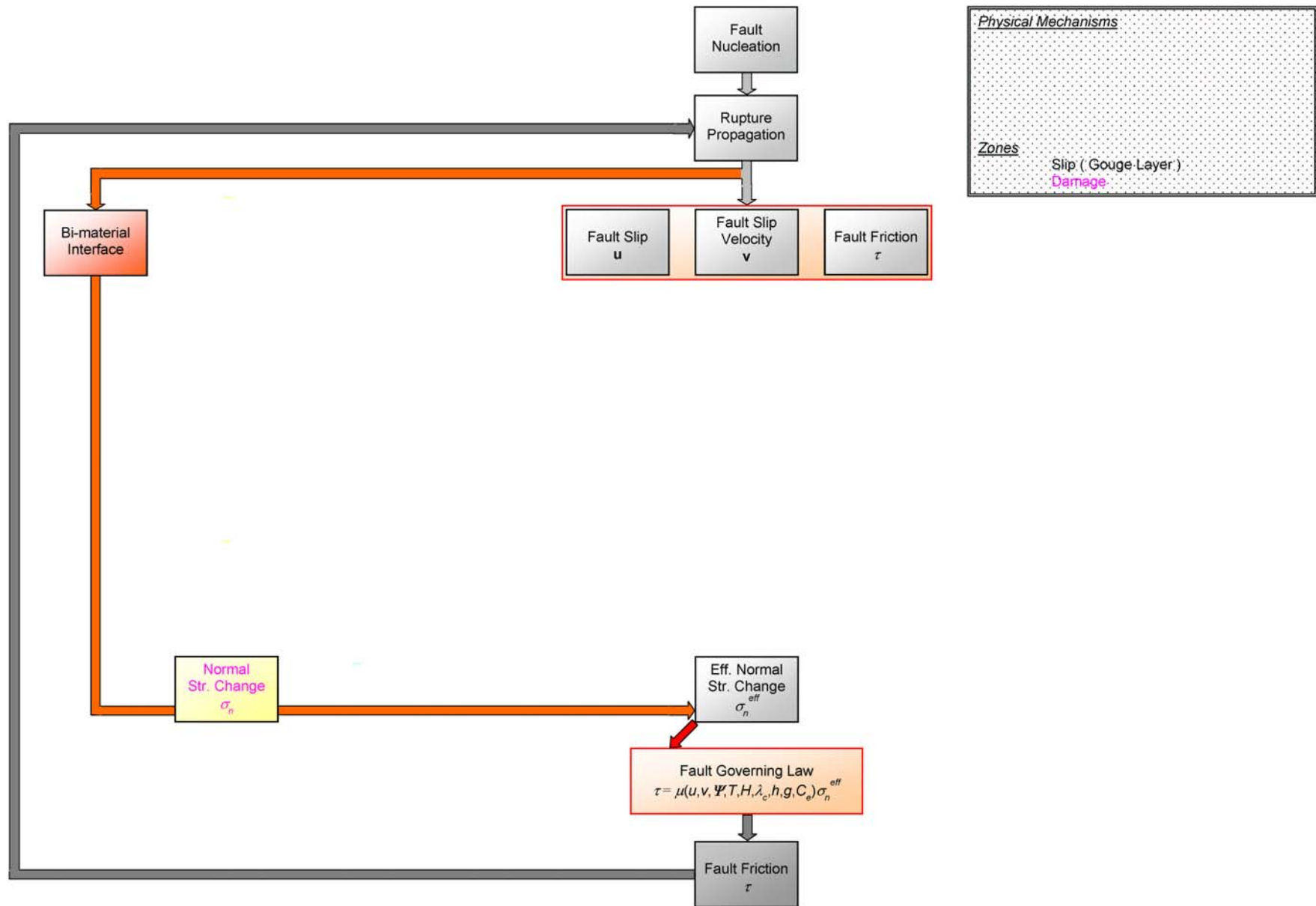




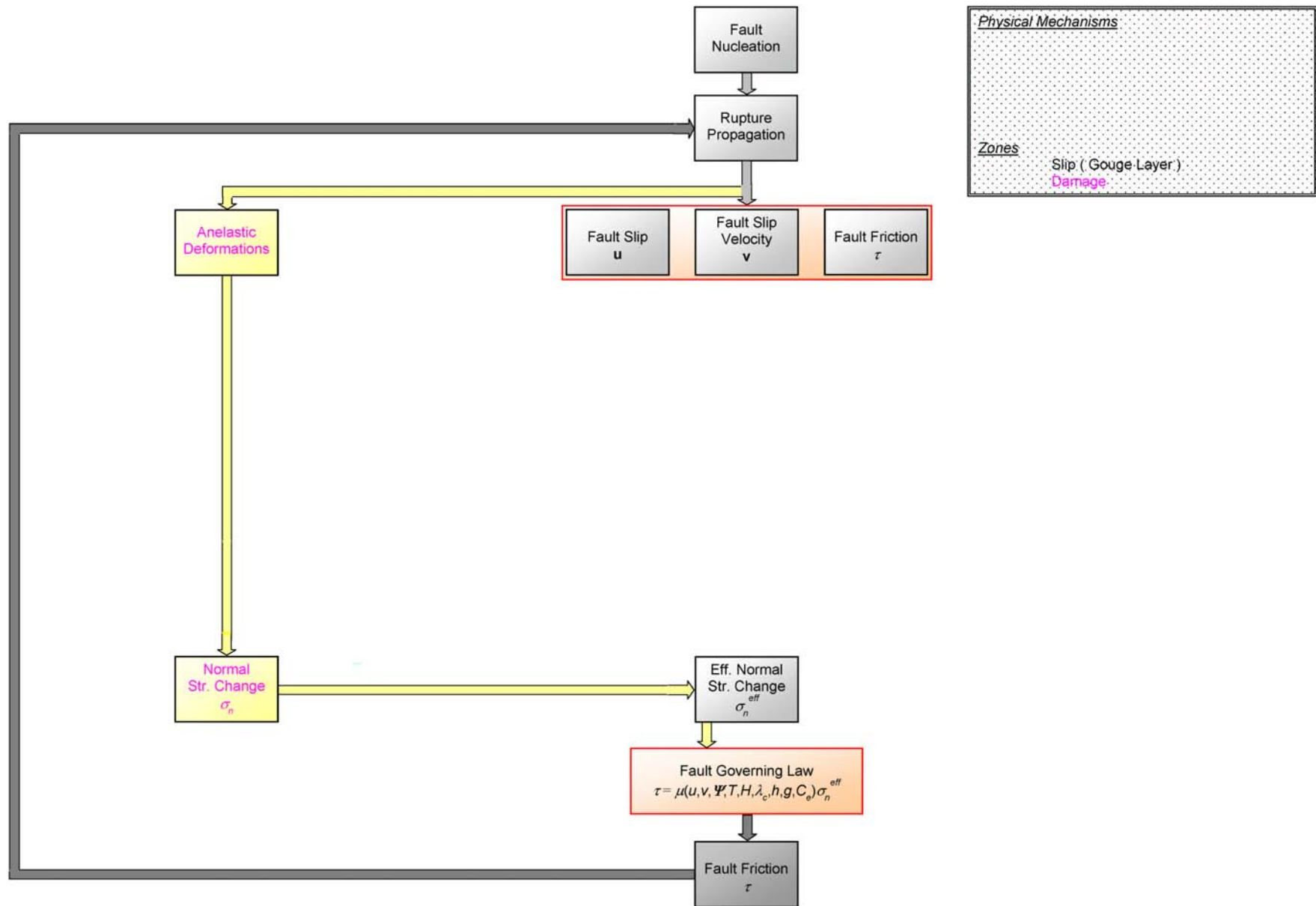


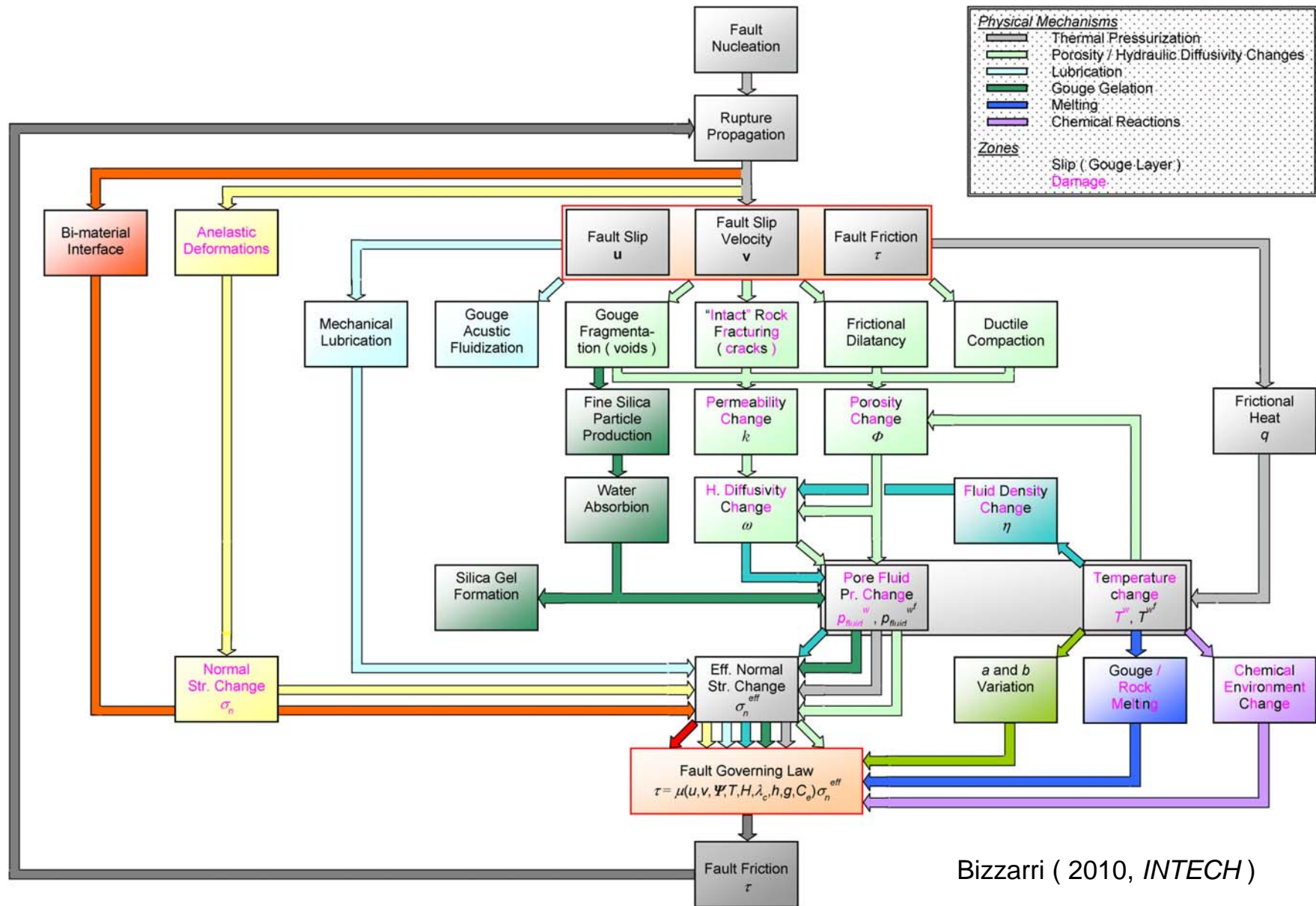














# Fracture Criteria & Constitutive Laws

## 1. FRACTURE CRITERION

- Condition that specify, at a given fault point and at a given time, if there is a rupture or not.
- It can be expressed in terms of **energy**, in terms of **maximum frictional resistance**, and so on.
- It is based on (i) the *Benioff* ( 1951 ) hypothesis: The fracture occurs when the stress in a volume reaches the rock strength  
or, analogously,  
(ii) the *Reid* ( 1910 ) statement: The fracture takes place when the stress attains a value greater than the rock can endure.

## 2. CONSTITUTIVE LAW

- Analytical relation existing between the components of the stress tensor and physical observable(s), like the slip, the slip velocity, the state variable, etc..
- From a mathematical point of view it is a **Fault Boundary Condition ( FBC )** that controls earthquake dynamics and its complexity in space and in time.
- Its simplest form consider only **two frictional levels**,  $\tau_u$  and  $\tau_f$ ; it accounts for stress drop ( or stress release ), but the process is instantaneous: there is a singularity at crack tip. 
- **Cohesive zone models**: *Barenblatt ( 1959a, 1959b )*, *Ida ( 1972 )*, *Andrews ( 1976a, 1976b )*. In these models the singularity is removed and the stress release occurs over a breakdown zone distance  $X_b$  and in a breakdown zone time  $T_b$ . 
- Friction laws ( Rate and State dependent f. l. ): *Dieterich ( 1976 )*, *Ruina ( 1980, 1983 )*. They accounts for fault spontaneous nucleation, re – strengthening, healing, etc..

## ***CONSTITUTIVE LAW ( continues )***

- “ The central issue is *whether* faults obey simple friction laws, and if so, what is the friction coefficient associated with fault slip “ ( *Scholz and Hanks, 2004* ).

## CONSTITUTIVE LAW ( continues )

- In full of generality we can express the constitutive ( or governing ) as:

$$\tau = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e) \sigma_n^{eff}(\sigma_n, p_f)$$



where:

1st – order dependencies

- $u$  is the Slip ( i. e. displ. disc. ) modulus, ←
- $v$  is the Slip Velocity modulus ( its time der. ), ←
- $\Psi = ( \Psi_1, \dots, \Psi_N )$  is the State Variable vector, ←
- $T$  is the Temperature ( accounting for Ductility, Plastic Flow, Melting and Vaporization ),
- $H$  is the Humidity,
- $\lambda_c$  is the Characteristic Length of surface ( accounting for Roughness and Topography of asperity contacts ),
- $h$  is the Hardness,
- $g$  is the Gouge ( accounting for Surface Consumption and Gouge formation ),
- $C_e$  is the Chemical Environment



# Strength & Constitutive Laws

## 1. THE STRENGTH PARAMETER

- Historically introduced by *Das and Aki* ( 1977a, 1977b ) to have a quantitative estimate of the ability to fracture for a fault
- Its expression can be generalized as:

$$S = (\mu_u \sigma_n^{eff} - \tau_0) / (\tau_0 - \mu_f \sigma_n^{eff})$$

where  $\mu$  are the friction coefficient.

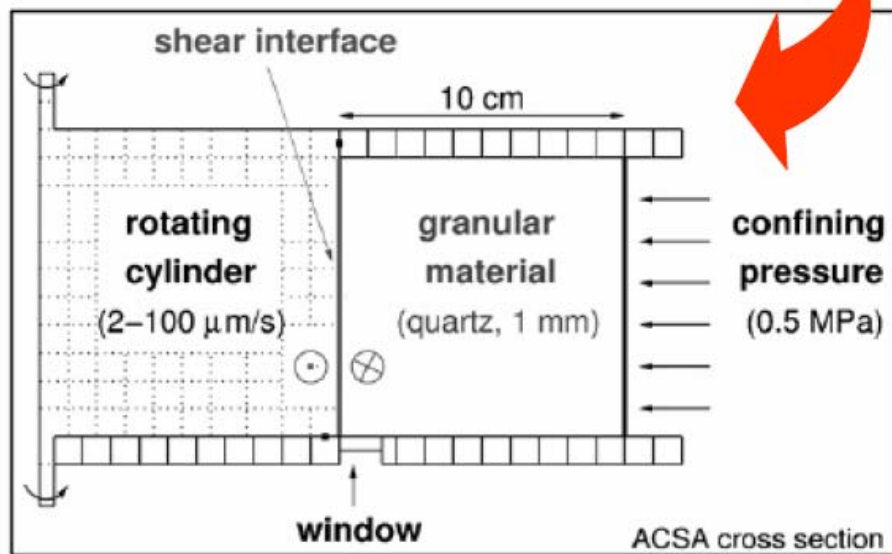
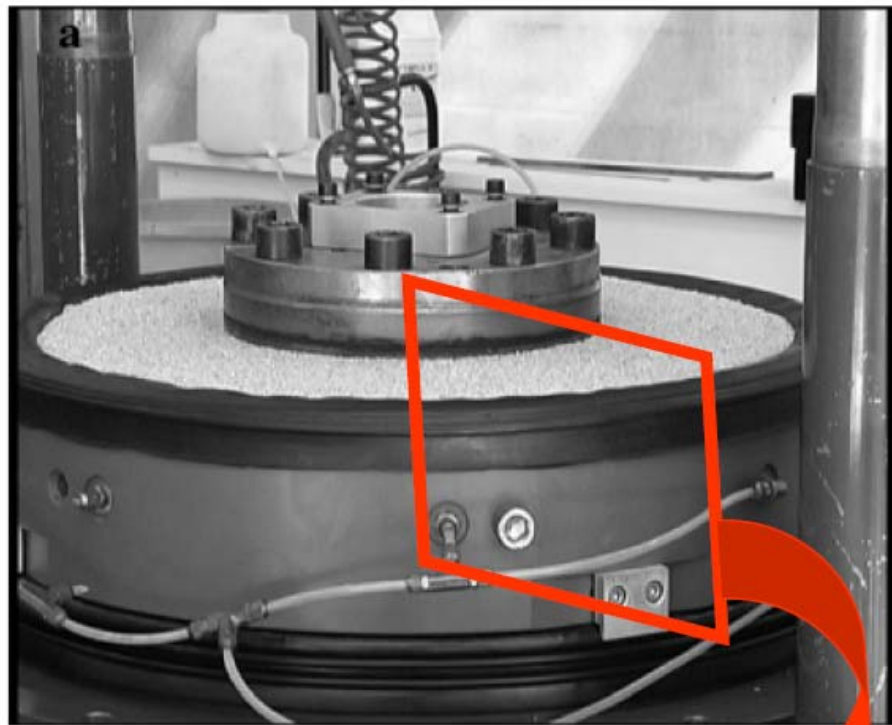
- We can also define

## 2. THE FAULT STRENGTH

- Is the parameter that quantify the Strength in the more general case, in which a fault is described by a rhealistic friction laws

$$S^{fault} = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e) \sigma_n^{eff}(\sigma_n, p_{fluid})$$

# Annular simple shear apparatus



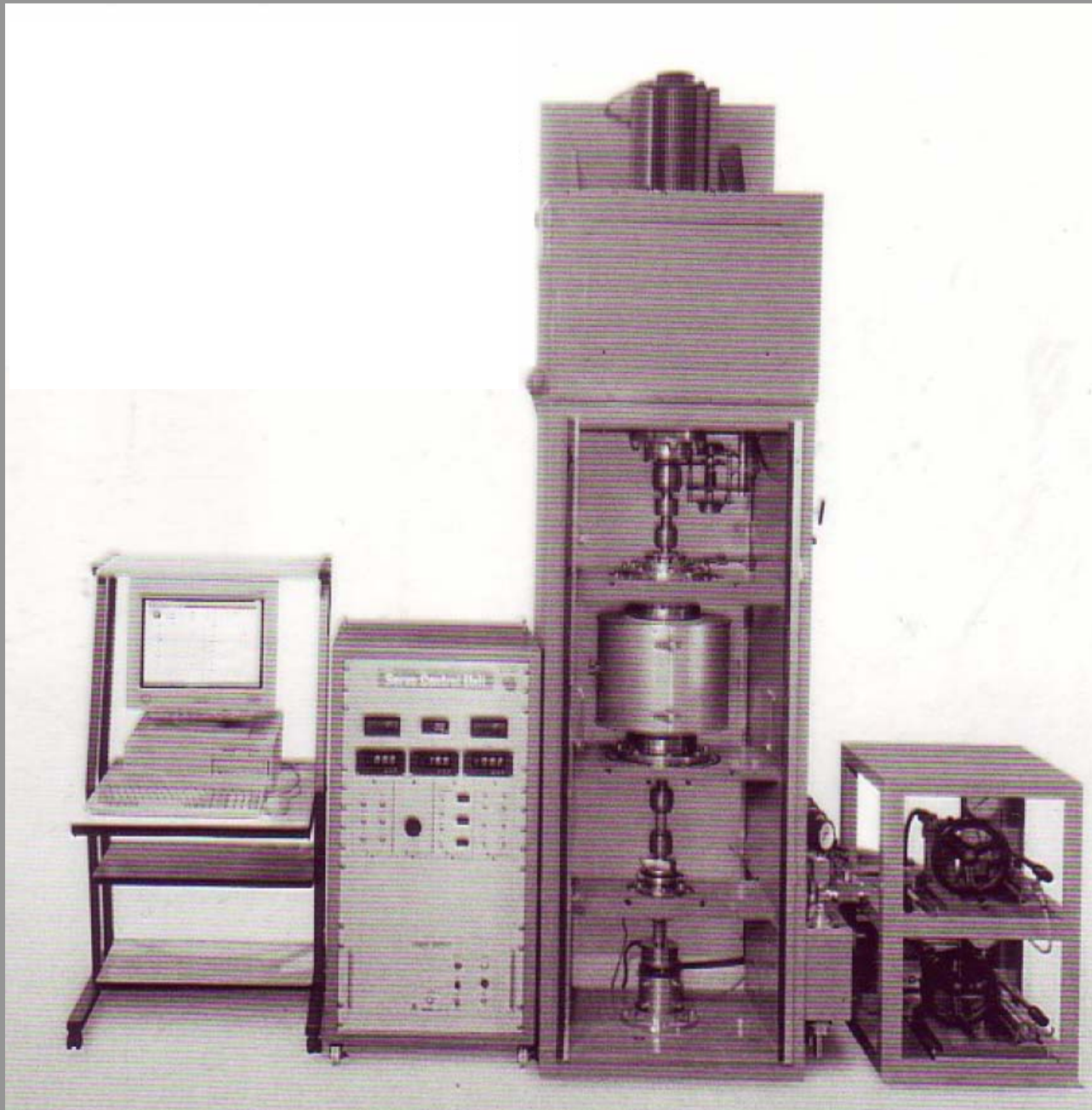
$$u_{tot} < 50 \text{ m}$$

$$v = 1 \mu\text{m/s} - 0.1 \text{ mm/s}$$

$$\sigma_n^{eff} < 1 \text{ MPa}$$

Chambon et al. ( 2006a, 2006b, *JGR*, 111, B09308, B09309 )

# High velocity rotary friction apparatus



$U_{tot} = \text{infinite}$

$v = 0.1 \mu\text{m/s} - 10 \text{ m/s}$

$\sigma_n^{eff} < 20 \text{ MPa}$

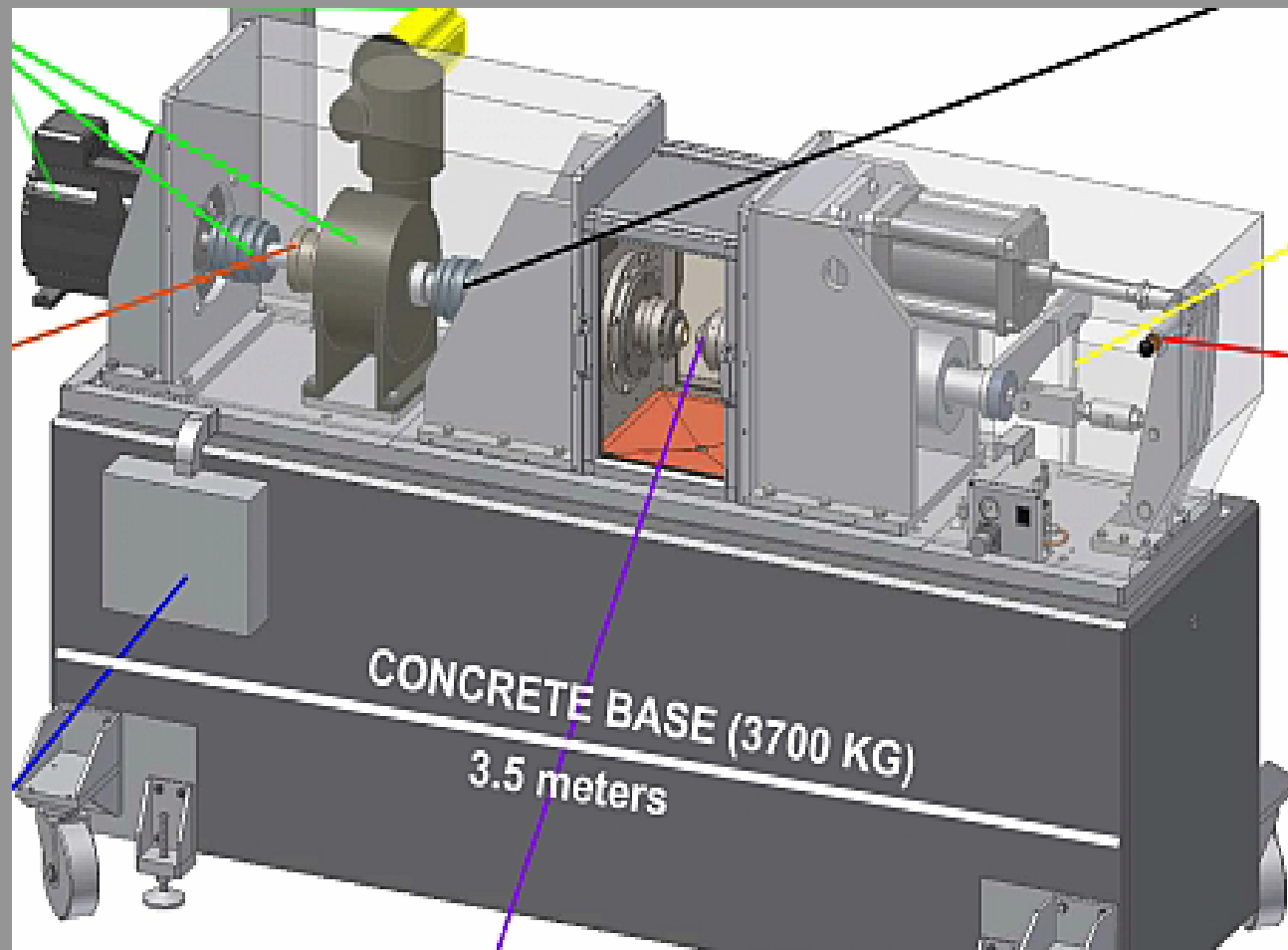
Shimamoto and Tsutumi ( 2004,  
*Str. Geol.*, **39** )

# High velocity rotary friction apparatus @ INGV

$U_{tot} = \text{infinite}$

$v = 1 \mu\text{m/s} - 9 \text{ m/s}$

$\sigma_n^{eff} < 70 \text{ MPa}$



Niemeijer et al. ( 2009, *AGU Fall Meeting* )

# Time - weakening Friction Law

$$\tau = \begin{cases} \left[ \mu_u - (\mu_u - \mu_f) \frac{(t - t_r)}{t_0} \right] \sigma_n^{eff} & , t - t_r < t_0 \\ \mu_f \sigma_n^{eff} & , t - t_r \geq t_0 \end{cases}$$

ilaw = 11

**TW**

$t_r = t_r(\xi)$  is the rupture onset time in every fault point  $\xi$  (when  $u > 0$ ).

Andrews ( 1985 ), Bizzarri et al. ( 2001 ) and other following Bizzarri' s papers

$t_0$  is the characteristic time – weakening duration.

# Position - weakening Friction Law

$$\tau = \begin{cases} \left[ \mu_u - (\mu_u - \mu_f) \frac{x}{R_0} \right] \sigma_n^{eff} & , -R_0 < x < 0 \\ \mu_f \sigma_n^{eff} & , -L < x < -R_0 \end{cases}$$

PW

$x$  is the position on the fault (extending up to  $-L$ ).

Palmer and Rice (1973)

$R_0$  is the characteristic position – weakening distance.



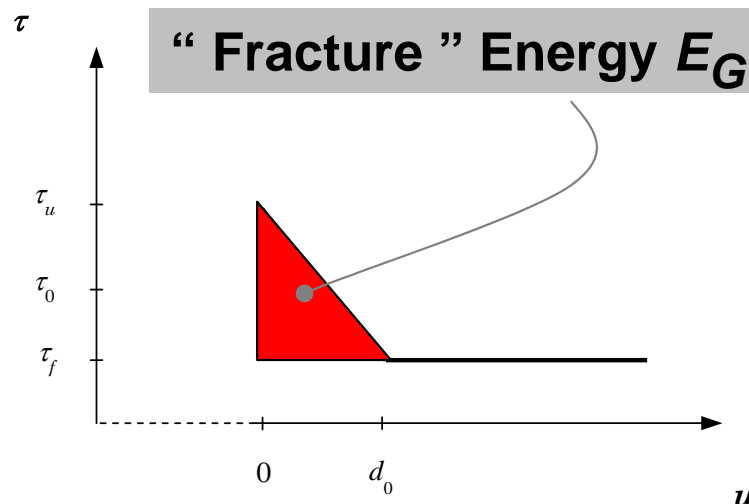
# Slip - Dependent Friction Laws

## 1. LINEAR SLIP – WEAKENING LAW

$$\tau = \begin{cases} \left[ \mu_u - (\mu_u - \mu_f) \frac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

ilaw = 21

SW



Barenblatt ( 1959a, 1959b ), Ida ( 1972 ), Andrews ( 1976a, 1976b ), and many authors thereafter

$d_0$  is the characteristic slip – weakening distance

## 2. NON – LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[ \mu_u - \frac{\mu_u - \mu_f}{d_0} \left( u - \frac{(1 - p_{IW})d_0}{2\pi} \sin\left(\frac{2\pi u}{d_0}\right) \right) \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

Ionescu and Campillo ( 1999 )

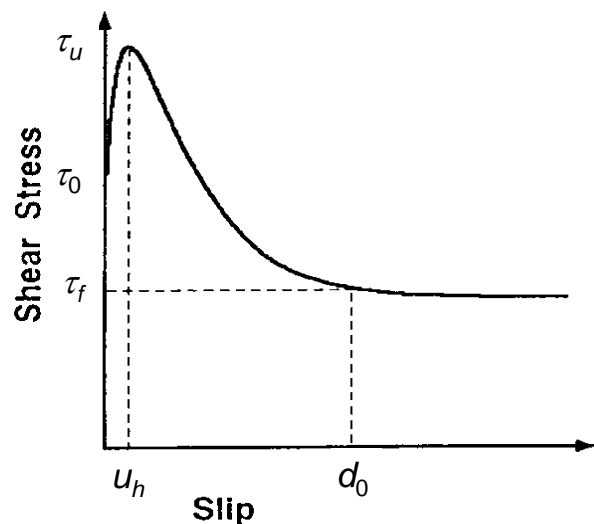
### 3. NON LINEAR SLIP – WEAKENING LAW WITH SLIP – HARDENING

$$\tau = \left\{ \left[ \left( \frac{\tau_0}{\sigma_n^{eff}} - \mu_f \right) \left( 1 + \alpha_{OY} \ln \left( 1 + \frac{u}{\beta_{OY}} \right) \right) \right] e^{-\frac{u}{d_0}} + \mu_f \right\} \sigma_n^{eff}$$

ilaw = 23

OW

$$u_h : \left. \frac{d\tau}{du} \right|_{u_h} = 0; \quad \begin{cases} u_h = r d_0 \quad (\text{e.g. } r = 0.1) \\ \tau(u_h) = \tau_u \end{cases}$$



Ohnaka and Yamashita (1989) and the following papers by Ohnaka and coworkers

$u_h$  is associated with the preparatory phase of the imminent macroscopic failure in the cohesive zone. It accounts for micro-cracking

## 4. NON LINEAR SLIP – WEAKEING LAW WITH EXPONENTIAL DECAY

$$\tau = \left[ (\mu_u - \mu_f) e^{-\frac{u}{d_0}} + \mu_f \right] \sigma_n^{eff}$$

ilaw = 24

**EW**

## 5. POWER LAW SLIP – WEAKENING

$$\tau = \left\{ \mu_u - (\mu_u - \mu_f) \left[ \left( \frac{p_{PW}}{p_{PW} + 1} \right) \frac{u}{d_0} \right]^{p_{PW}} \right\} \sigma_n^{eff}$$

ilaw = 25

PW

# Rate - Dependent Friction Law

$$\tau = \frac{v_*}{v + v_*} \mu_u \sigma_n^{eff}$$

*Burrige and Knopoff ( 1967 ),  
Carlson and Langer ( 1989 ),  
Madariaga and Cochard ( 1994 ),  
Cochard and Madariaga ( 1994 )*



# Rate - and State - Dependent Friction Laws



## 1. DIETERICH IN REDUCED FORMULATION

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - a \ln \left( \frac{v_*}{v} \right) + b \ln \left( \frac{\Psi v_*}{L} \right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = 1 - \frac{\Psi v}{L} \end{array} \right.$$

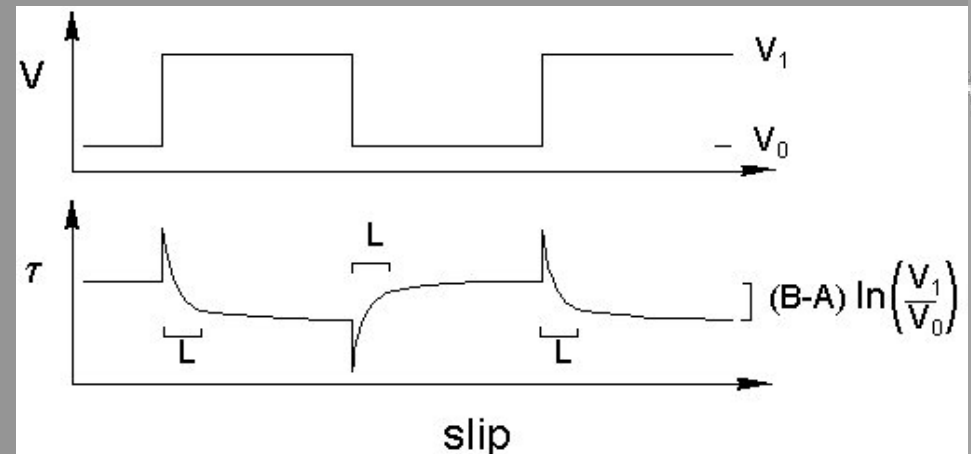
ilaw = 31

DR

However, while in velocity stepping experiments the traction response following the velocity variation is directly controlled by the parameter  $L$ , its effects are much less evident during the dynamic rupture propagation.

Bizzarri and Cocco (2005)

**Response to an abrupt jump in load**



## 2. RUINA – DIETERICH

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - a \ln\left(\frac{v_*}{v}\right) + b \ln\left(\frac{\Psi v_*}{L}\right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = -\frac{\Psi v}{L} \ln\left(\frac{\Psi v}{L}\right) \end{array} \right.$$

ilaw = 32

RD

Ruina ( 1980, 1983 ), Beeler et al. ( 1984 ), Roy and Marone ( 1996 )

### 3. DIETERICH – RUINA WITH VARYING NORMAL STR.

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - a \ln\left(\frac{v_*}{v}\right) + b \ln\left(\frac{\Psi v_*}{L}\right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = 1 - \frac{\Psi v}{L} - \left( \frac{\alpha_{LD} \Psi}{b \sigma_n^{eff}} \right) \frac{d}{dt} \sigma_n^{eff} \end{array} \right.$$

ilaw = 31

decis10=T

**DR**

Linker and Dieterich ( 1992 ), Dieterich and Linker ( 1992), Bizzarri and Cocco ( 2006b, 2006c )

## 4. RUINA – DIETERICH WITH VARYING NORMAL STR.

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - a \ln\left(\frac{v_*}{v}\right) + b \ln\left(\frac{\Psi v_*}{L}\right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = -\frac{\Psi v}{L} \ln\left(\frac{\Psi v}{L}\right) - \left(\frac{\alpha_{LD} \Psi}{b \sigma_n^{eff}}\right) \frac{d}{dt} \sigma_n^{eff} \end{array} \right.$$

ilaw = 32

decis10=T

**RD**

Linker and Dieterich ( 1992 ), Bizzarri and Cocco ( 2006b, 2006c )

## 5. DIETERICH IN REDUCED FORM REGULARIZED

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - a \ln \left( \frac{v + v_*}{v + v_r} \right) + b \ln \left( \frac{\Psi(v + v_r)}{L} + 1 \right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = 1 - \frac{\Psi(v + v_r)}{L} \end{array} \right.$$

ilaw = 33

**DE**

$v_r$  is a regularization fault slip velocity

Perrin et al. (1995), Cocco et al. (2004)

## 6. RUINA REGULARIZED

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - \alpha \ln \left( \frac{v_* - v_r}{v + v_r} \right) + \frac{\Psi}{\sigma_n^{eff}} \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = -\frac{v + v_r}{L} \left( \Psi + b \ln \left( \frac{v + v_r}{v_* - v_r} \right) \right) \end{array} \right.$$

ilaw = 34

**RE**

$v_r$  is a regularization fault slip velocity

Bizzarri (2002, unpublished work)

## 7. DIETERICH IN REDUCED FORM WITH HEALING

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - a \ln \left( \frac{v^*}{v} + 1 \right) + b \ln \left( \frac{\Psi v^*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = \frac{\gamma_{fh} - \Psi}{t_{fh}} - \frac{\Psi v}{L} \end{array} \right.$$

ilaw = 35

DH

$$\gamma_{fh} = 1 \text{ s}$$

$t_{fh}$  is the time for healing (slip duration)

Evolution law proposed by Nielsen et al. (2000) and by Nielsen and Carlson (2000). Used in this form by Cocco et al. (2004)

## 9. PRAKASH – CLIFTON

$$\left\{ \begin{array}{l} \tau = \left[ \mu_* - a \ln\left(\frac{v_*}{v}\right) + b \ln\left(\frac{\Psi v_*}{L}\right) \right] \left( \frac{d}{dt} \Psi_1 + \frac{d}{dt} \Psi_2 \right) \\ \frac{d}{dt} \Psi = 1 - \frac{\Psi v}{L} \\ \frac{d}{dt} \Psi_1 = -\frac{v}{L_1} \left( \Psi_1 - \alpha_{PC_1} \sigma_n^{eff} \right) \\ \frac{d}{dt} \Psi_2 = -\frac{v}{L_2} \left( \Psi_2 - \alpha_{PC_2} \sigma_n^{eff} \right) \end{array} \right.$$

ilaw = 37

PC

$\Psi_1$  and  $\Psi_2$  are additional state variables accounting for the coupling with effective normal stress. The formulation of friction law is not based on the Amontons – Coulomb law.

Coupling with effective normal stress proposed by Prakash and Clifton (1993) and Prakash (1998). Used in this form by Bizzarri (2005, unpublished work)



# Slip - and State - Dependent Friction Law

$$\tau = \begin{cases} \left[ (\mu_u - \Delta\mu) \left( 1 - \frac{u}{d_1} \right) \right] \sigma_n^{eff} & , u < d_1, \Psi \geq \Psi_1 \\ 0 & , u \geq d_1, \Psi \geq \Psi_0 \\ \mu_{sp} \left( 1 - \frac{\Psi}{\Psi_0} \right) \sigma_n^{eff} & , \Psi < \Psi_0, \Psi < \Psi_1 \end{cases}$$

$$\frac{d}{dt} \Psi = - \frac{\beta_{CM}}{d_0} (\Psi - v)$$

CM

$\Delta\mu$  is an initial artificial stress drop

$$\Psi_1 \equiv \Psi_0 (u - u_1) / (d_1 - u_1)$$

$$U_1 \equiv - d_1 (\mu_{sp} - \mu_u + \Delta\mu) / (\mu_u - \Delta\mu)$$

$d_0$  and  $d_1$  are characteristic lengths

$\mu_{sp} = 0 \Rightarrow$  linear SW with  $d_1$  as characteristic length

Cochard and Madariaga ( 1994 )

# Free Volume Friction law

$$\left\{ \begin{array}{l} \tau = \sigma_d \operatorname{Arcsinh} \left( \frac{v}{v_*} \frac{e^{\frac{f_* + \chi_s + \chi_h}{\chi}}}{1 - m_0} \right) \\ \frac{d}{dt} \chi = -R_c e^{-\frac{\chi_c}{\chi}} + \alpha_{FV} \tau v \\ m_0 = \begin{cases} 1 & , \tau \leq \tau_0 e^{\frac{\chi_h}{\chi}} \\ \frac{\tau_0}{\tau} e^{\frac{\chi_h}{\chi}} & , \tau > \tau_0 e^{\frac{\chi_h}{\chi}} \end{cases} \end{array} \right.$$

ilaw = 51

**FV**

$\chi \equiv \Phi - \Phi_0$  free volume variable

$\chi_s$  reference value of  $\chi$  for shearing

$\chi_h$  FV value required to create a Shear Transformation Zone ( STZ )

$\chi_c$  FV value for compaction

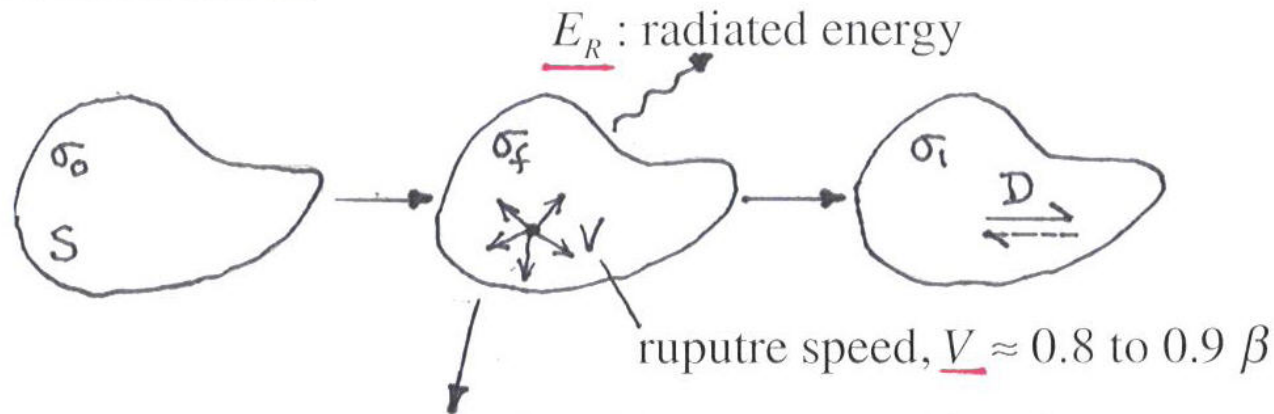
$R_c$  rate of compaction

$\alpha_{FV}$  scaled dilatancy coefficient

*Falk and Langer ( 1998, 2000 );  
Lemaitre ( 2002 ); Daub and Carlson  
( 2008 )*

# How to relate relevant quantities to constitutive parameters

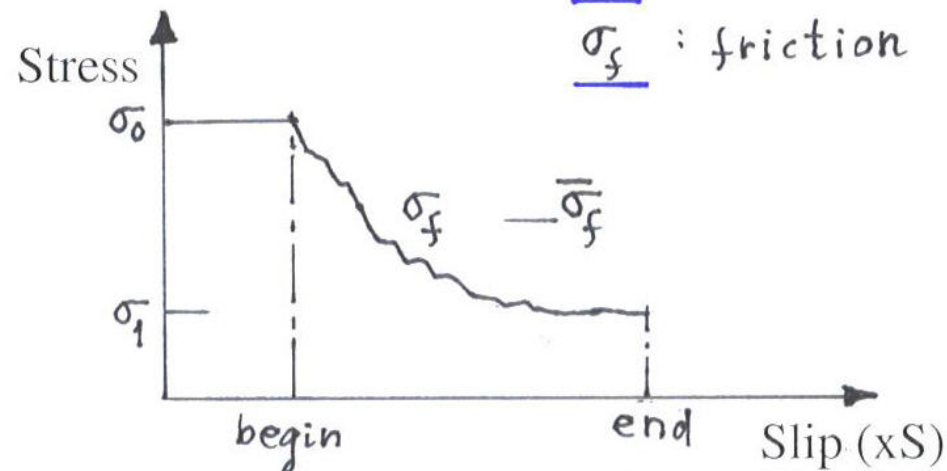
## Dynamic Parameters



$E_{NR} = E_F + E_G + \dots$  : non-radiated energy

$E_F$  : friction (heat),  $E_G$  : fracture energy

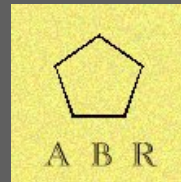
$\sigma_f$  : friction



$\Delta\sigma_s = \sigma_0 - \sigma_1$  : static stress drop

$\Delta\sigma_d = \sigma_0 - \bar{\sigma}_f$  : dynamic stress drop

**This slide is empty intentionally.**



# **Support Slides: Parameters, Notes, etc.**

*To not be displayed directly. Referenced above.*

## **Thermal pressurization:**

*Sibson ( 1973 ); Lachenbruch ( 1980 ); Mase and Smith ( 1985, 1987 );  
Andrews ( 2002 ); Bizzarri and Cocco ( 2006b, 2006c ) .*

*Morrow et al. ( 1984 ) show that gouge contains water*

## **Gouge behaviour:**

*Marone et al. ( 1990 ); Marone and Kilgore ( 1993 ); Mair and Marone ( 1999 );  
Mair et al. ( 2002 )*



## Frictional melting:

*Jeffreys (1942 ); McKenzie and Brune ( 1972 ); Richards ( 1977 ); Sibson ( 1977 ); Cardwell et al. ( 1978 ); Allen ( 1979 )*



Pseudo -  
tachylyte: Fault  
vein ( *Sibson,*  
*1975* )



**Mechanical lubrication:**

*Spray ( 1993 ); Brodsky and Kanamori ( 2001 ); Kanamori and Brodsky ( 2001 )*

**Acoustic fluidization:**

*Melosh ( 1979, 1996 )*

**Gouge gelation:**

*Goldbsy and Tullis ( 2002 ); Di Toro et al. ( 2004 )*

## **Bi – material interface:**

*Andrews and Ben – Zion ( 1997 ); Harris and Day ( 1997 ); Andrews and Harris ( 2005 )*



MTL: Fractured mylonite,  
cataclasite and gouge

**Humidity effects:**

*Dieterich and Conrad ( 1984 )*

**Characteristic length of surface effects:**

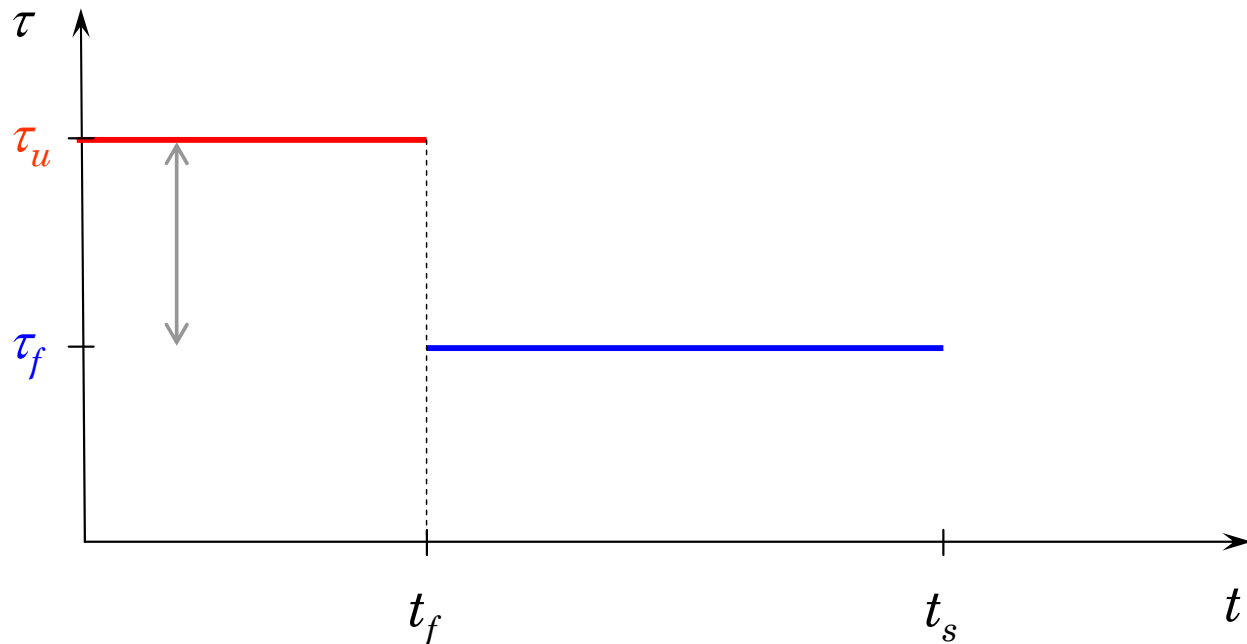
*Ohnaka and Shen ( 1999 ); Ohnaka ( 2003 )*

# Simplest friction models

At a particular fault point  $\xi$  ( following *Savage and Wood, 1971; Scholz, 1990* )

Maximum ( or upper, or yield ) stress

Kinetic ( or frictional ) stress



Strength excess:  $\tau_u - \tau_0 = 0$

Dynamic stress drop:  $\Delta\tau_d = \tau_0 - \tau_f$

In the Dugdale' s model ( *Dugdale, 1960; Barenblatt, 1962* ) the drop occurs when  $u = d_0$ .

Failure time ( or rupture onset )

Rupture arrest

# Simplest friction models

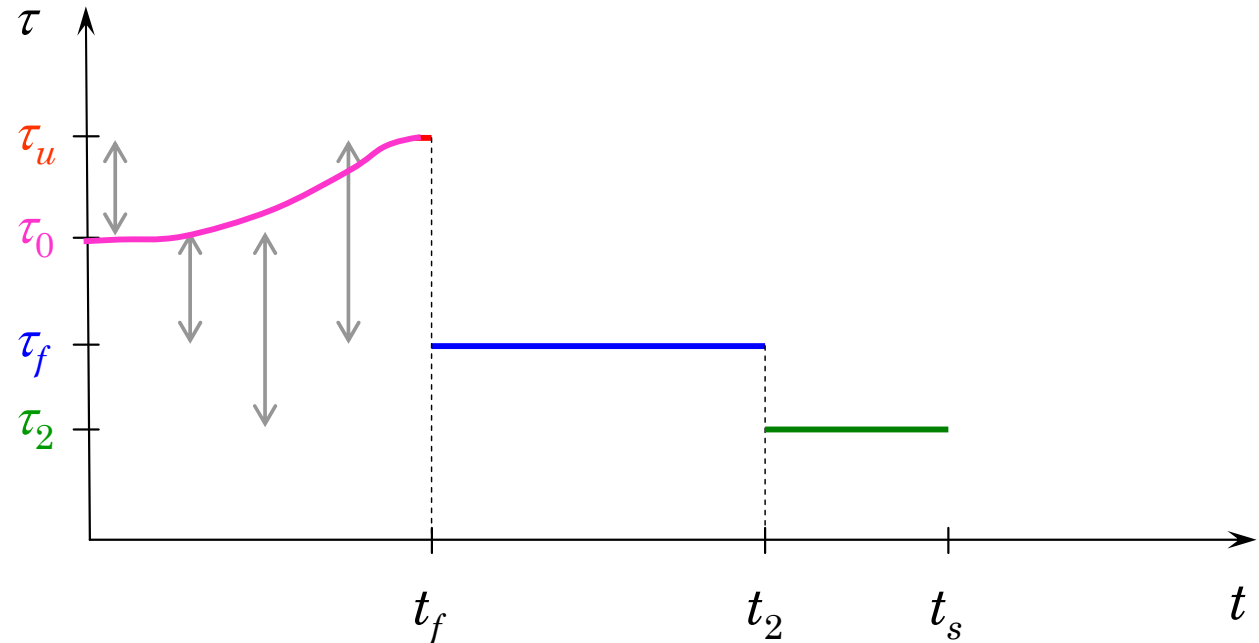
At a particular fault point  $\xi$  ( following *Savage and Wood, 1971; Scholz, 1990* )

Maximum ( or upper, or yield ) stress

Initial stress

Kinetic ( or frictional ) stress

Residual stress



Strength excess:  $\tau_u - \tau_0$

Dynamic stress drop:  $\Delta\tau_d = \tau_0 - \tau_f$

Static stress drop:  $\Delta\tau_s = \tau_0 - \tau_2$

Breakdown str. drop:  $\Delta\tau_b = \tau_u - \tau_f$

Failure time ( or rupture onset )

Dynamic overshoot

Rupture arrest



- *Savage and Wood ( 1971 )* also define:

Mean stress:  $\langle \tau \rangle = \frac{1}{2} (\tau_u + \tau_2)$

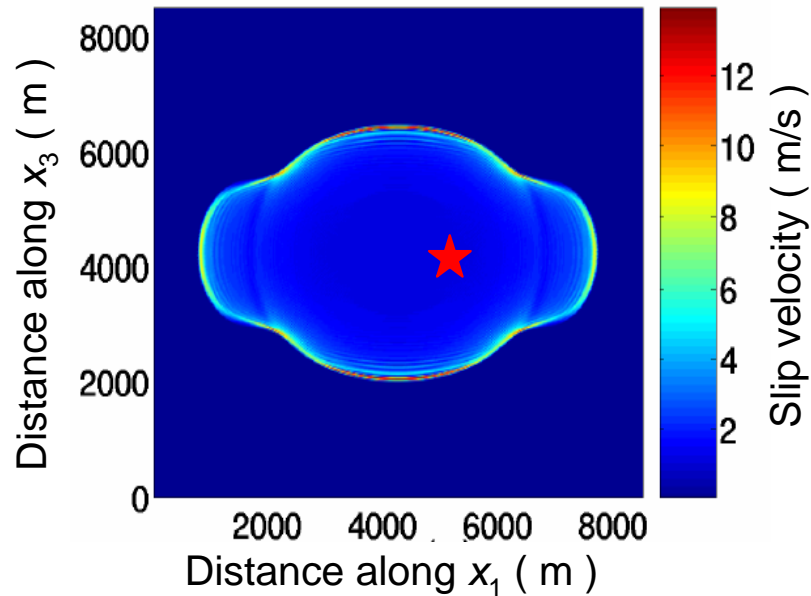
Seismic efficiency:  $\eta = E_s/E$ , where:  $E_s$  is the seismic energy  
 $E$  is the total available energy

Apparent stress:  $\tau_a = \eta \langle \tau \rangle$

- Direct observation of the absolute stress near an earthquake is not feasible, but it is possible ( *Wyss and Brune, 1968* ) calculate  $\tau_a$  and stress drop from physical observables.

# The cohesive zone

Time snapshot  
( $t = 0.8 \text{ s}$ ) - SW law



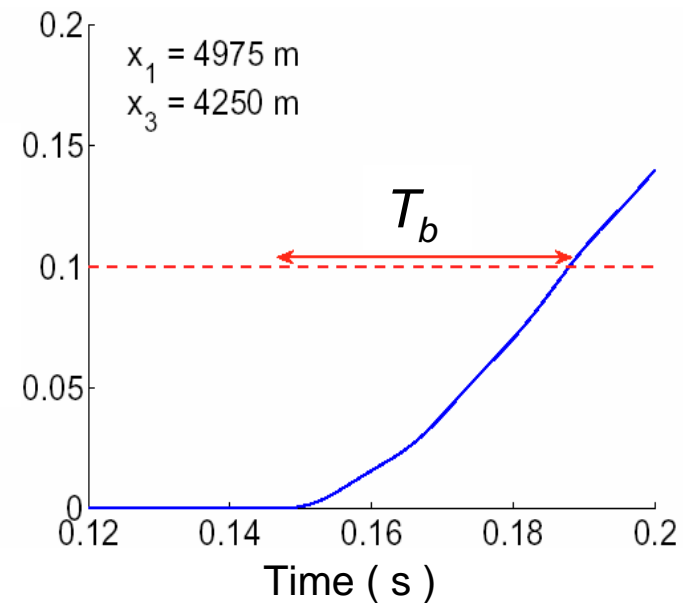
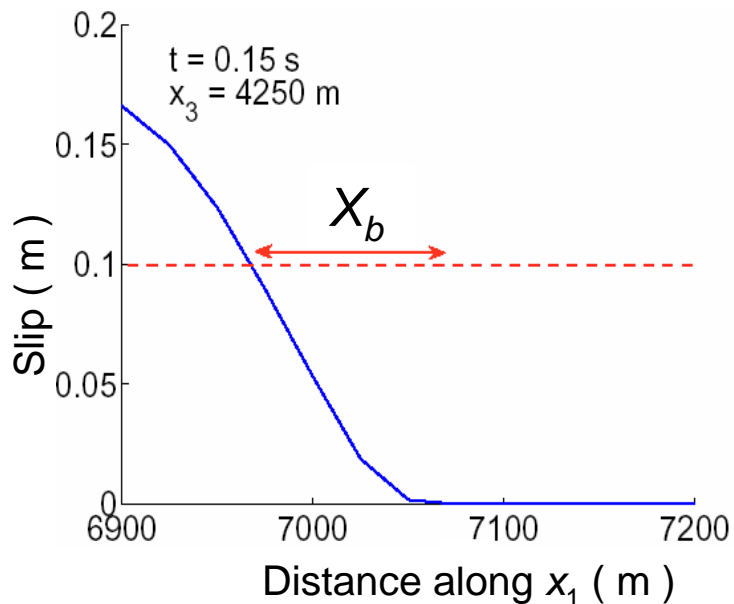
In the target location we can estimate:

$$X_b = 105 \text{ m} \quad T_b = 0.04 \text{ s}$$

From these quantities:

$$V_{rupt} = X_b / T_b = 2625 \text{ m/s}$$

Local estimate





# Slip - hardening effect



- \* The slip – hardening ( **SH** ) phenomenon has been also found in seismological inversion studies ( e. g. *Quin, 1990; Miyatake, 1992; Mikumo and Miyatake, 1993; Beroza and Mikumo, 1996; Ide, 1997; Bouchon, 1997* ).

# Interpretation of the state variable

