Università degli Studi di Bologna
Dottorato di Ricerca in Geofisica – XXVIII Ciclo

MODELS OF SEISMIC RUPTURES

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Index

1. EARTHQUAKE SOURCE DYNAMICS

- Elasto – dynamic problem
- Rupture description
- Dislocation vs. crack models
- Forward modeling scheme
- Rupture stages

2. FAULT GOVERNING LAWS (CONSTITUTIVE EQUATIONS)

- Fault models
- Physical phenomena in faulting
- Fracture criteria and constitutive laws
- Strength and constitutve laws
- Slip – dependend friction laws
- Rate – and state – dependent friction laws

3. EARTHQUAKE NUCLEATION

4. 1 – D SPRING – SLIDER MODEL

- Numerical method
- Fault seismic cycle modeling
- Analytical stress perturbations
- Rhealistic stress perturbations

5. RUPTURE PROPAGATION IN 2 – D FAULT MODELS

- Numerical methods
- BIE vs. FD
- Slip – weakening vs. Dieterich – Ruina law
- The cohesive zone and the breakdown processes
- Theoretical interpretations and correspondency formula
- The estimate of d_0 and related problems
- The importance of the evolution equation

6. ***RUPTURE PROPAGATION IN A TRULY 3 – D FAULT MODEL***

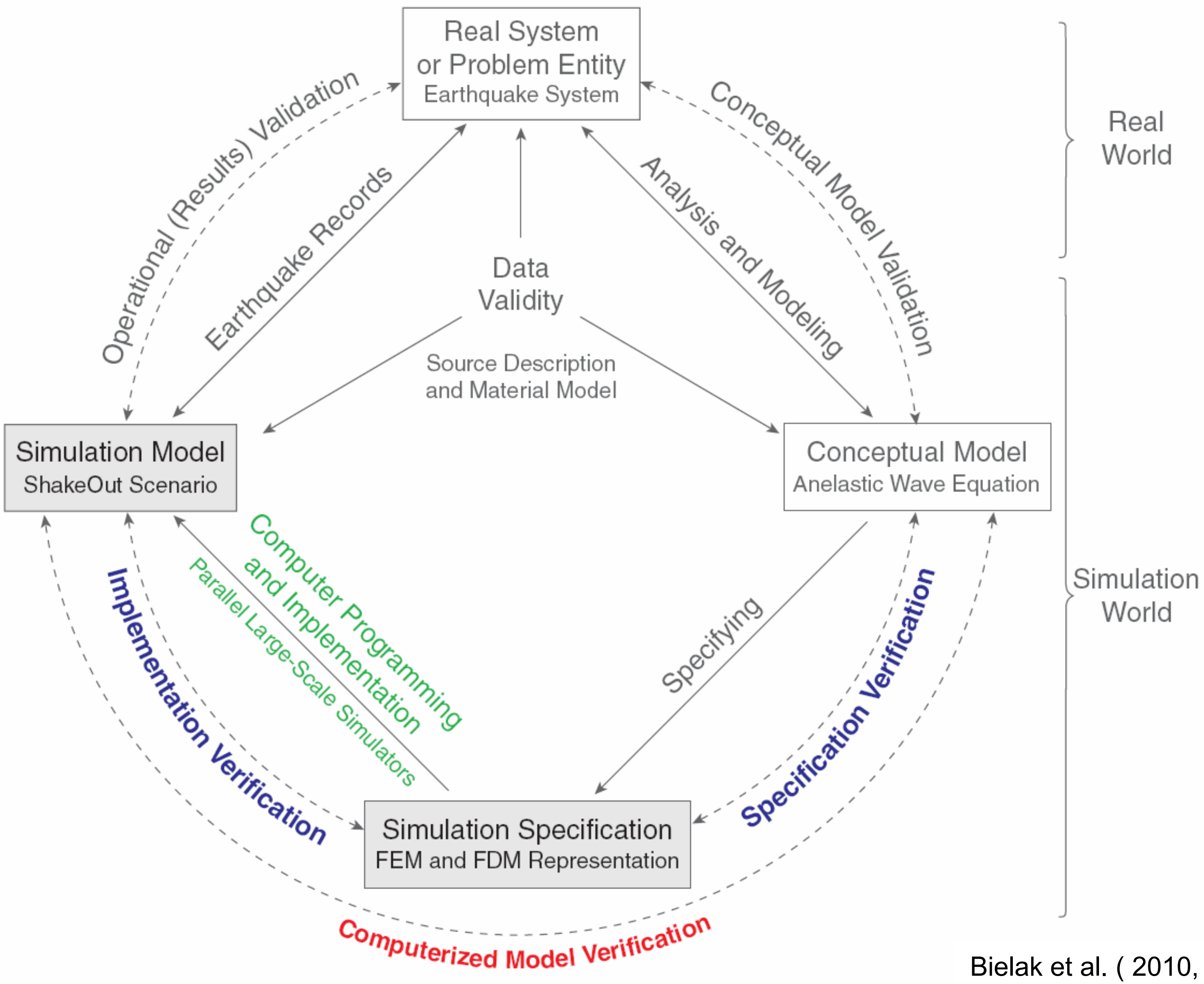
- The numerical method
- The reference case. Comparison between 2 – D and 3 – D models
- Coupling of two modes of propagation. The rake variation
- Dependence on the absolute stress level. Simmetry leak
- Heterogeneous configurations

7. ***RHEOLOGICAL HETEROGENEITIES, CRACK ARREST AND HEALING PHENOMENA***

- The crack and arrest models
  - The barrier – healing
  - The pseudo self – healing
- The pulse
  - Different materials
  - Analytical modifications to friction laws
  - The Nielsen' s model

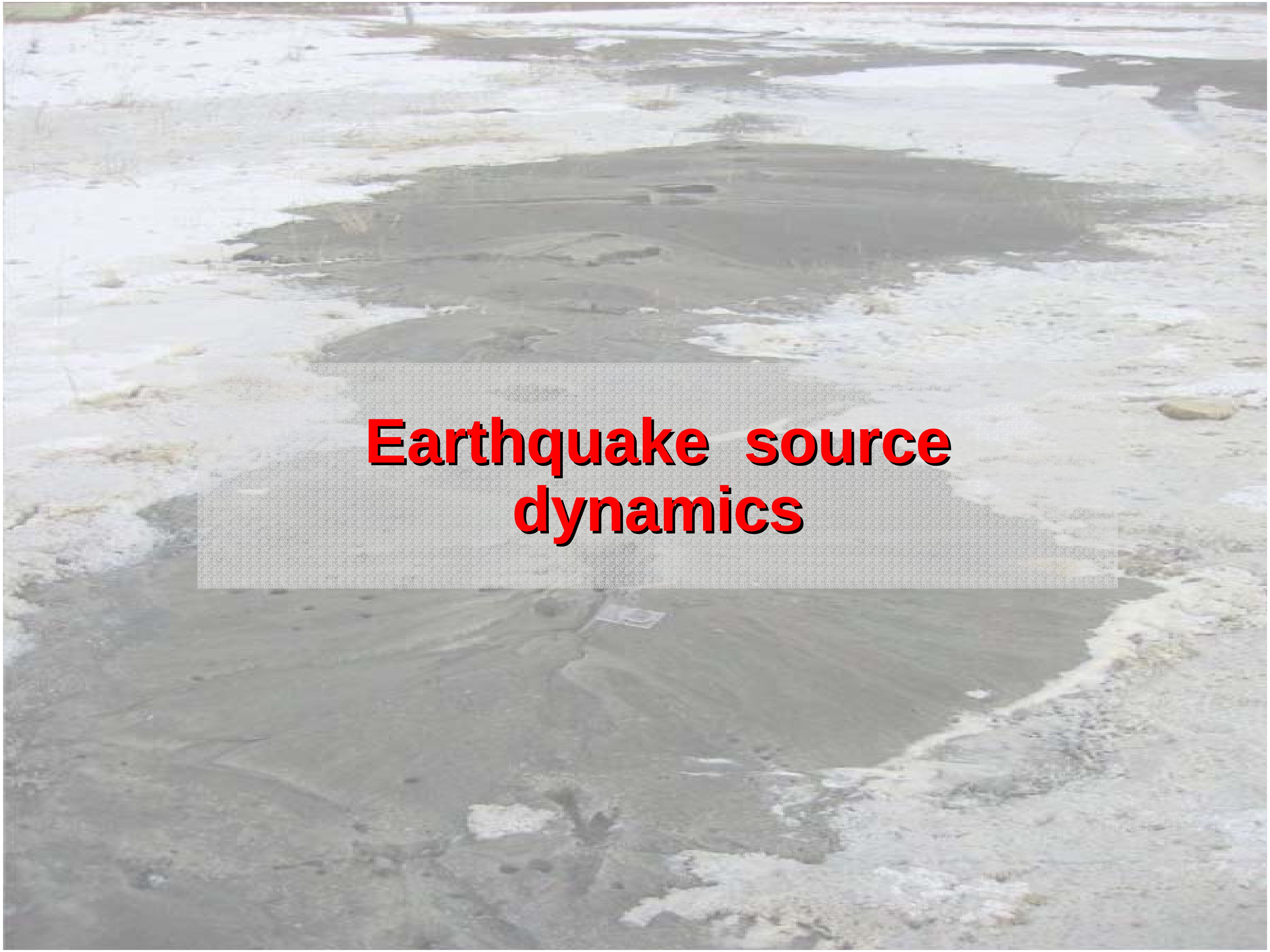
8. CONVERGENCE

General overview



Bielak et al. (2010, *GJI*)

Earthquake source dynamics



Elasto - dynamic problem

* ***Solution of the fundamental elasto – dynamic equation (i. e. the II law of dynamic for continuum media):***

$$\rho(d^2/dt^2)U_i = \sigma_{ij,j} + f_i \qquad ; i = 1, 2, 3$$

where:

ρ is the mass cubic density,

U is the particle displacement vector (**U = x' – x**),

$\{\sigma_{ij}\}$ is the stress tensor; $\sigma_{ij} = C_{ijkl}e_{kl}$; $i,j,k,l = 1, 2, 3$, where C_{ijkl} is the elastic constant tensor, accounting for the rheology of the medium and e_{kl} is the strain tensor ($e_{kl} = ½ (U_{k,l} + U_{l,k})$),

f is the body force vector.

* *Choice of the* ***dimensionality*** ***d*** *of the problem (1 – D, 2 – D, 3 – D).*

*(**d** = rank of the* **U** *array, i. e. number of equations)*

1. ***Wave propagation*** ***problem: Hyperbolic PDE D' Alembert wave equation:***

$$\nabla^2 \mathbf{U} - (1/c_0)\, (\partial^2/\partial t^2)\mathbf{U} = 0$$

where c_0 is the wave speed.

2. ***Rupture*** ***problem***

" It is a considerably more difficult task to study the focus than to study the medium " [Kostrov, 1970]

Rupture Description

Following *Scholz (1990)* the rupture can be described by using:

* ***CRACK MODELS***:

  The energy dissipation at crack edge (or crack tip) is paramount. Describe explicitely the crack propagation.

* ***FRICTION MODELS***:

  The effects at the edges are not explicitley considered. Explicitely allow for the calculation of the evolution of stress tensor components in terms of material properties of the fault.

Dislocation vs. Crack Models

DISLOCATION MODELS

- Study of **displacement discontinuity**
- **Slip** is assumed to be constant on the fault;
  The fault evolution is represented by unilateral or bilateral motion (rectangular dislocations: Haskell' s model)
- Kinematic description: it accounts for time evolution of rupture front and it neglects dynamics of faulting

⇧ **Long period** seismic waves modeling ($\lambda \geq L_{fault}$)

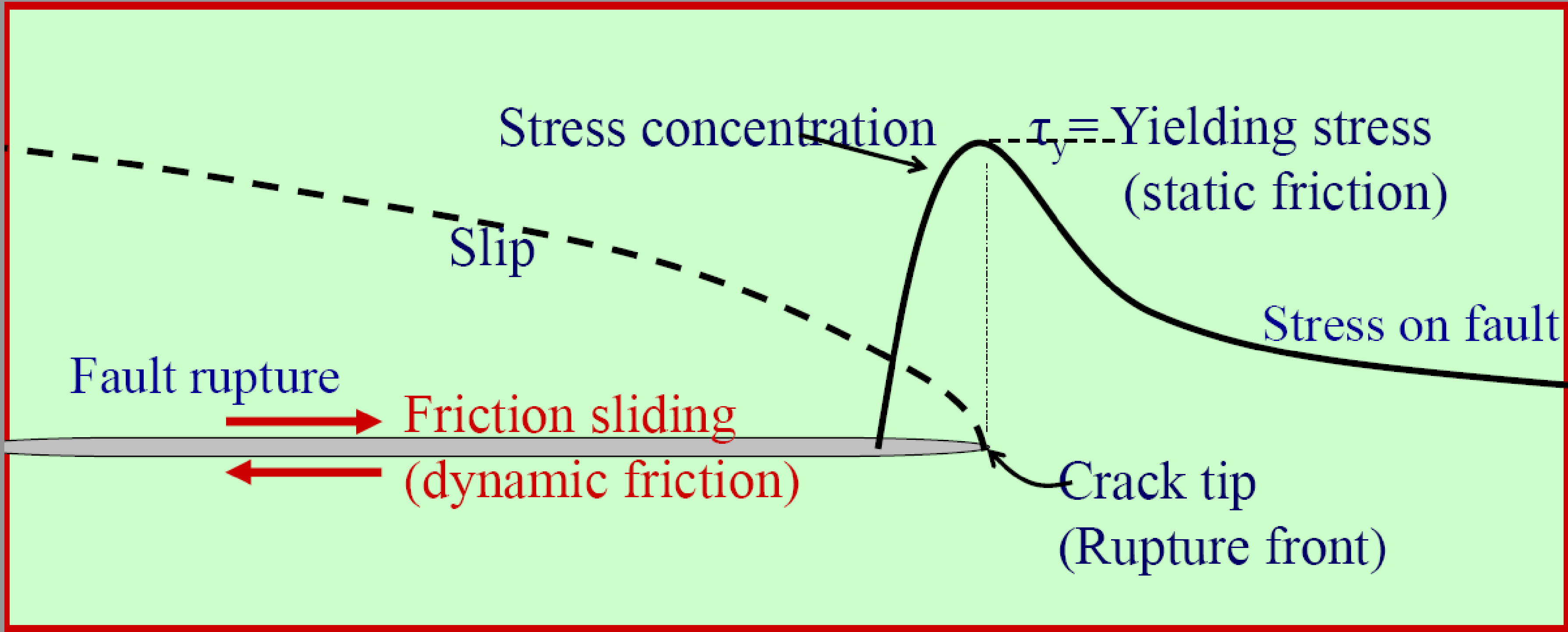
⇩ **constant dislocation** is **inadmissible**;
strain **energy** at crack tip is **unbounded**;
stress drop is **infinite**

CRACK MODELS

- * Impose **finite energy flow** into the rupture
- * **Slip** is **not prescribed**,

  but it is calculated from the stres drop and from the fault strength S^{fault}
- * Dynamic description: the shear stress drops inside the crack (after nucleation processes), increases the stress outiside the crack near the crack tip) and tends to facilitate further grow of the rupture
- ⇧ The motion is determined by fracture criterion (and eventually by the assumed constitutive law on the fault)
- ⇧ The problem is characterized by assuming the boundary conditions on the fault plane. It has mixed b. c.: slip assigned outside the crack tip and stress tensor components inside the crack tip

The concept of crack tip



Cracked portion of the surface (propagating rupture)	**Locked portion of the surface**

Forward modeling scheme

1. Fault model:

- **Fault geometry** (orientation, planar or non – planar, …)
- **Fault system** (multiple segments, multiple faults, …)

2. Medium surrounding the fault surface(s)

- **Properties of the medium** surrounding the fault(s): cubic mass density structure, velocity structure, anysotropy, attenuation

3. Choice of the dimensionality d′ of the problem (1 – D, 2 – D, 3 – D, 4 – D).

(d′ = number of the independent variables in the solutions)

4. Choice of the representation

5. *Choice of the numerical method*

- (**FE**, **FD**, **BE**, **BIE**, **SE**, **hybrid**)

6. *Specification of the Boundary Conditions*

- **Domain** Boundaries Conditions (DBCs)
- **Fault** Boundary Condition (FBCs)
- **Auxiliary** Conditions (ACs)

7. *Specification of the Initial Conditions*

- Initial conditions **on the fault**: (initial slip, slip velocity, state variable, pre – stress);
- Initial conditions **outside the fault**: (tectonic load, (state of neighbouring faults: the fault is <u>not</u> an isolated system))

8. *Evaluation of the solutions*

- Convergence analysis (**consistency** + **stability**)

Rupture stages

1. Nucleation (quasi – static to dynamic evolution)

- *How can we simulate nucleation?*
- *How can we promote fault instability?*

2. Propagation

- *What is the fault constitutive equation (governing law)?*

3. Healing

- *What type of healing occurs?*
- *What controls fault healing?*

4. Rupture arrest

- *What is responsible of rupture arrest?*
- *How can we represent it? Earthquake energy balance?*

5. Fault re – strengthening

- *How can we model further instabilities episodes on the fault?*

Radiation Damping Approximation

The solution of the fundamental elasto – dynamic equation is written as (Rice, 1993):

$$\tau_i = -\sum K_{ij}(u_j - v_{plate}t) - (G/2v_S)(\mathrm{d}/\mathrm{d}t)u_i; \qquad i, j = 1, 2, 3$$

where:

τ_i	is the i – component of shear traction,
u_i	is the the i – component of fault slip,
K_{ij}	is static kernel calculated from Chinnery (1963),
v_{plate}	is the lading velocity (i.e., the plate velocity),

The last term (the radiation damping term) makes this approach quasi – dynamic, since it accounts for elastodynamic effects.

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Support Slides:
Parameters, Notes, etc.

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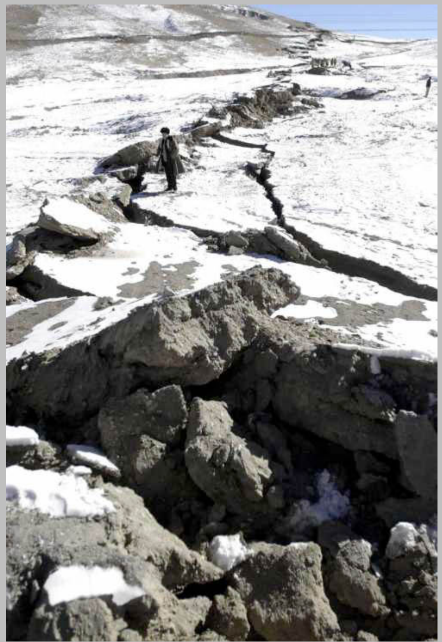
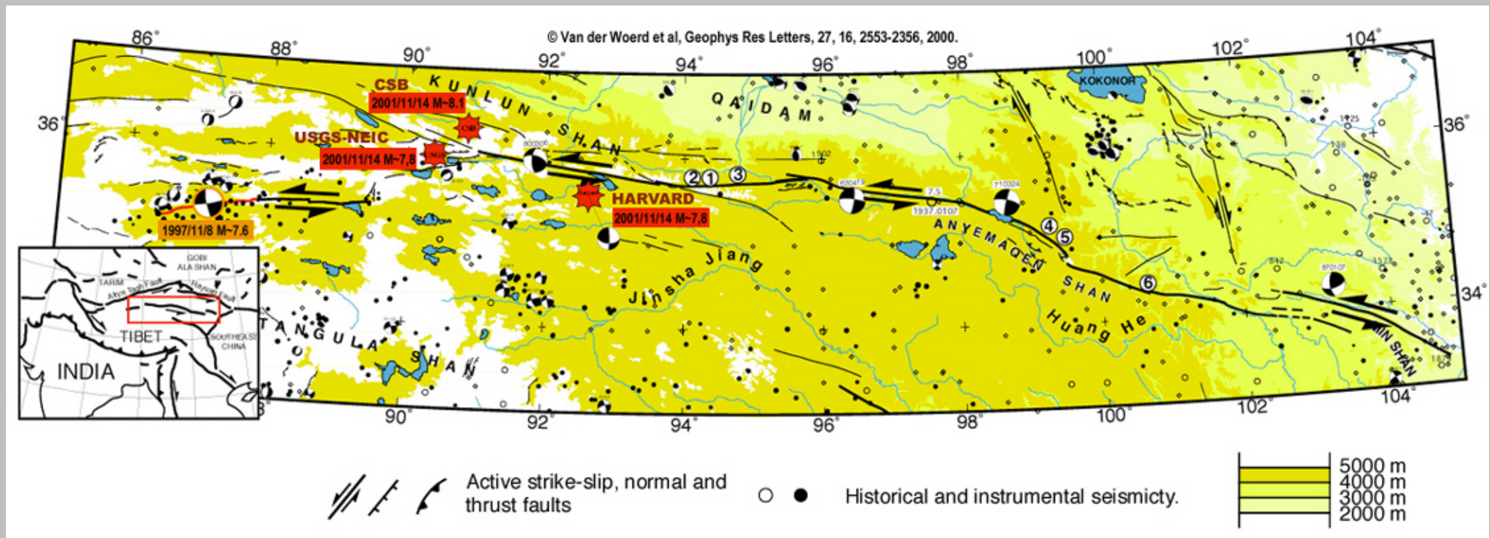
Geometrical complexity

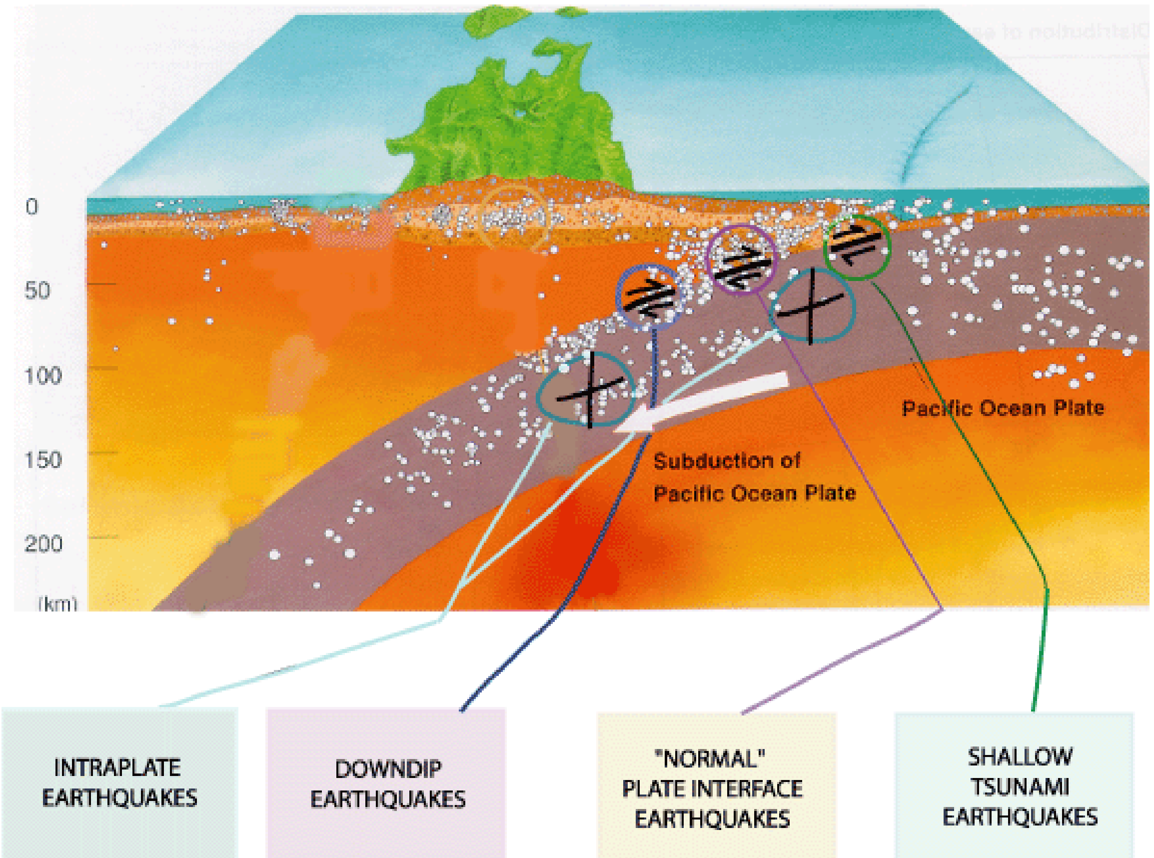


Kokoxili
***M_w* 7.9**
earthquake
(Qinghai
Province,
China)



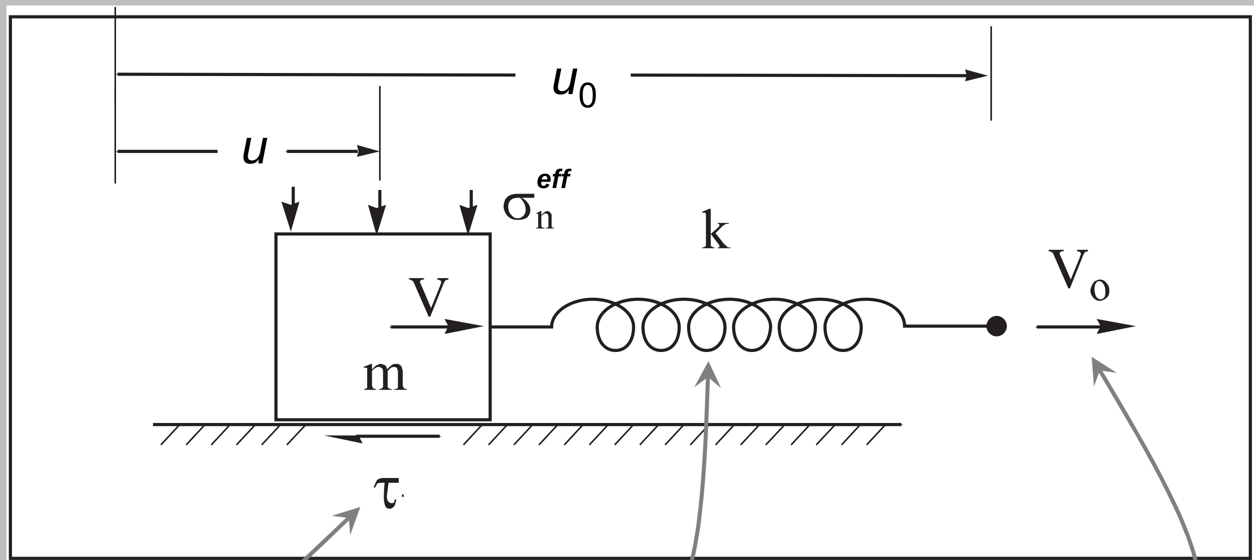
Different types of earthquakes

1. Interplate
2. Tsunami
3. Crustal
4. Downdip
5. Intraplate
6. Deep



Dimensionality *d'*

1 – D Sping – Slider (mass – spring) model



Frictional sliding

(↔ rheological properties)

Elastic behaviour

(↔ surrounding medium)

Loading velocity

(↔ tectonic load)

Let we recall basics concepts on dislocation theory.

If $U_{i,j}$ is continuous in the integration domain $\int_{P_1}^{P_2} U_{i,j}\,\mathrm{d}x_j$ then does not depends on the integration path and therefore:

$$\oint_C \mathrm{d}U_i = 0, \qquad \forall C.$$

On the contrary, when $\oint_C \mathrm{d}U_i = b_i$ the considered body contains a **dislocation** and the circuit C contains at least one curve (the **dislocation curve**) on which the tensor $U_{i,j}$ is not defined.

The vector $U_i(\mathbf{x})$ can be reduced to a one – valued function if we produce in the body and starting from the dislocation curve a cut (the **dislocation surface**), through which we assume an explicit discontinuity of **U**. Being ζ the coordinate normal to the dislocation surface we can write:

$$\Delta U_i \equiv u_i = \lim_{\zeta \to 0^+} U_i(\mathbf{x}) - \lim_{\zeta \to 0^-} U_i(\mathbf{x}) = b_i$$

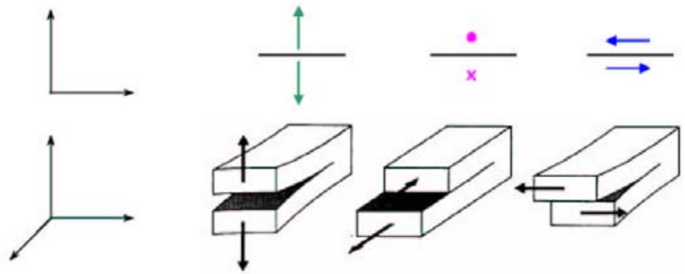
where **b** = (b_1,b_2,b_3) is called Burgers' s vector.

In this framework the dislocation is described from a *microscopic / crystallographic point of view*.

Fracture propagation modes

Elasotdynamics Fondam. Eq.: $\boxed{\rho\, \ddot{u}_i = f_i + \sigma_{ij,j}}$

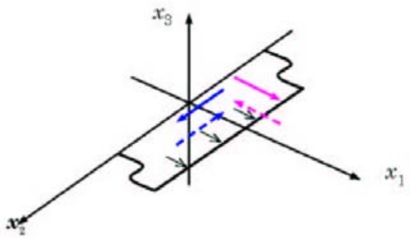
Solution: $\mathbf{u}(\mathbf{x},t)$ (misture of shear crack and opening crack)



	mode I	shear (mode II)	shear (mode III)
DISLOCATION	tensile	di bordo (edge)	a vite (screw) elicoidale

Geometrical Characterization

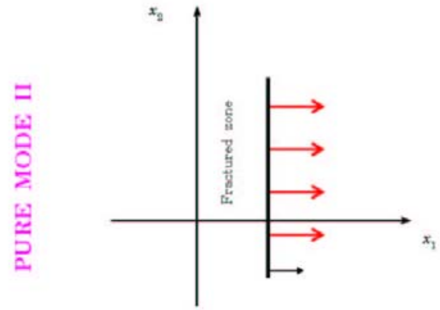
- opening cracks (mode I) $\mathbf{u} = (\,0,\,0,\,u_3(\mathbf{x},t)\,)$ 4 – D
- shear cracks $\mathbf{u} = (\,u_1(\mathbf{x},t),\,u_2(\mathbf{x},t),\,0\,)$ 4 – D
  - Planar fault surface ($x_3 = 0$) $\Rightarrow$ on – fault coordinates: x_1, x_2
    $\mathbf{u} = (\,u_1(x_1,x_2,t),\,u_2(x_1,x_2,t),\,0\,)$ *truly* 3 – D
  - Propagation direction: x_1



- mixed mode $\mathbf{u} = (\,u_1(x_1,t),\,u_2(x_1,t),\,0\,)$ *pseudo* 3 – D
- mode II (in – plane) $\mathbf{u} = (\,u_1(x_1,t),\,0,\,0\,)$ 2 – D
- mode III (anti – plane) $\mathbf{u} = (\,0,\,u_2(x_1,t),\,0\,)$ 2 – D

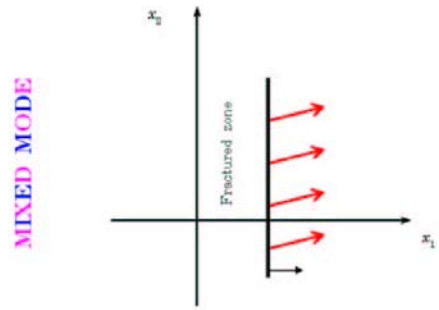
Analytical Characterization

Shear rupture on a planar fault surface ($x_3 = 0$)
Snapshots at fixed time t



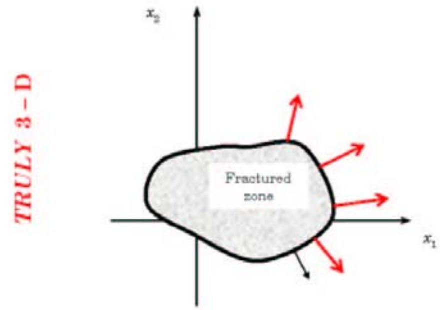
Dependence on x_1
Independence on x_2

$\Rightarrow u_1(x_1,t)$



Dependence on x_1
Independence on x_2

$\Rightarrow u_1(x_1,t)$
$u_2(x_1,t)$



Dependence on x_1
Dependence on x_2

$\Rightarrow u_1(x_1,x_2,t)$
$u_2(x_1,x_2,t)$

Crack tip

Local displacement

Local crack enlargement direction

Representation

1. *INTEGRAL REPRESENTATION*

Source integral rapresentation (*Betti*' s theorem, Integration in time (*Green – Volterra*' s relation), limit in fault surface, Lamb' s problem):

$$u_n(\mathbf{x},t)=\int_{-\infty}^{+\infty}\mathrm{d}t'\int_{S(t')}\mathrm{d}\xi\, G_{n\alpha}(\mathbf{x}-\xi,t-t')\,\sigma_{\alpha 3}{}^{p}(\xi,t')\,\xi\quad n=1,2,3;\alpha=1,2;\mathbf{x},\xi\in \mathrm{R}^3$$

First neighbours decoupling (in the case of a 2 – D, pure in – plane rupture):

Traction

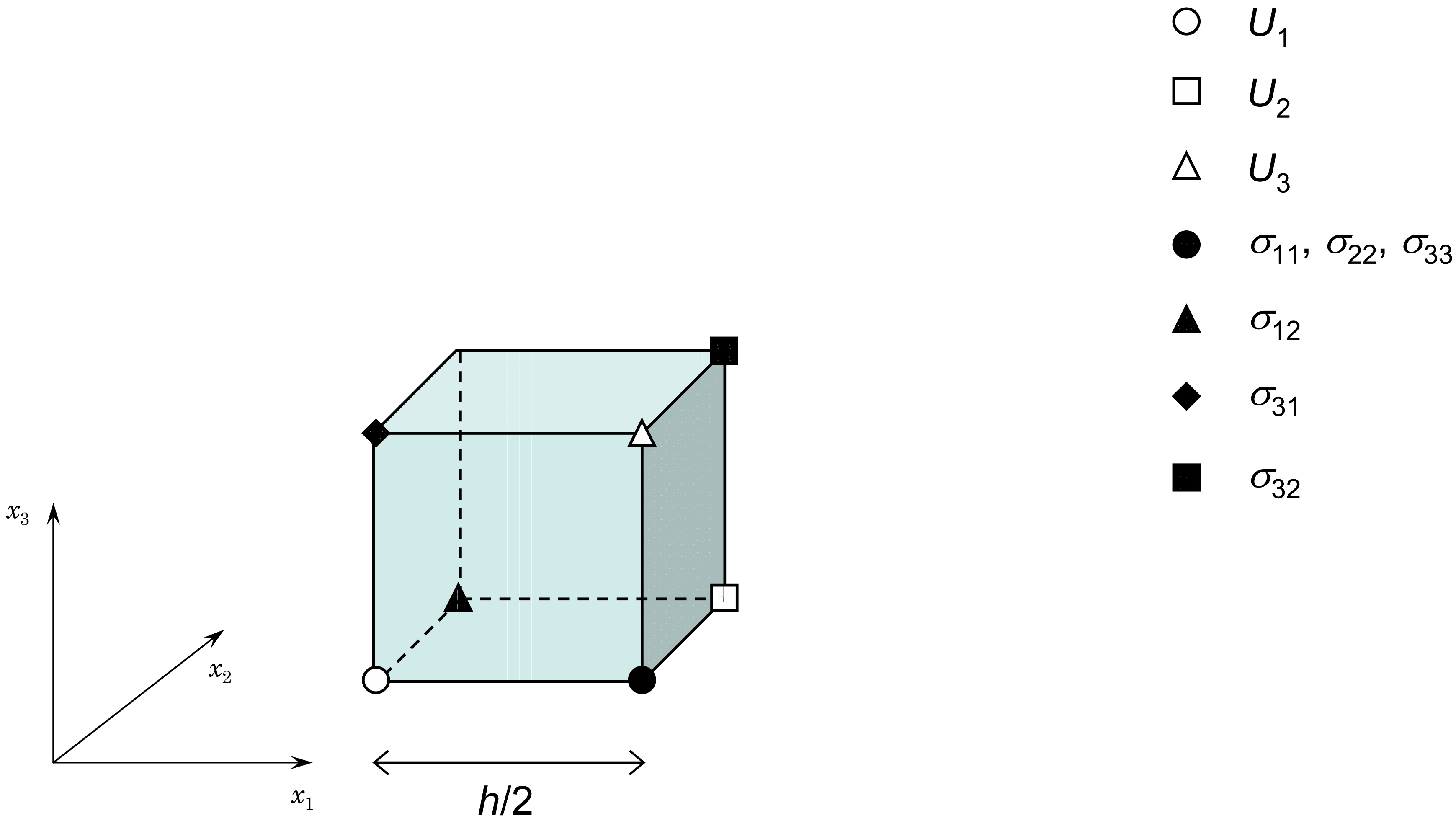
$$\begin{cases} u_1(x_1,t)+C\,\boxed{\tau_1{}^{p}(x_1,t)}=L_1(x_1,t) \\ \tau_{0_1}+\boxed{\tau_1{}^{p}(x_1,t)}=\boxed{\mu\sigma_n{}^{eff}} \end{cases}$$

Friction

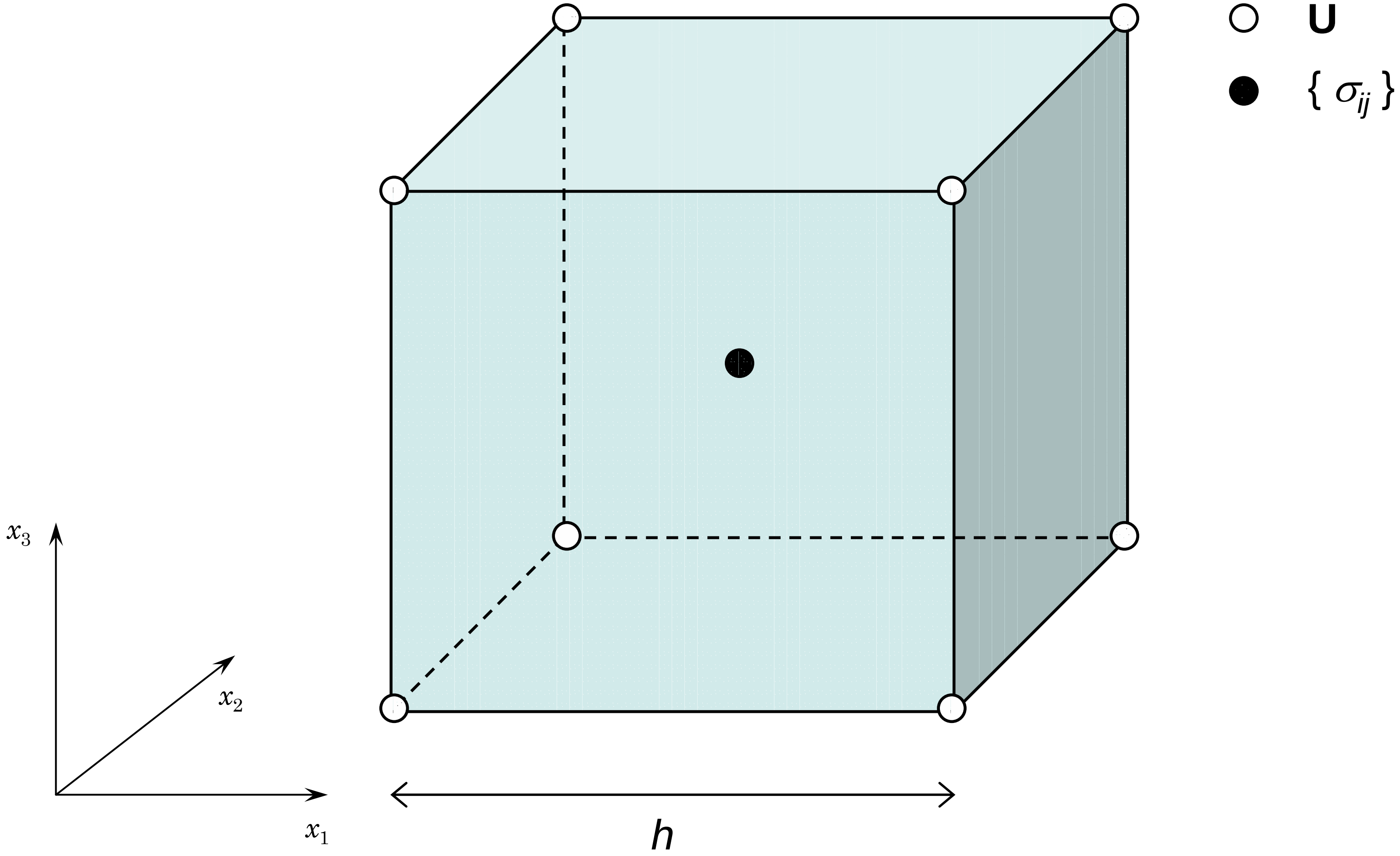
2. DISCRETIZATION OF EQUATIONS (FE, FD APPROACHES)

- **Choice of the grid type**

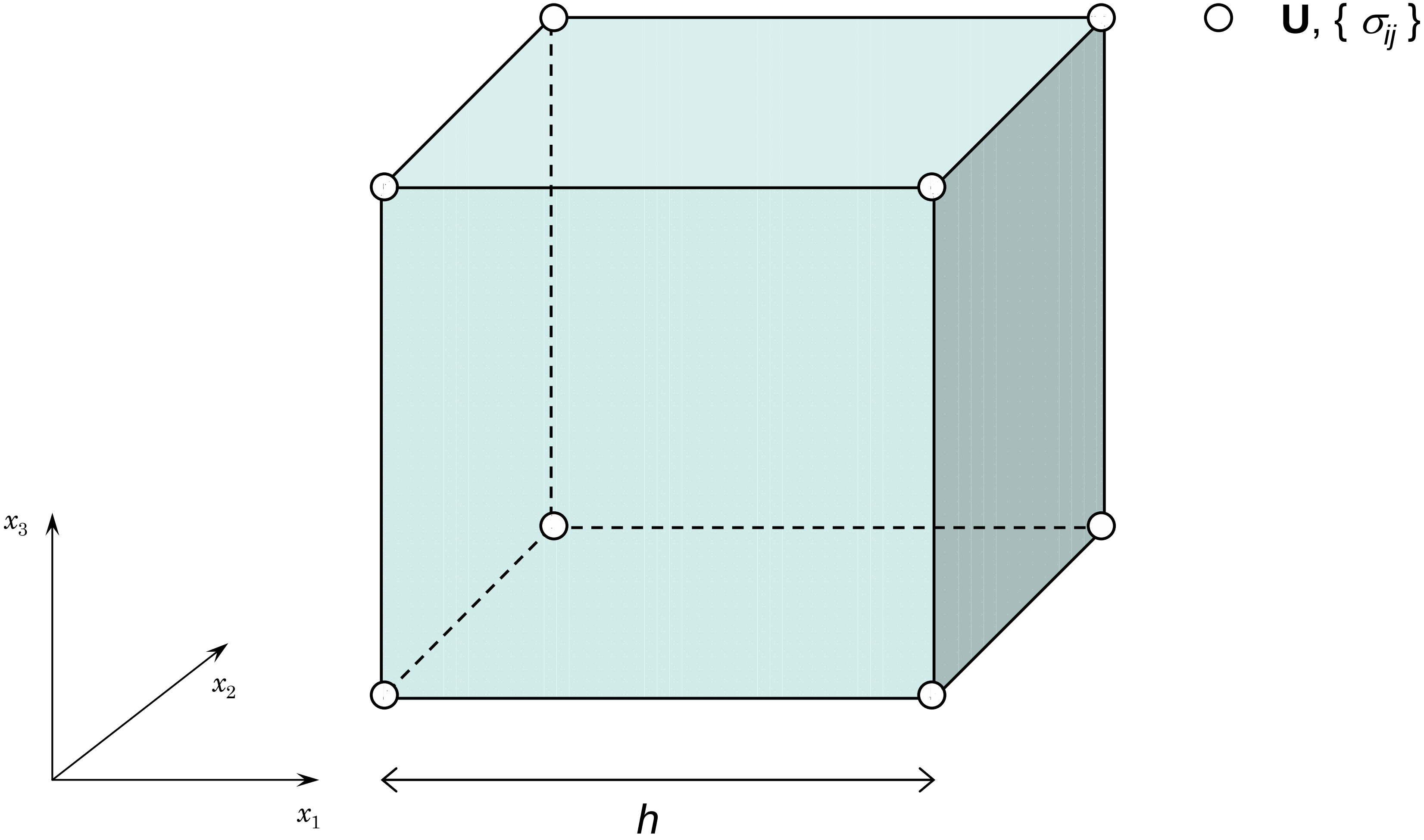
- ***Staggered – grid (SG)***:



- ***Partly Staggered – grid (PSG)***:



- ***Conventional – grid (CG)***:



Domain Boundaries Conditions

* ***BOUNDARY:***
  - **Bottom**
    - Fixed
    - Absorbing
  - **Top**
    - Free surface
    - Topography
    - Coasts
  - **Lateral**
    - Cyclic
    - Absorbing

- Let us consider a boundary perpendicular to the *i* – axis. Indeces *i*, *j* and *k* identify node location along x_1, x_2 and x_3 axes, respectively. Apex *m* indicates the actual time level, while index *l* stands for vector component (*l* = 1, 2, 3).

- ***Fixed Boundary (FB)***:

$$U^m_{1jk_l} = 0, \qquad \dot{U}^m_{1jk_l} = 0$$

$$U^m_{i_{end}jk_l} = 0, \quad \dot{U}^m_{i_{end}jk_l} = 0$$

(Conditions $\dot{U}^m_{1jk_l} = 0$ and $\dot{U}^m_{i_{end}jk_l} = 0$ represent Dirichlet and Neumann boundary conditionsm, respectively).

- ***Absorbing Boundary (AB)***:

Left boundary:

$$
\begin{aligned}
\dot{U}^{m}_{1jk_l} &= && A_{01}\dot{U}^{m}_{2jk_l} &&+ A_{02}\dot{U}^{m}_{3jk_l} \\
&+ A_{10}\dot{U}^{m-1}_{1jk_l} &&+ A_{11}\dot{U}^{m-1}_{2jk_l} &&+ A_{12}\dot{U}^{m-1}_{3jk_l} \\
&+ A_{20}\dot{U}^{m-2}_{1jk_l} &&+ A_{21}\dot{U}^{m-2}_{2jk_l} &&+ A_{22}\dot{U}^{m-2}_{3jk_l}
\end{aligned}
$$

Right boundary:

$$
\begin{aligned}
\dot{U}^{m}_{i_{end}jk_l} &= && A_{01}\dot{U}^{m}_{i_{end}-1\,jk_l} &&+ A_{02}\dot{U}^{m}_{i_{end}-2\,jk_l} \\
&+ A_{10}\dot{U}^{m-1}_{i_{end}jk_l} &&+ A_{11}\dot{U}^{m-1}_{i_{end}-1\,jk_l} &&+ A_{12}\dot{U}^{m-1}_{i_{end}-2\,jk_l} \\
&+ A_{20}\dot{U}^{m-2}_{i_{end}jk_l} &&+ A_{21}\dot{U}^{m-2}_{i_{end}-1\,jk_l} &&+ A_{22}\dot{U}^{m-2}_{i_{end}-2\,jk_l}
\end{aligned}
$$

In the previous compact representation of ABCs (that follows *Moczo*, *1998*):

- the coefficients $\{A_{pq}\}_{p,q\,=\,1,\,2,\,3}$ depend on the choice of ABC scheme (e. g. *Clayton and Engquist*, *1977*; *Reynolds*, *1978*; *Emerman and Stephen*, *1983*; *Higdon*, *1991*; *Peng and Toksöz*, *1994*, *1995*; *Liu and Archuleta*, *2000*, …);

- displacement components at actual time level *m* are derived by numerical integration from particle velocity components, after update;

- values in edges and in corners are derived from algebraic averaging of values of quantities belonging to walls;

- as it is a special boundary condition, there is no need to consider any rheology in the updated point.

Our choice:

$$\mathbb{A} = \begin{pmatrix} 0 & -Q_X - R_X & -Q_X R_X \\ -Q_T + R_X & Q_X R_X - Q_T R_X - Q_{XT} - 1 & Q_X - R_X Q_{XT} \\ Q_T R_X & Q_T + R_X Q_{XT} & Q_{XT} \end{pmatrix}$$

where

$$Q_X = \frac{W_b(1 + N_I) - N_I}{(1 + N_I)(1 - W_b)}$$

$$R_X = (1 - \omega_{CFL})/(1 + \omega_{CFL})$$

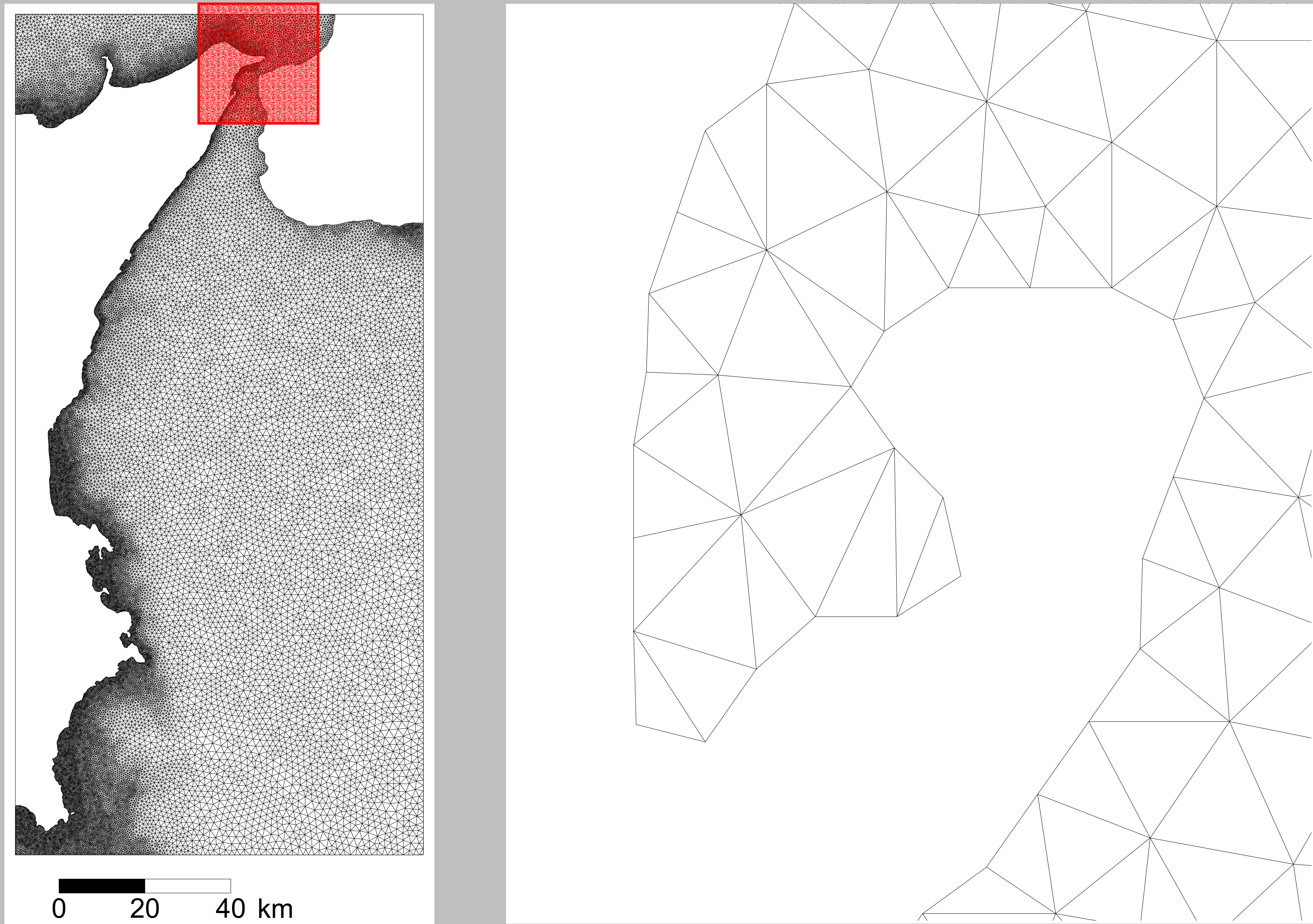
$$Q_T = \frac{W_b(1 + N_I) - 1}{(1 + N_I)(1 - W_b)}$$

$$N_I = v_P \Delta t/\Delta x$$

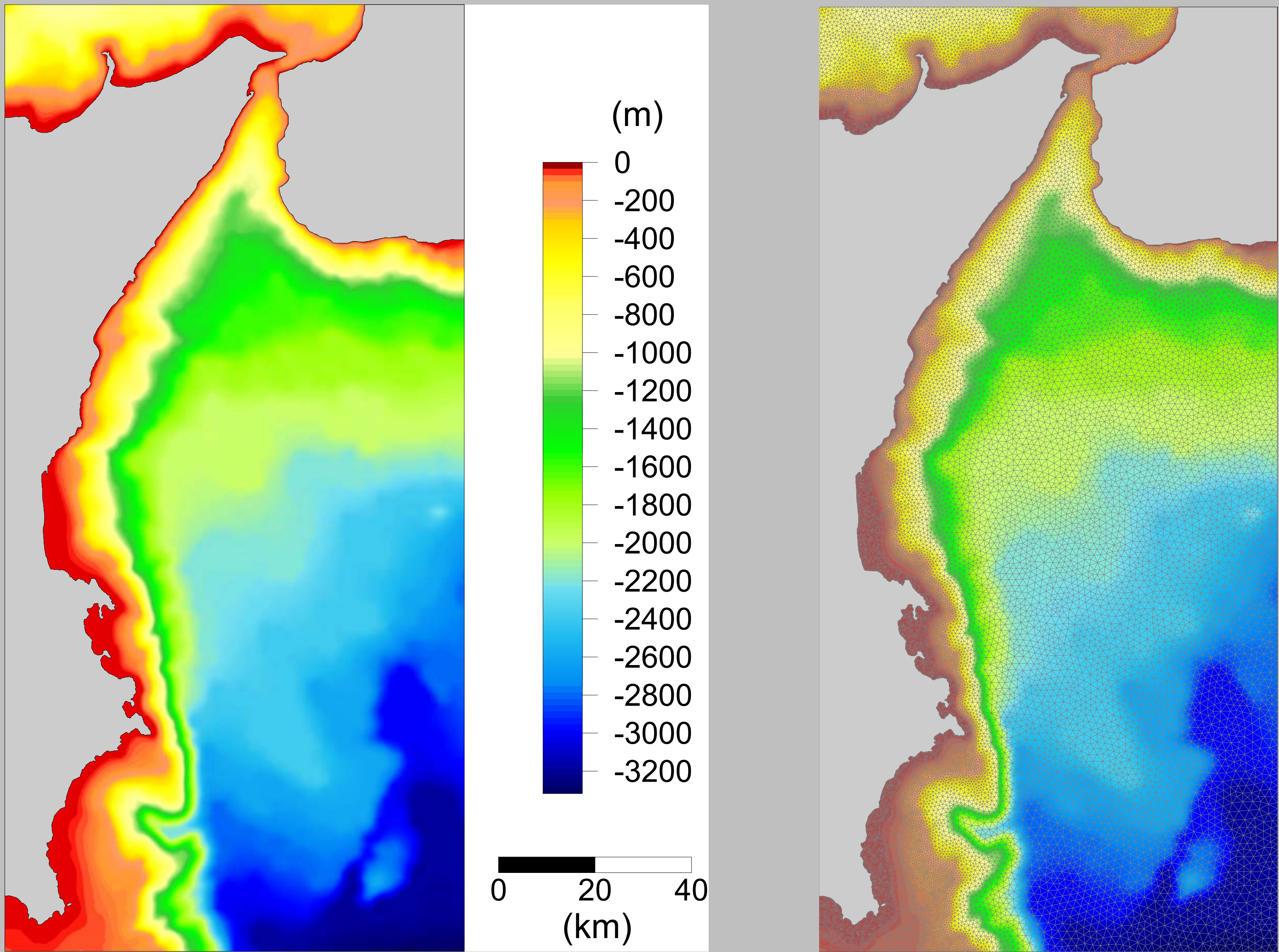
$$Q_{XT} = W_b/(W_b - 1)$$

$$\omega_{CFL} = v_S \Delta t/\Delta x$$

$$W_b = 0.4$$



0
20
40 km


(m)
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
-1800
-2000
-2200
-2400
-2600
-2800
-3000
-3200
0
20
40
(km)

Number of nodes	30264
Number of elements	57733
Type of building block	Triangle
Minimum node distance	200 m
Maximum node distance	2000 m
References	Armigliato A., Tinti S., (2005), *EGU General Assebly*; Tinti S., Armigliato A., Bortolucci E. (2001), *J. Seismol.*, **5**, 41-61.

Fault Boundary Conditions

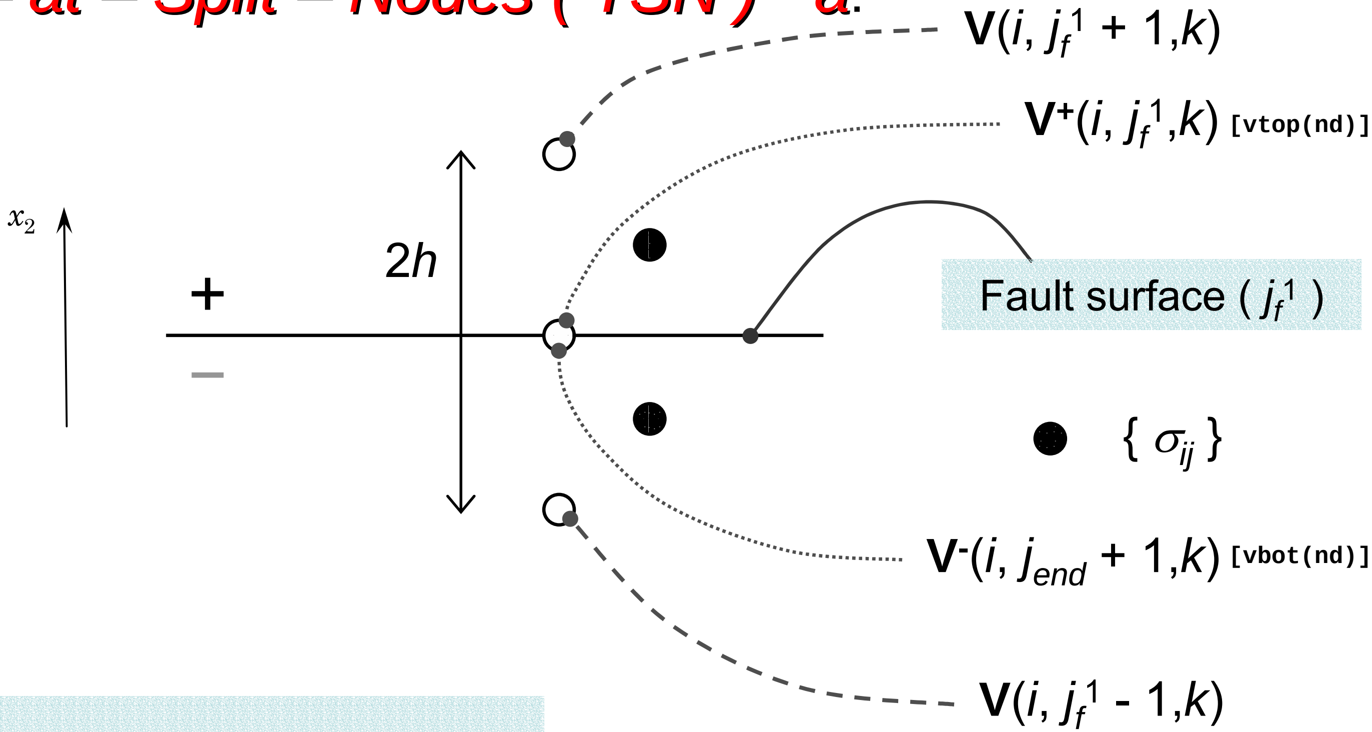
1. TYPE

- **Traction – at – Split – Nodes (TSN): in 2 – D by** *Andrews (1973)*; **in 3 – D by** *Day (1977), Archuleta and Day (1980), Day (1982a, 1982b), Andrews (1999), Bizzarri (2003), Bizzarri and Cocco (2005), Day et al. (2005)*
- **Stress – Glut (SG):** *Backus and Mulchay (1976), Andrews (1976)*
- **Thin – zone (TnZ):** *Virieux and Madariaga (1982)*
- **Thick – zone (TkZ):** *Madariaga et al. (1998)*

2. CONSTITUTIVE LAW

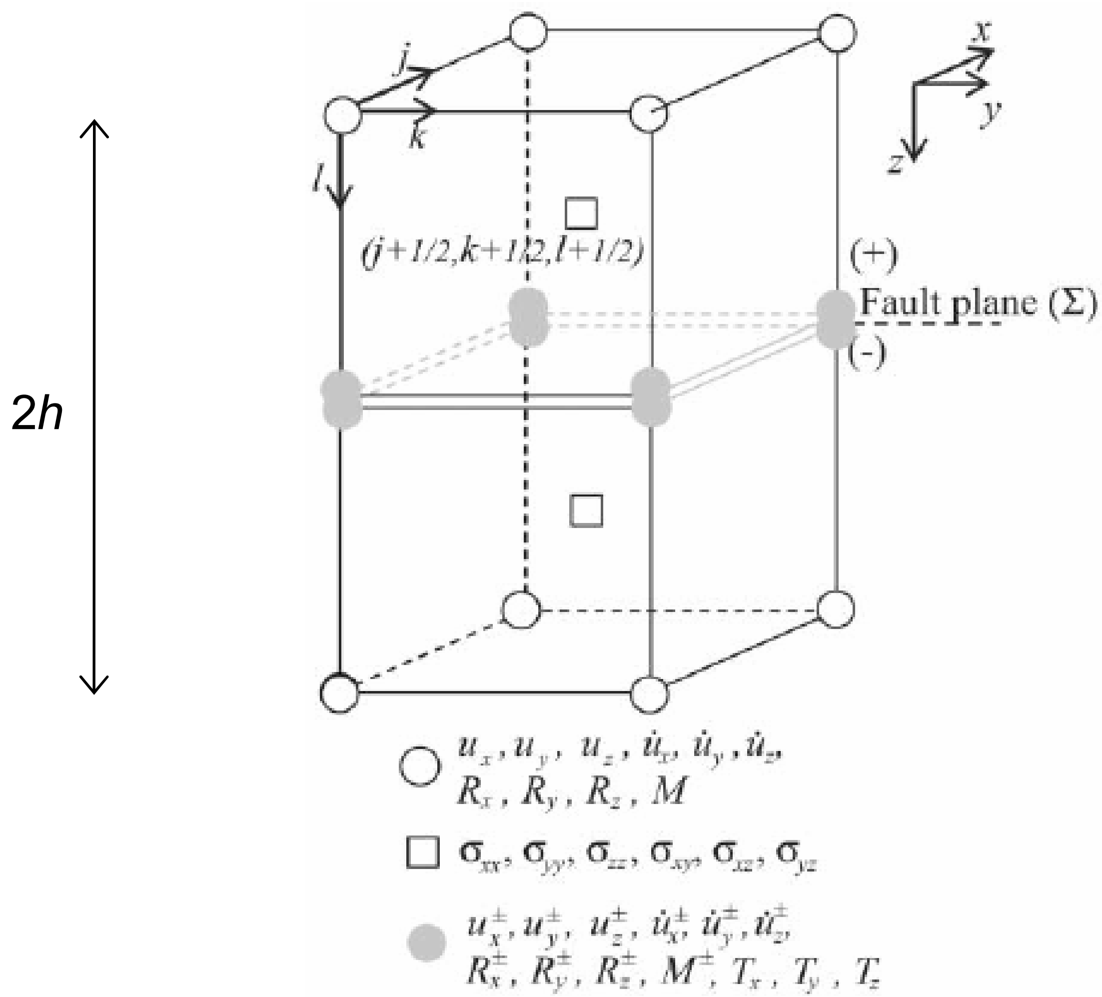
- **Accounts for fault rheology**
- **and different physical phenomena occurring during the rupture process**

- ***Traction – at – Split – Nodes (TSN) - a***:



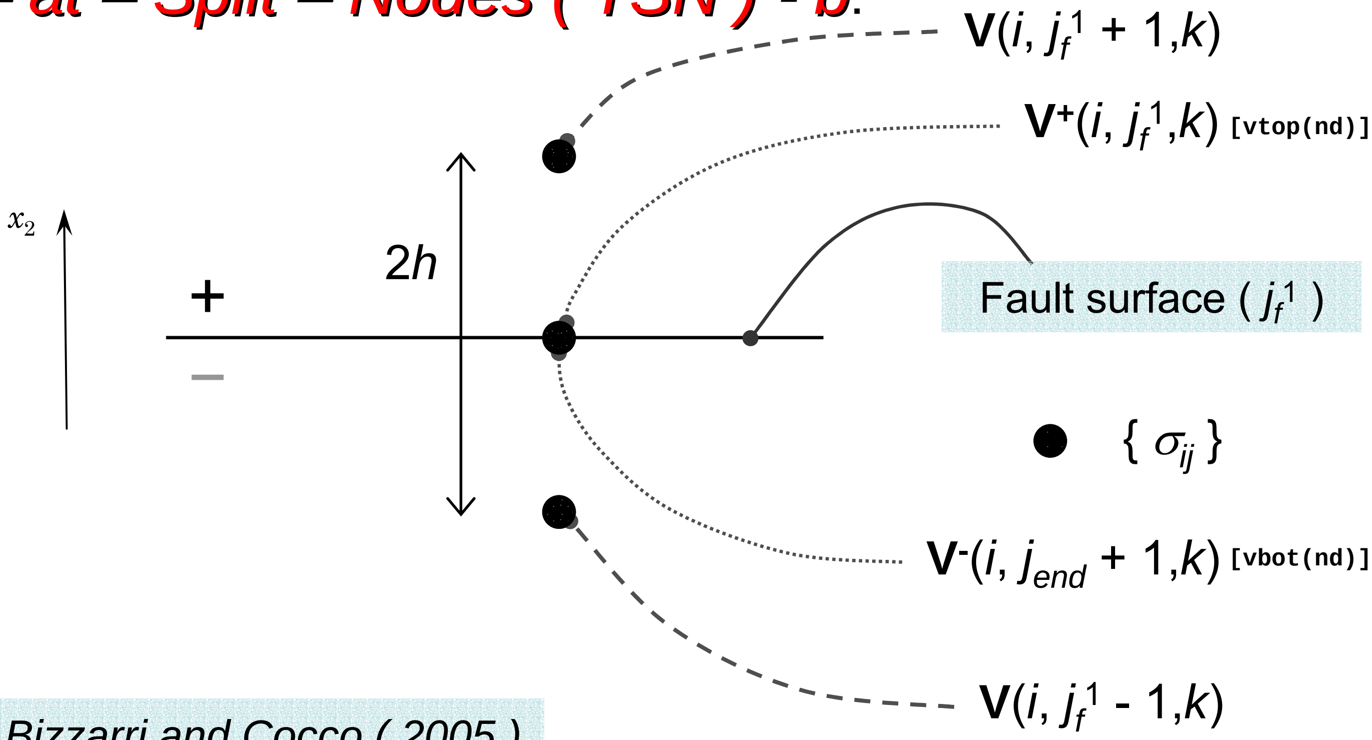
Day (1982b)

$\Delta\mathbf{V} = \mathbf{V}^{+} - \mathbf{V}^{-}$ relative motion of the " + " side with respect to the " – " side. This value is used to calculate fault slip velocity **v**, using the constitutive law



3 – D Partly Staggered – grid (SG) formulation

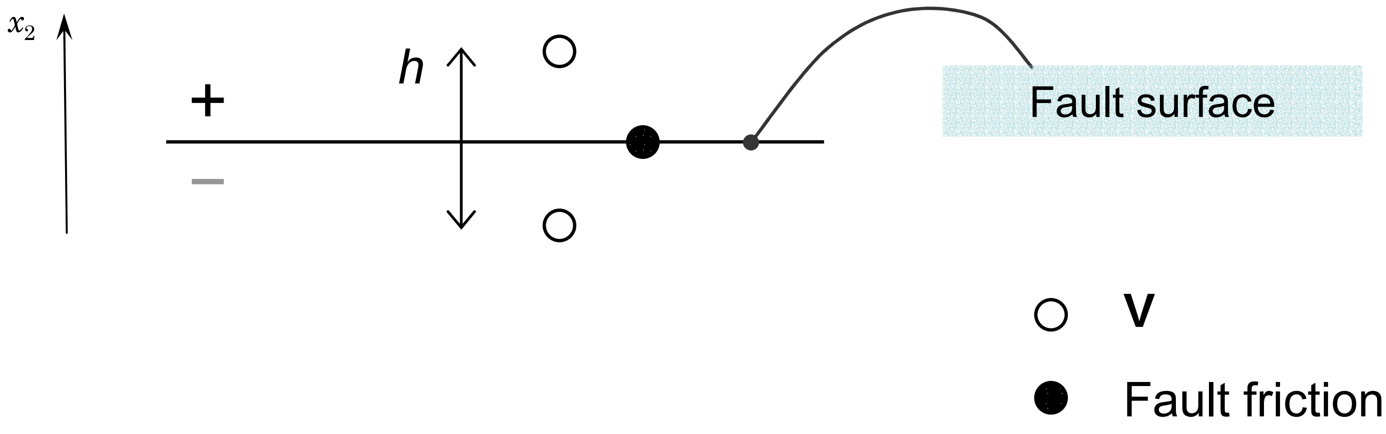
- ***Traction – at – Split – Nodes (TSN) - b***:



Andrews (1999), Bizzarri and Cocco (2005)

* **Discontinuum medium** (continuum mechanics equations of motion are applied to each half - space individually; the fault is an explicit discontinuity in displacement)

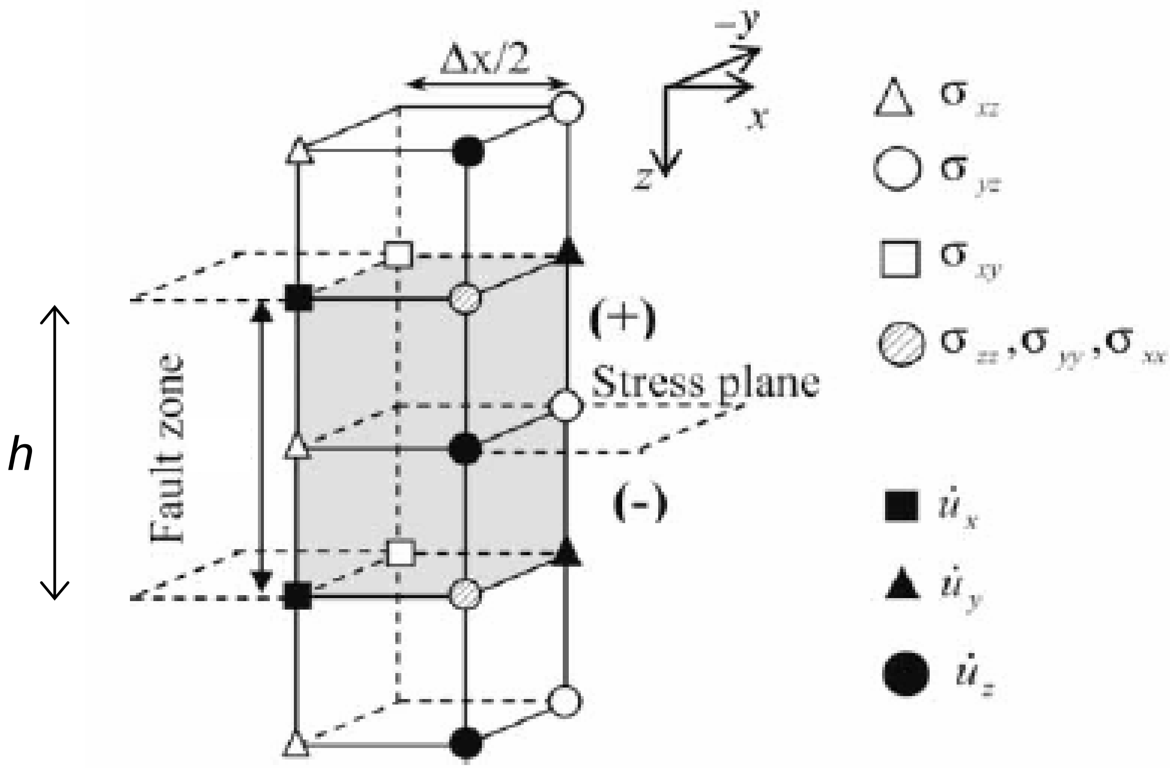
- ***Thin – zone (TnZ)***:



v = **V**$^+$ – **V**$^-$ relative motion of the " + " side with respect to the " – " side.

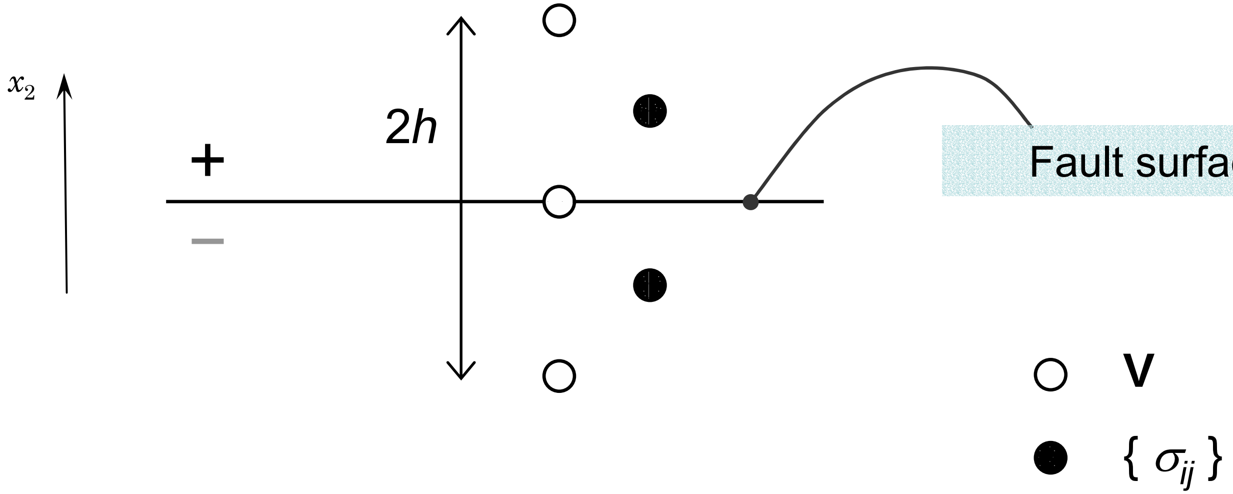
It is calculated over h

*** Continuum medium**



3 – D Velocity – Stress (VS) Staggered – grid (SG) formulation

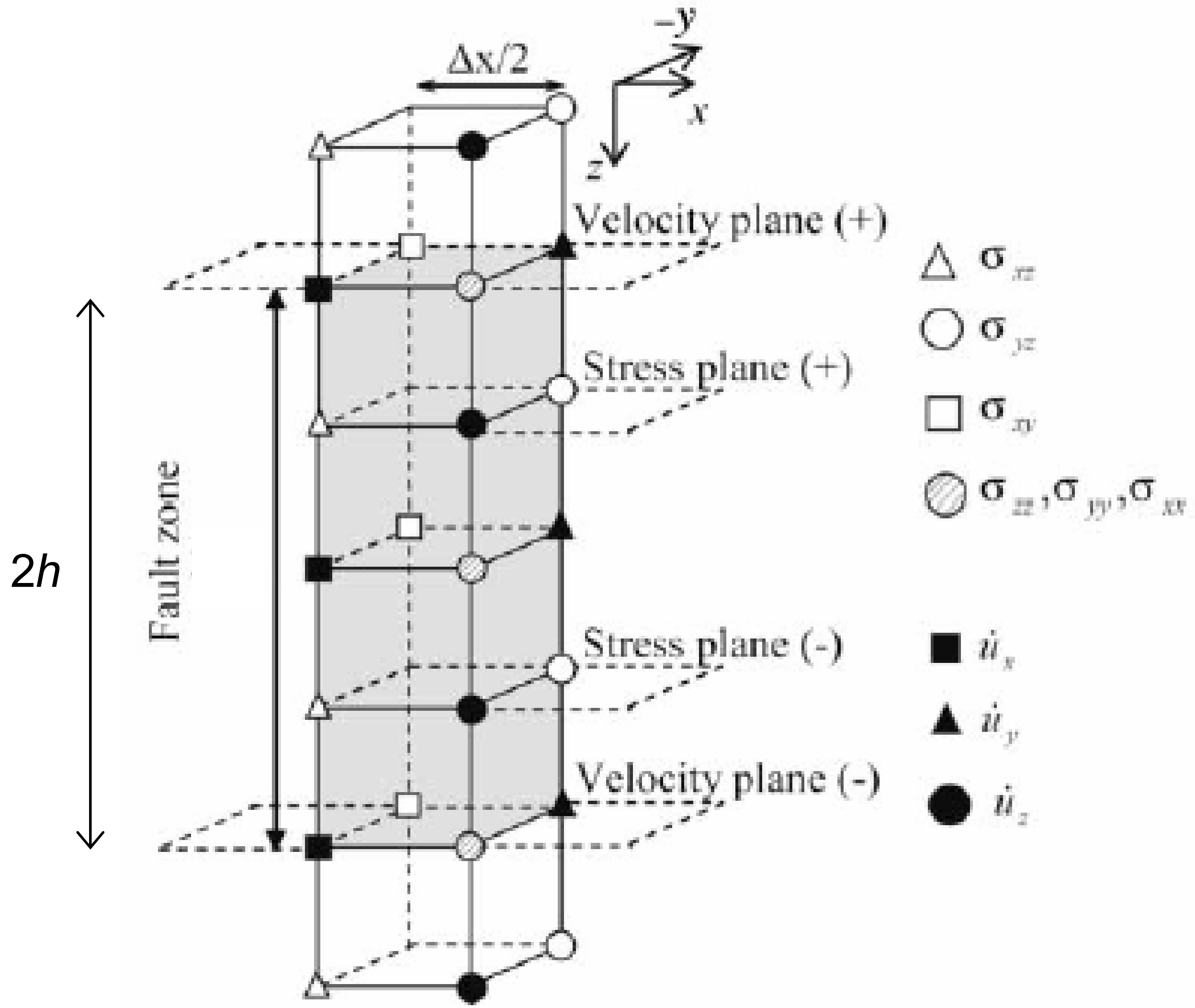
- ***Thick – zone (TkZ)***:



v = V⁺ – V⁻ relative motion of the " + " side with respect to the " – " side.

It is calculated over 2h

*** Continuum medium**



3 – D Velocity – Stress (VS) Staggered – grid (SG) formulation

Auxiliary Conditions

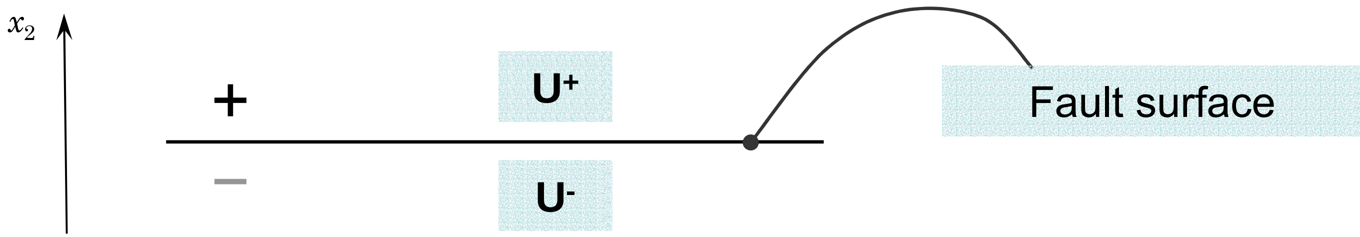
* *COLLINEARITY BETWEEN FAULT SHEAR TRACTION AND FAULT SLIP VELOCITY:*

$$\mathrm{T} \;/\!/\; \mathbf{v}$$

(i. e. $\hat{\mathrm{T}} = \dfrac{\mathbf{v}}{\|\mathbf{v}\|}$).

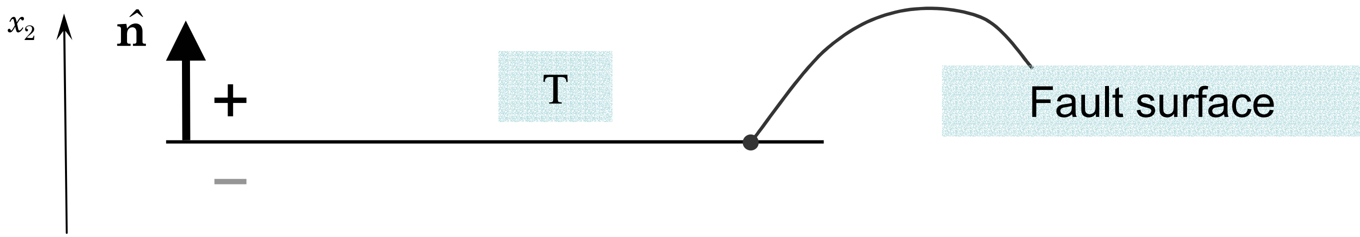
* COLLINEARITY VS. ANTIPARALLELISM

1. Definition of the fault slip **u** (i. e. displacement discontitunity):



$\mathbf{u} = \mathbf{U}^+ - \mathbf{U}^-$ relative motion of the " + " side with respect to the " – " side

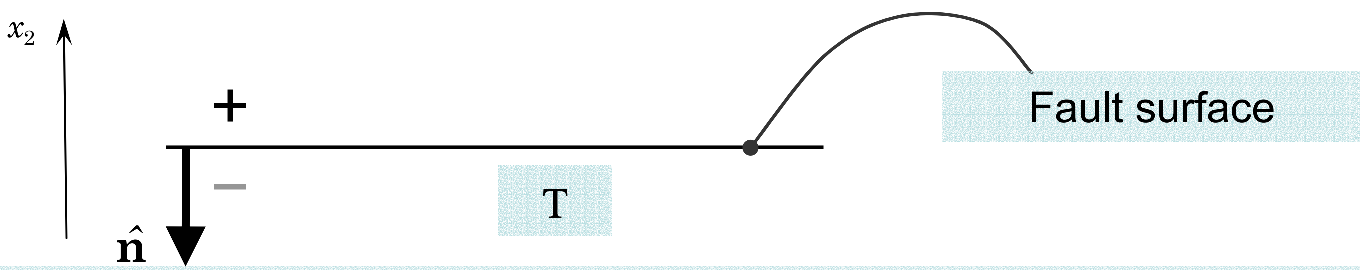
2. Fault surface orientation. Possibility A



T — Shear traction that a particle loacted on the " + " side exercised on particle located in the " – " side

In this case the traction vector T is collinear to the direction of motion (namely to the fault slip vector **u** and thererore to the fault slip velocity vector **v**).

Possibility B



T Shear traction that a particle loacted on the “ – “ side exercised on particle located in the “ + “ side

In this case the traction vector T is **antiparallel** to the direction of motion (namely to the fault slip vector **u** and thererore to the fault slip velocity vector **v**).