



**Rupture propagation in
2 – D fault models**

Earthquake parameters

Observed Parameters

Seismic Moment: M_0

Radiated Energy: E_R

Rupture Speed: v_r

Source Duration (Corner Frequency): T (f_c)

Inferred (sometimes Observed) Parameters

Static Stress Drop: $\Delta\sigma_s$

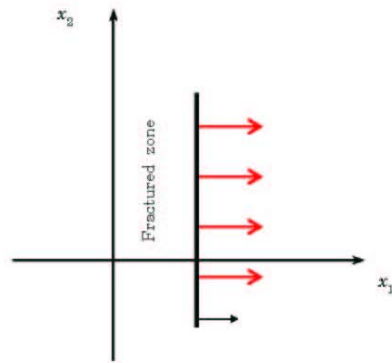
Particle Motion Velocity: \dot{U}

Source Dimension: L

Critical Slip: D_c

Remembering dimensionality ...

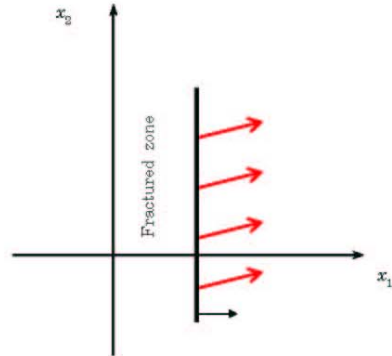
PURE MODE II



Dependence on x_1
Independence on x_2

$$\Rightarrow u_1(x_1, t)$$

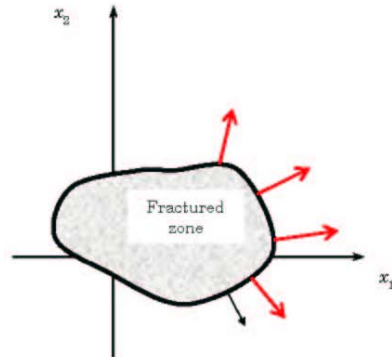
MIXED MODE



Dependence on x_1
Independence on x_2

$$\Rightarrow u_1(x_1, t) \\ u_2(x_1, t)$$

TRULY 3-D



Dependence on x_1
Dependence on x_2

$$\Rightarrow u_1(x_1, x_2, t) \\ u_2(x_1, x_2, t)$$

— Crack tip
→ Local crack enlargement direction

→ Local displacement



Numerical Method: BIE 2 - D

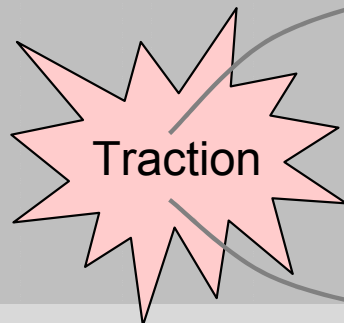
We solve the fundamental elastodynamic equation, neglecting body forces \mathbf{f}

$$\rho \ddot{U}_i = \sigma_{ij,j} + f_i$$

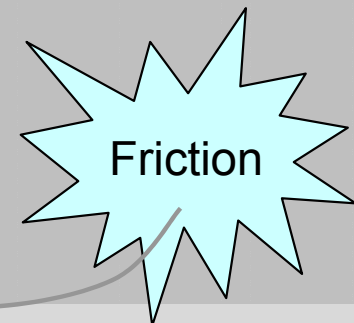
Source integral representation (*Betti's* theorem, Integration in time limit in fault surface, Lamb's problem):

$$u_n(\mathbf{x}, t) = \int_{-\infty}^{+\infty} dt' \int_{S(t')} d\xi G_{n\alpha}(\mathbf{x} - \xi, t - t') \sigma_{\alpha\beta}^p(\xi, t') \xi_n, n=1,2,3; \alpha=1,2; \mathbf{x}, \xi \in \mathbb{R}^3$$

First neighbours decoupling (in the case of a 2 - D, pure in - plane rupture):

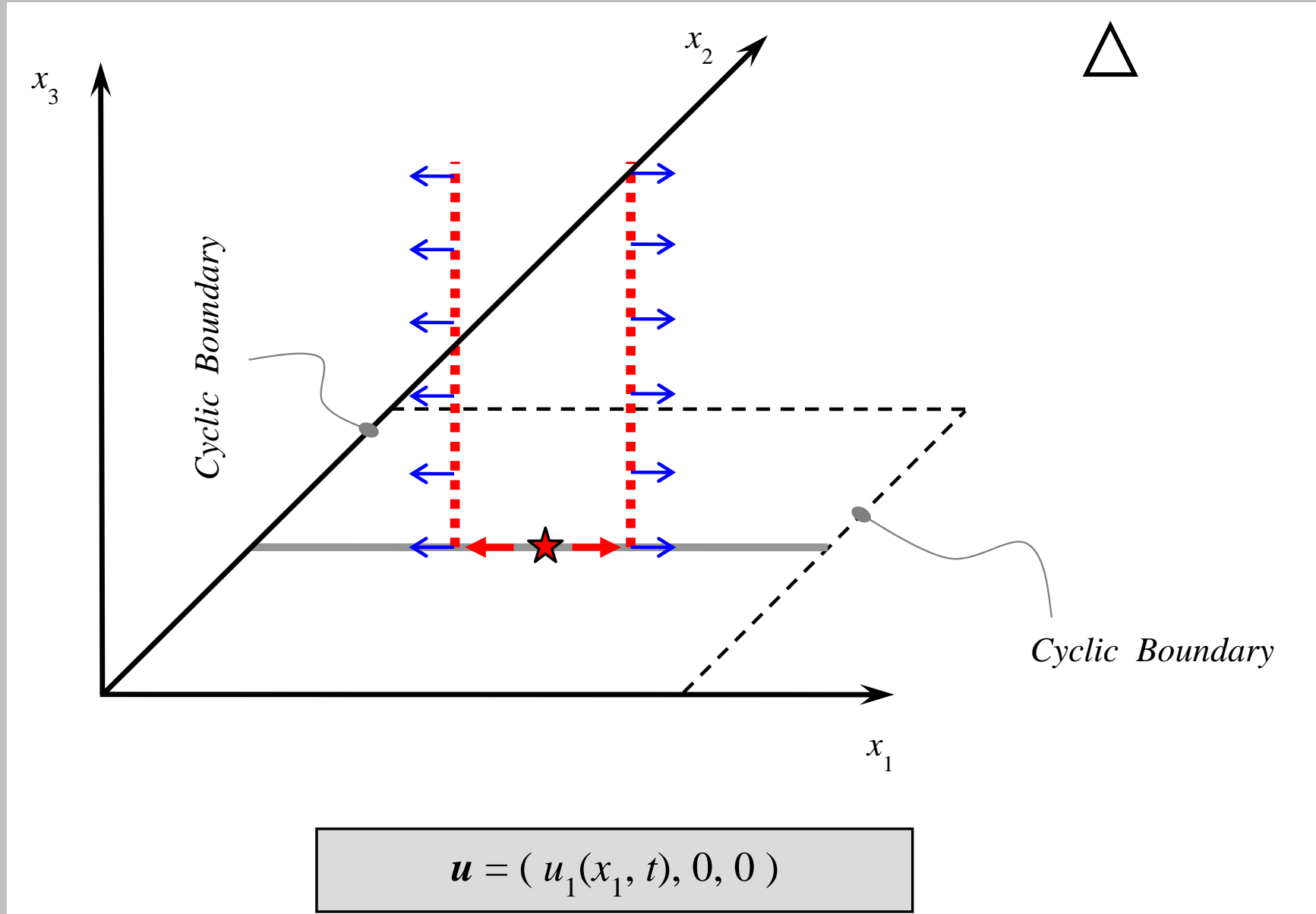


$$\begin{cases} u_1(x_1, t) + C \tau_1^p(x_1, t) = L_1(x_1, t) \\ \tau_{0_1} + \tau_1^p(x_1, t) = \mu \sigma_n^{eff} \end{cases}$$





Numerical Method: FD 2 - D



We solve the fundamental elastodynamic equation, neglecting body forces \mathbf{f}

~~$$\rho \ddot{u}_i = \sigma_{ij,j} + f_i$$~~

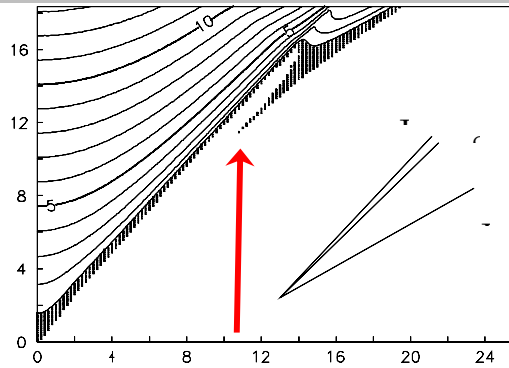
We discretize the x_1x_2 plane by using triangular cells (better performances)

$$\rho \frac{\partial}{\partial t} \dot{u}_1 = \frac{\partial}{\partial x_1} \Sigma_{11} + \frac{\partial}{\partial x_2} \Sigma_{12}$$

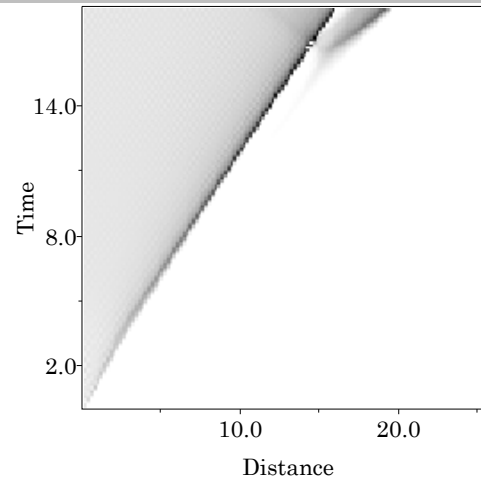
$$\rho \frac{\partial}{\partial t} \dot{u}_2 = \frac{\partial}{\partial x_1} \Sigma_{12} + \frac{\partial}{\partial x_2} \Sigma_{22}$$

The plane is linear and elastic except in the fault intersection line, where a Fault Boundary Condition (TSN scheme) is adopted. In this line a constitutive law is assumed to relate staggered stress with observables (slip, slip velocity, ...)

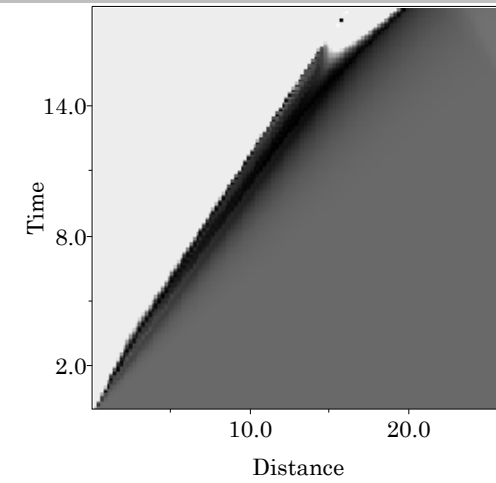
BIE



(a)

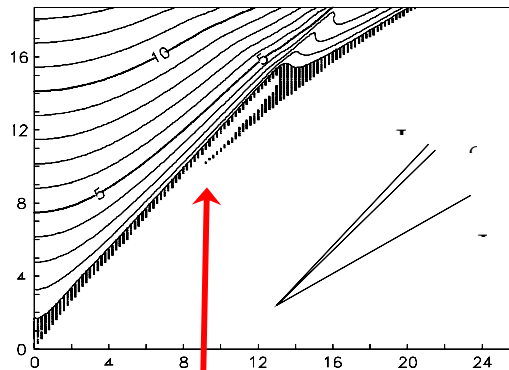


(b)

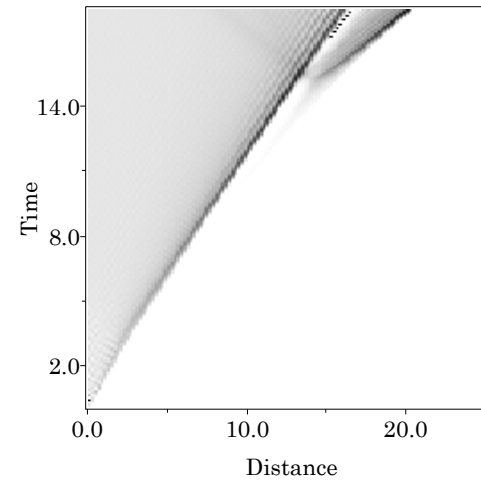


(c)

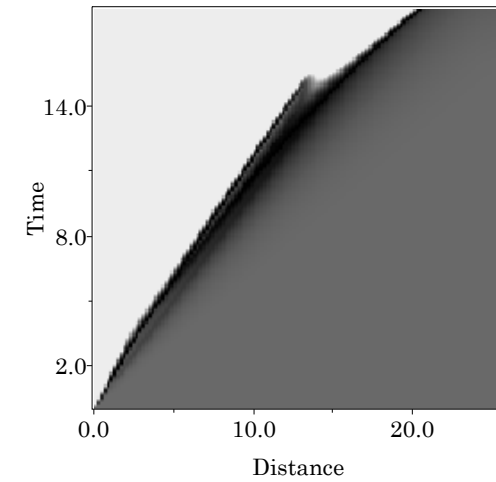
FD



(d)



(e)

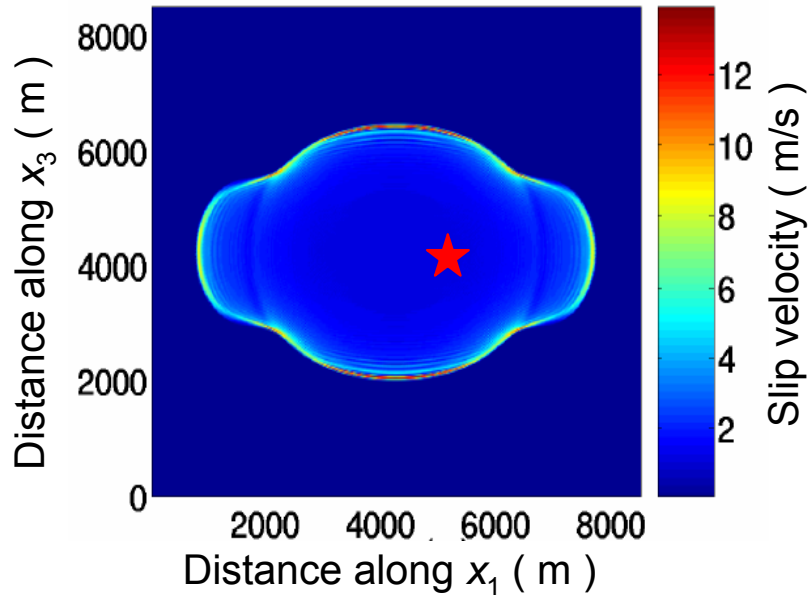


(f)



The cohesive zone

Time snapshot
($t = 0.8 \text{ s}$) - SW law



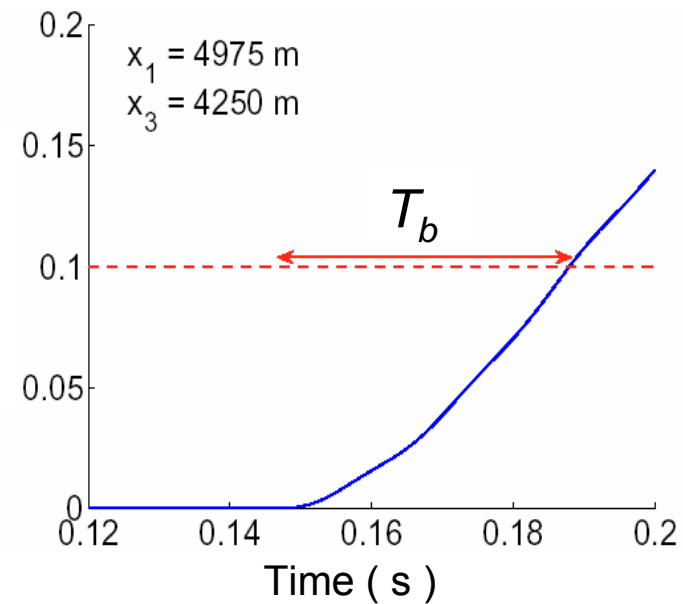
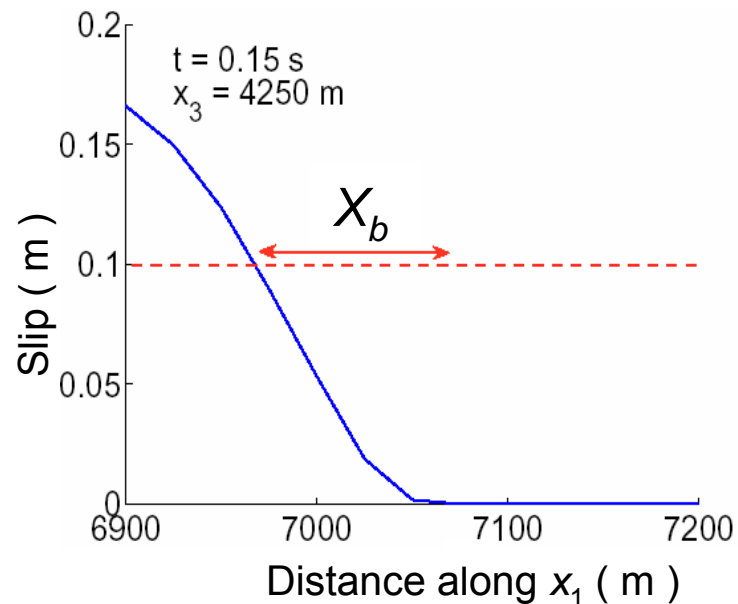
In the target location we can estimate:

$$X_b = 105 \text{ m} \quad T_b = 0.04 \text{ s}$$

From these quantities:

$$V_{rupt} = X_b / T_b = 2625 \text{ m/s}$$

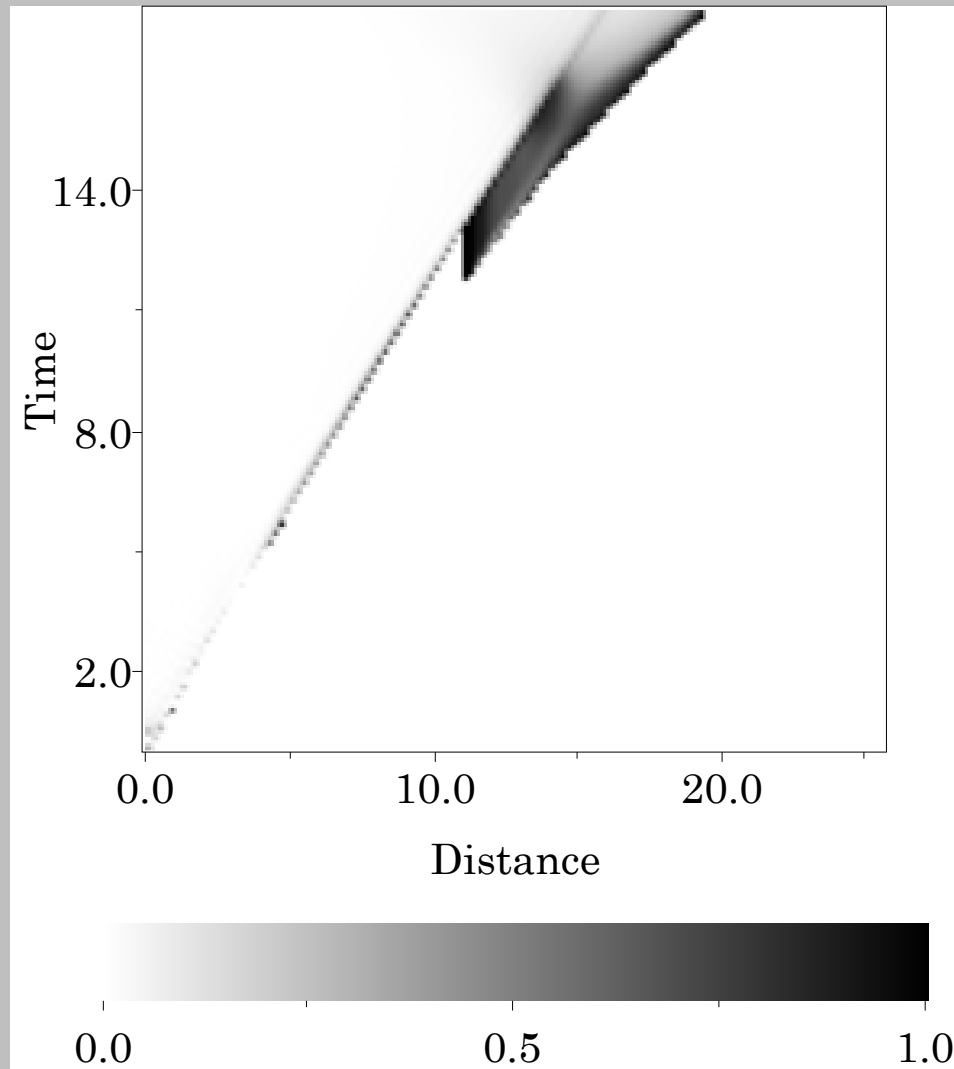
Local estimate





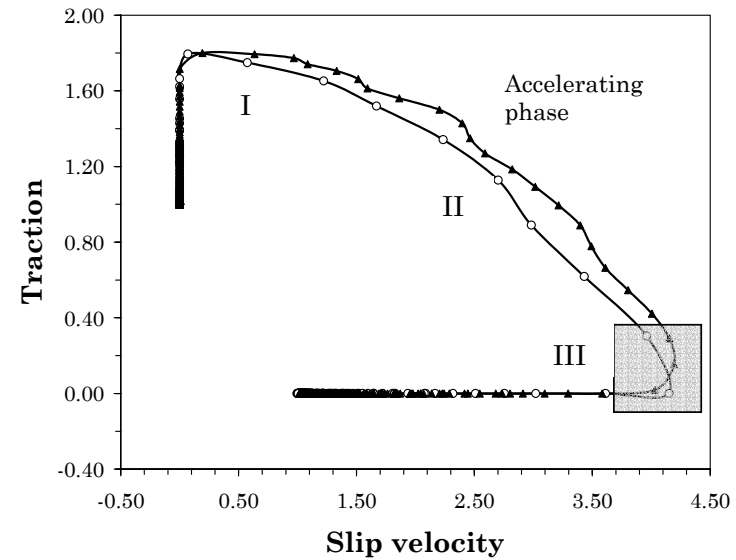
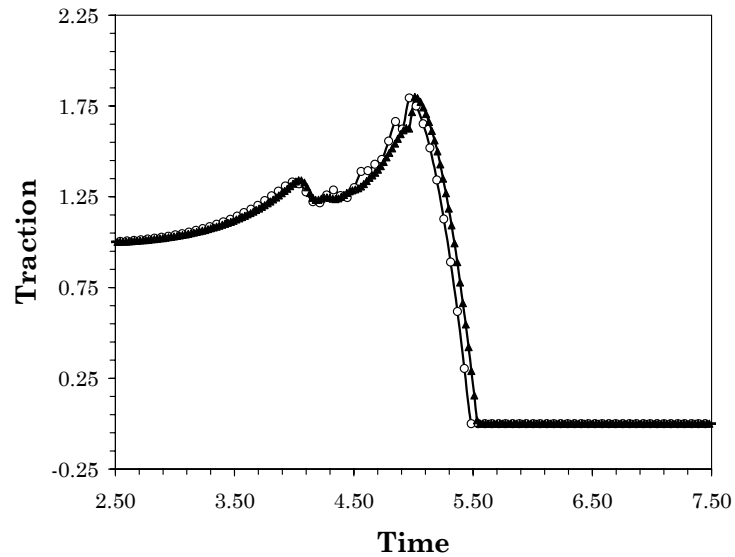
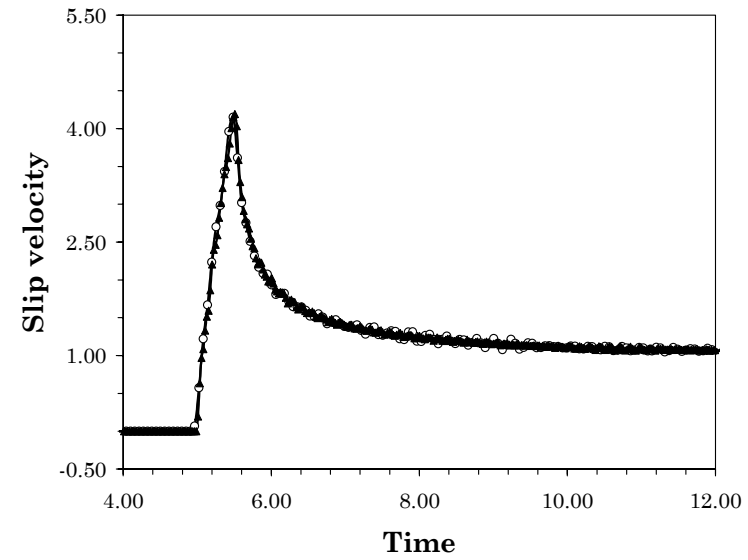
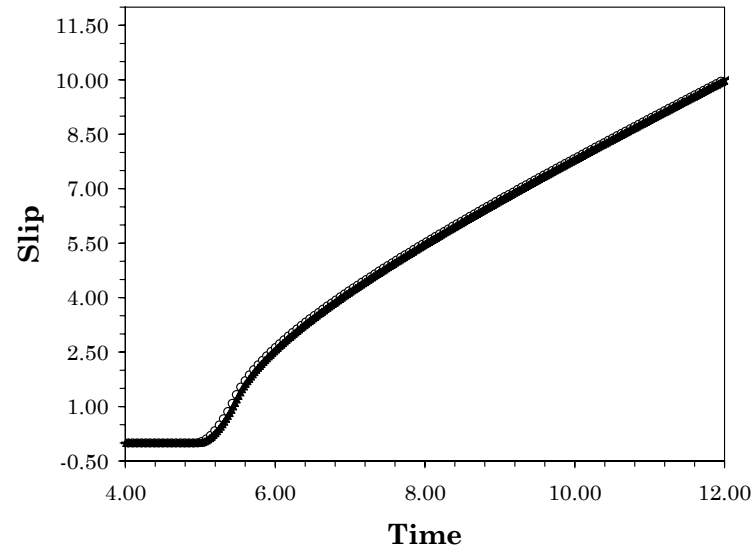
Misfit between slip modeled with BIE and FD

$$m(x_i, t_n) = \frac{|u^{(\text{BIE})}(x_i, t_n) - \tilde{u}^{(\text{FD})}(x_i, t_n)|}{|u^{(\text{BIE})}(x_i, t_n) + \tilde{u}^{(\text{FD})}(x_i, t_n)|}$$



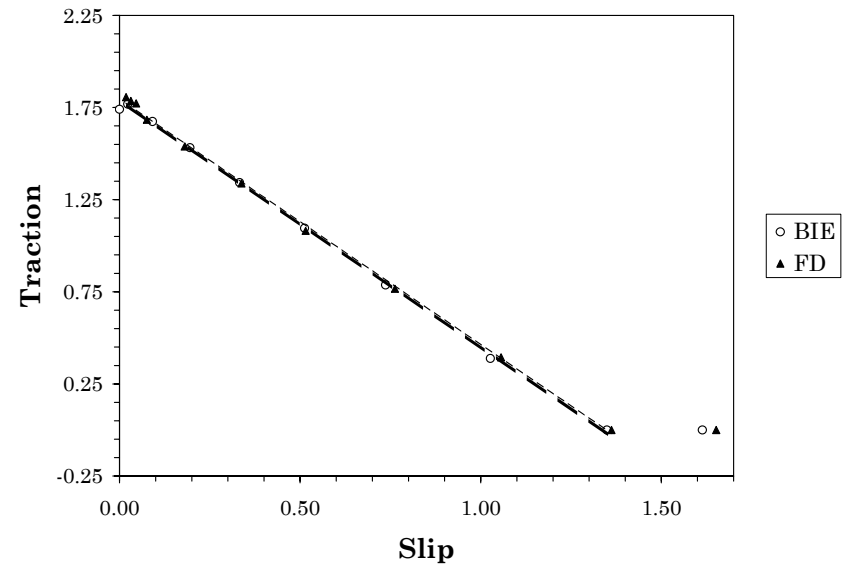
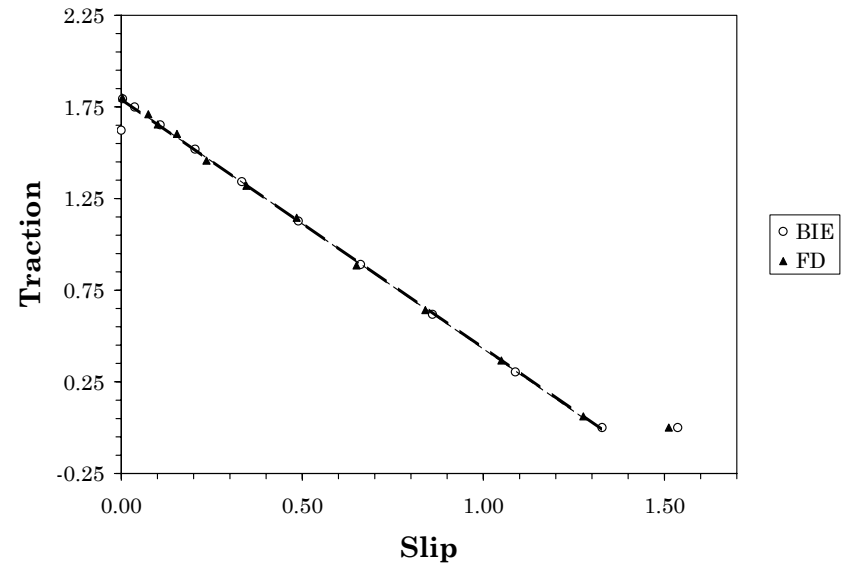


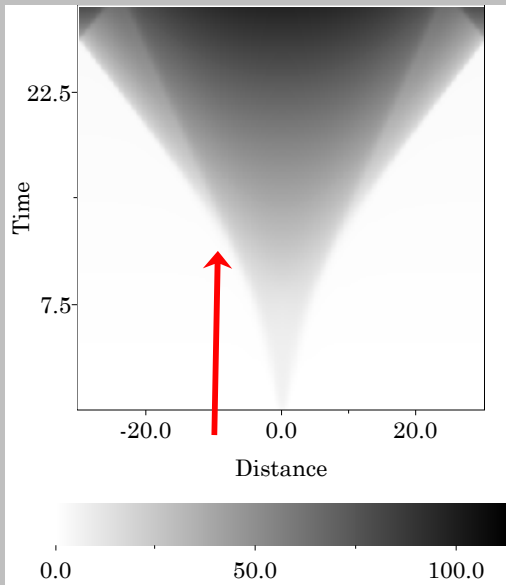
BIE vs. FD with SW #2



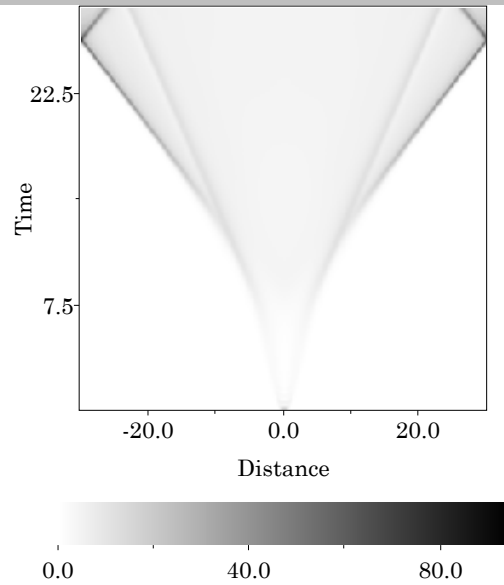


BIE vs. FD with SW #3

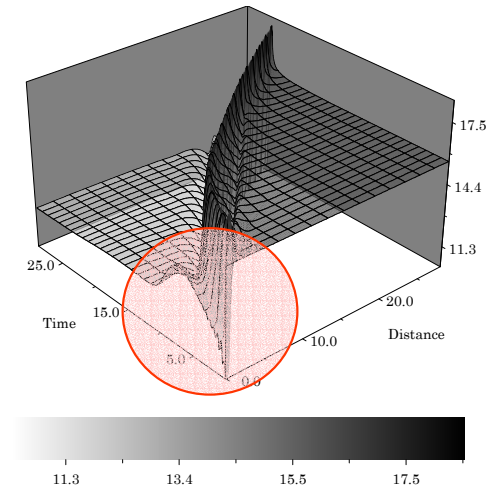




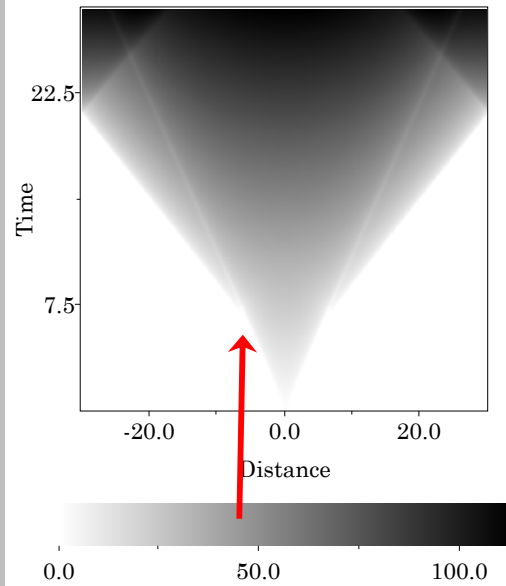
(a)



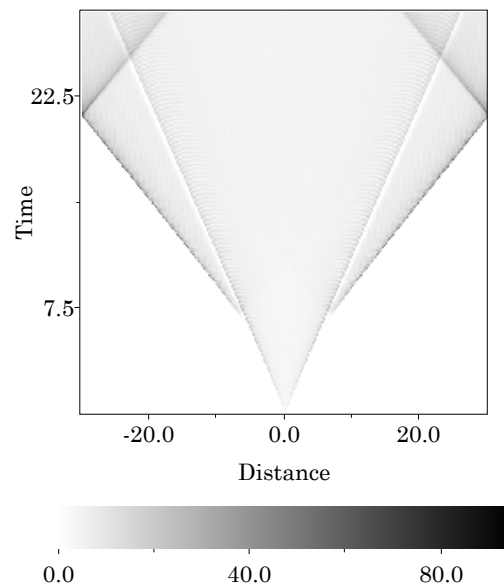
(b)



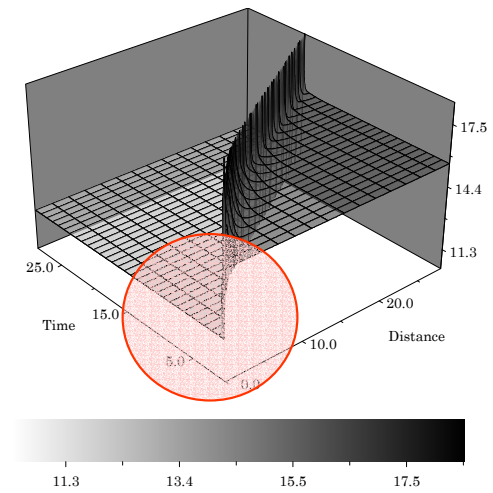
(c)



(d)



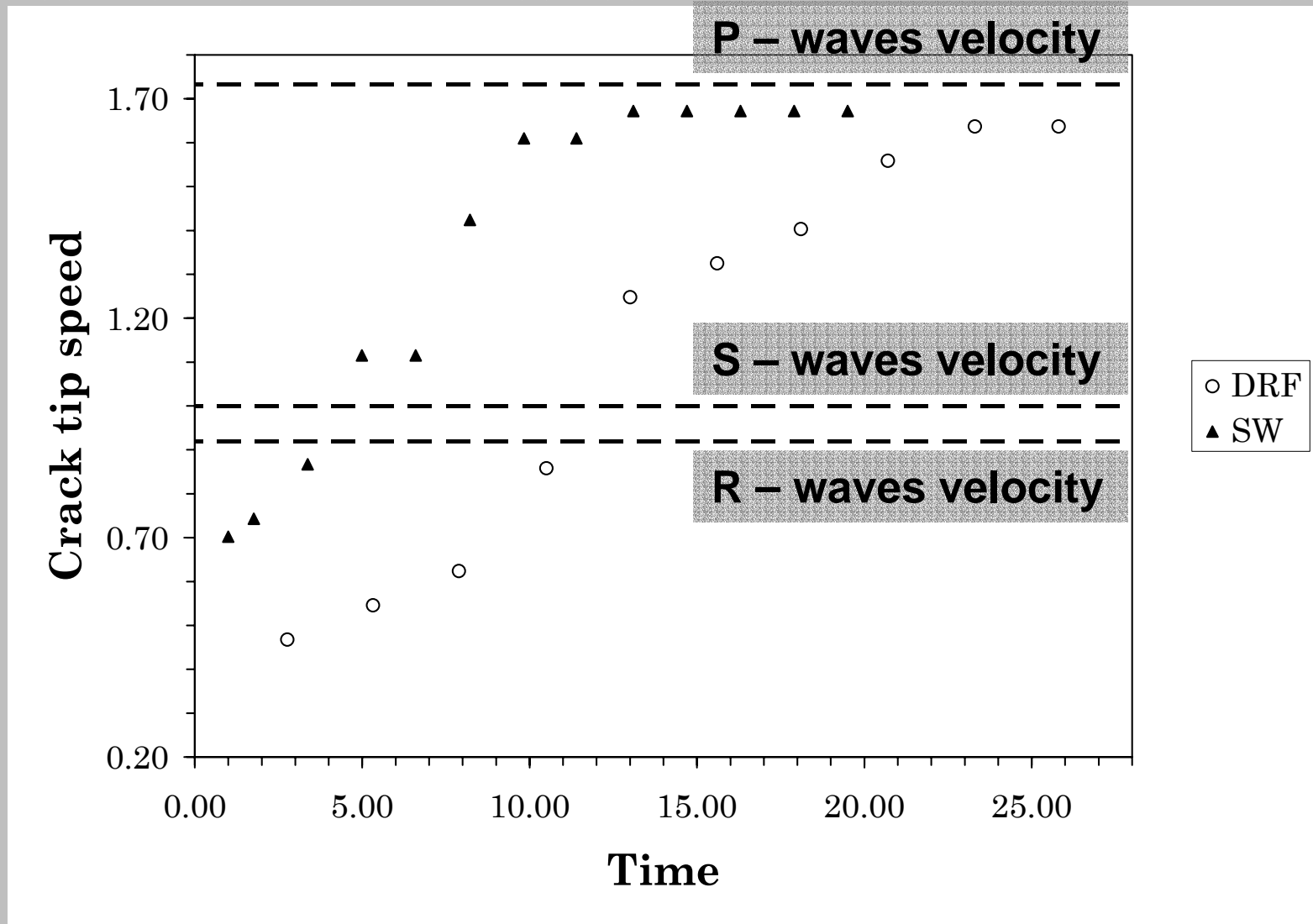
(e)



(f)

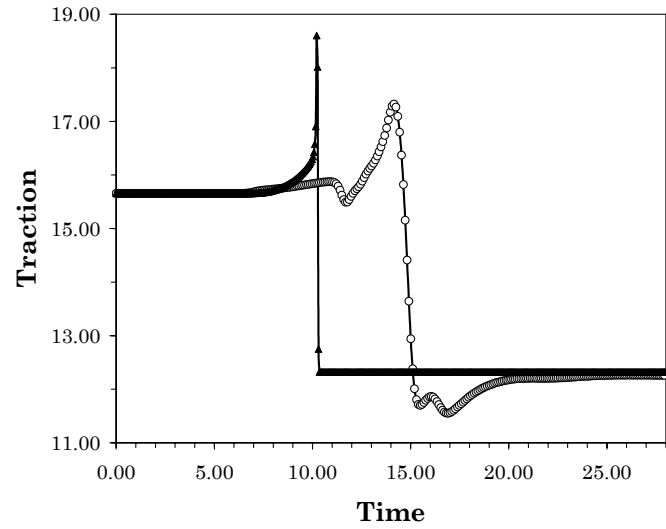


SW vs. DR law #2

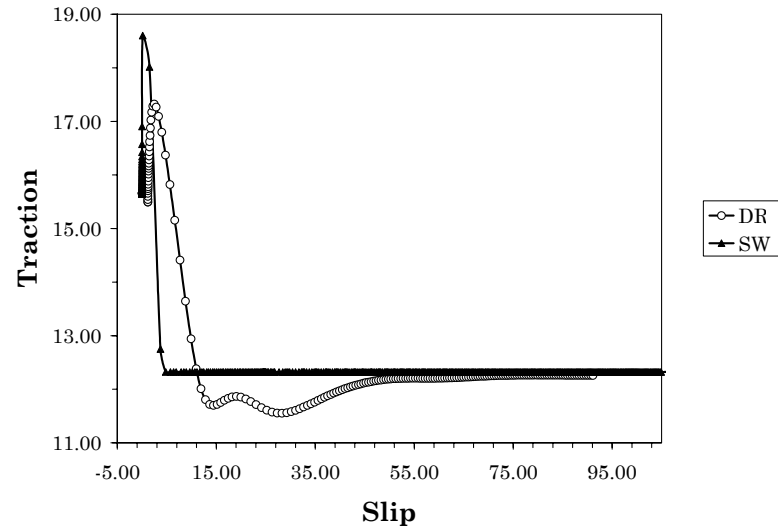




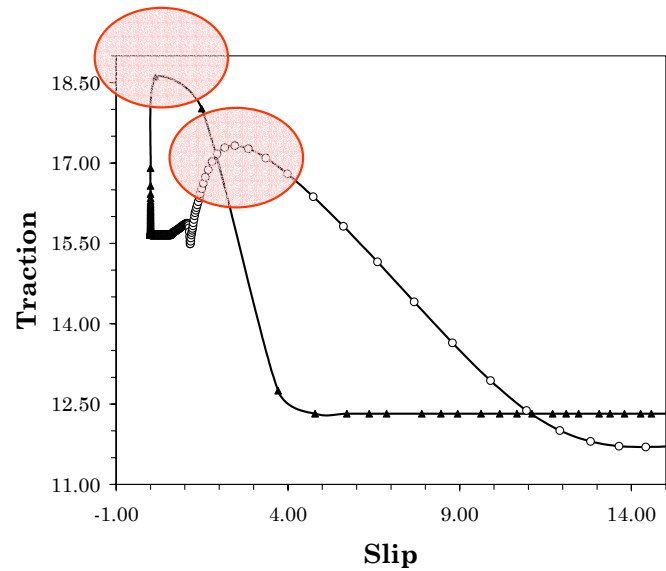
SW vs. DR law #3



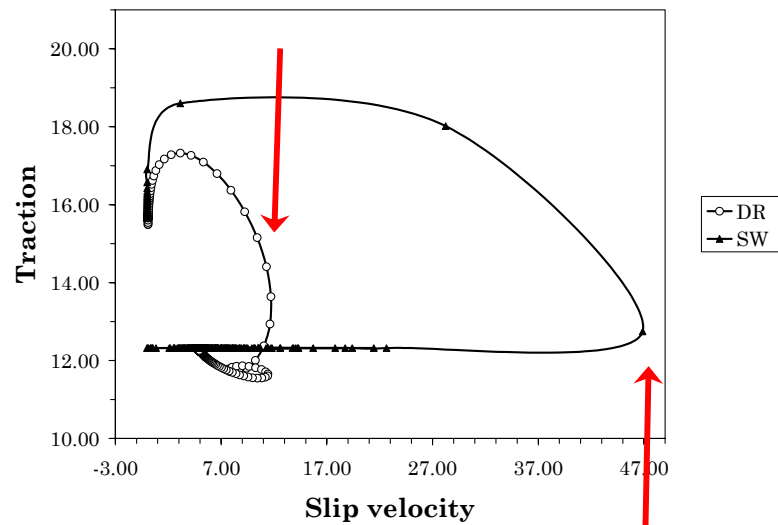
(a)



(b)



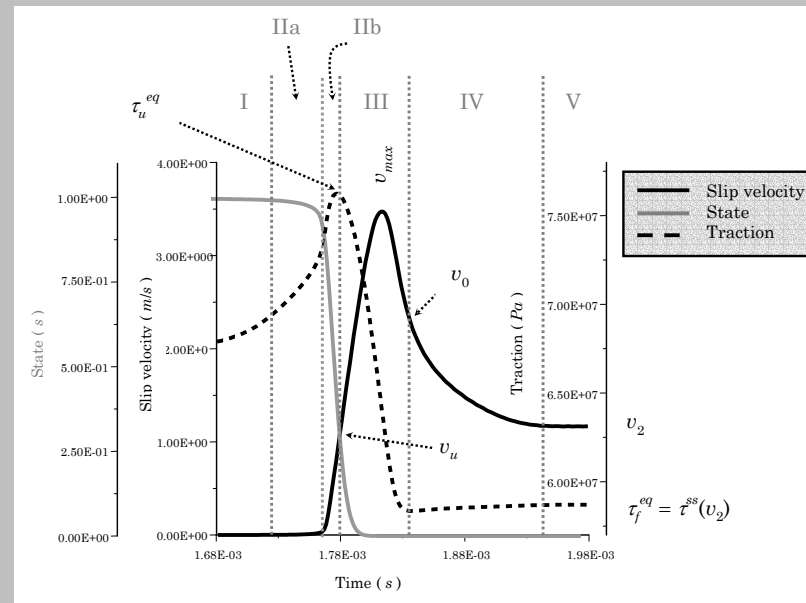
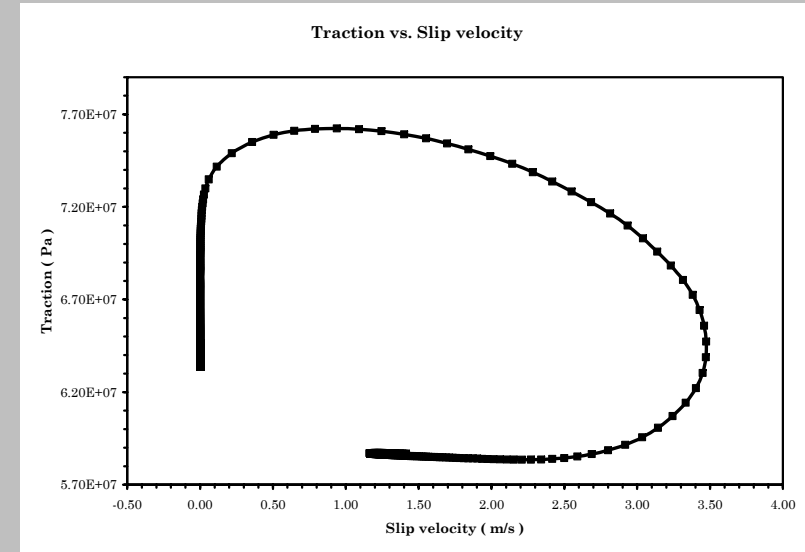
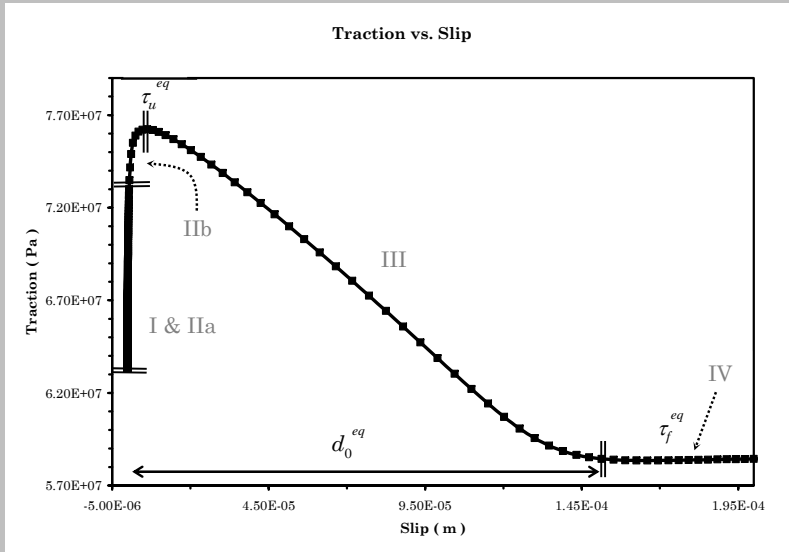
(c)



(d)

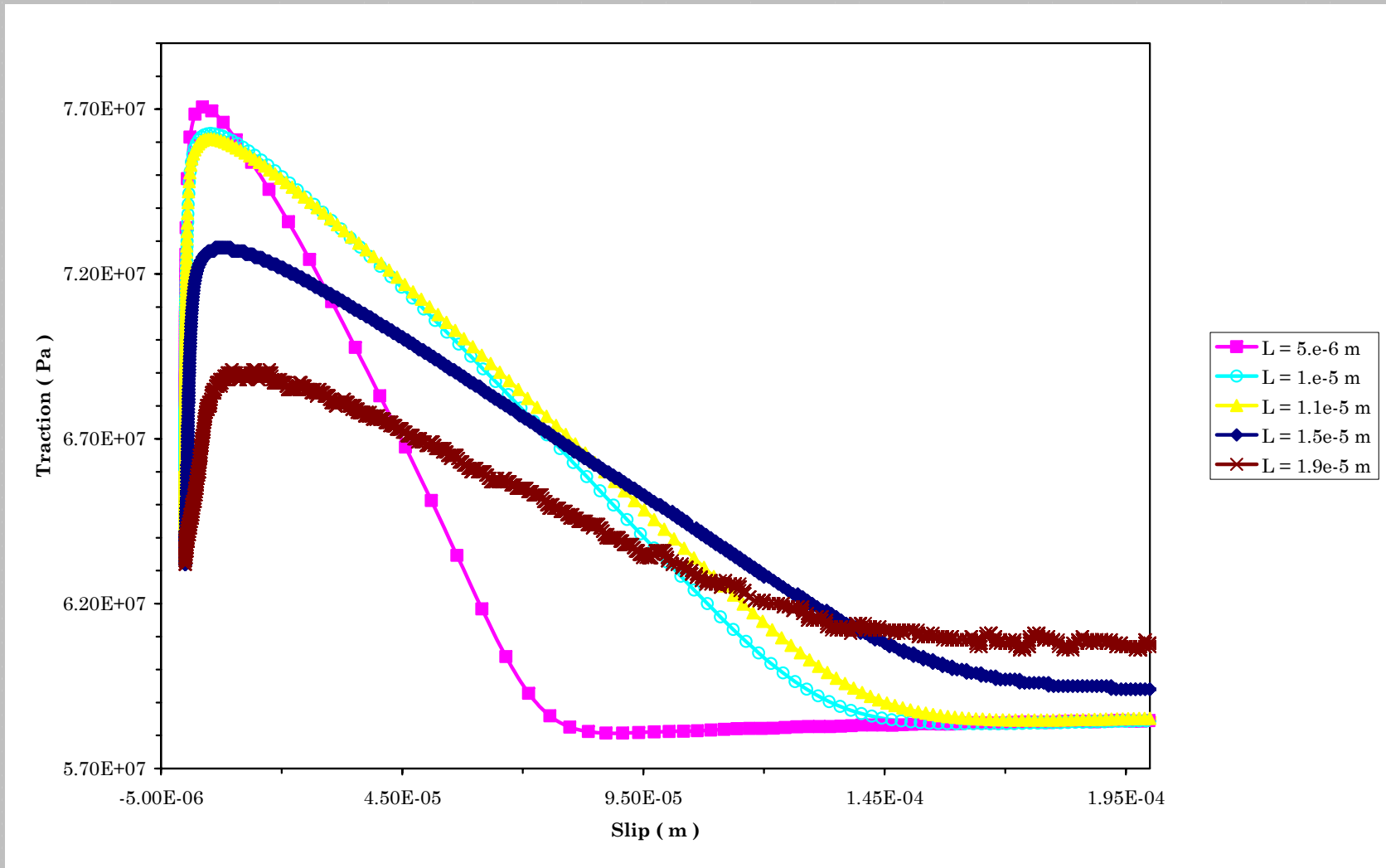


The dynamic propagation. The cohesive zone and the breakdown





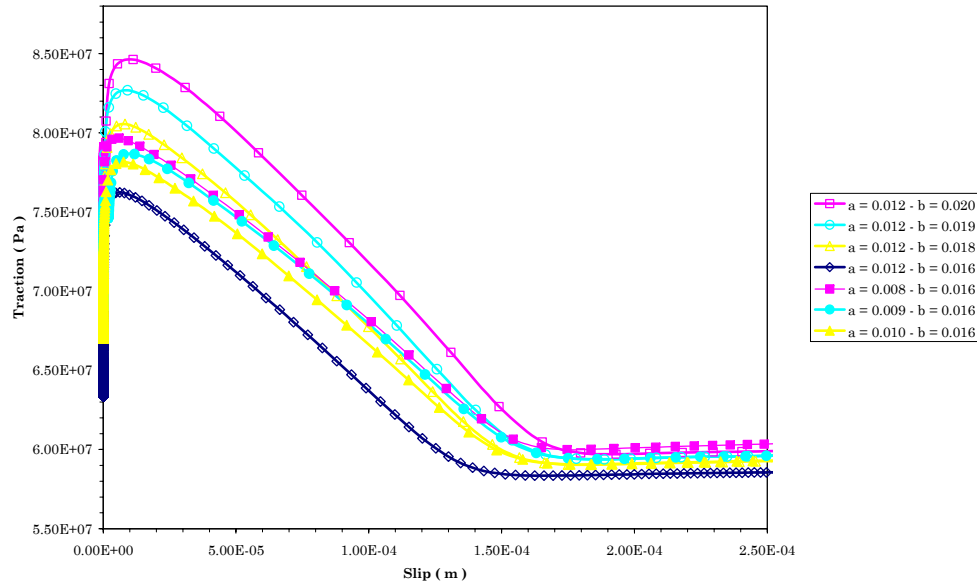
Dependence on L parameter



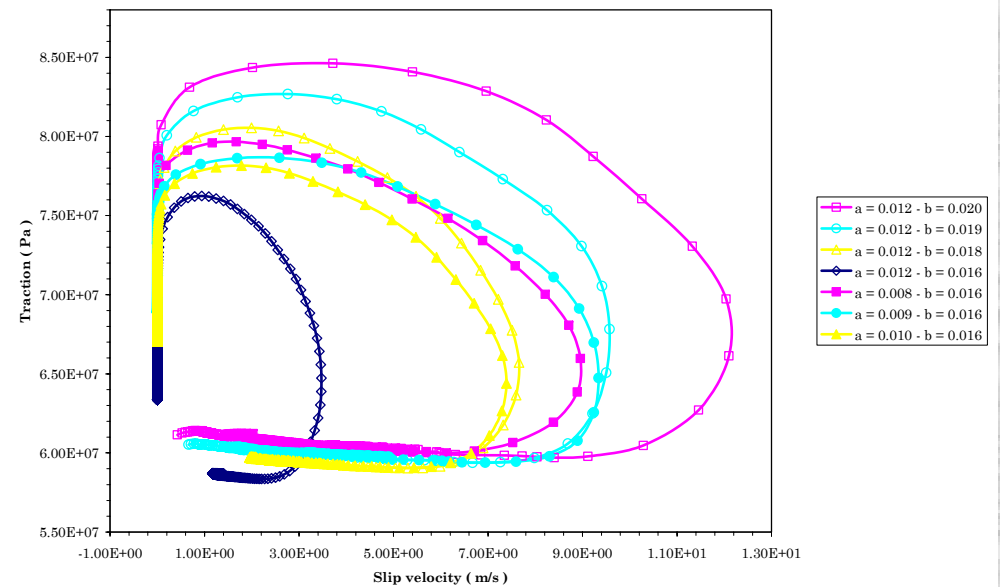


Dependence on a and b

Slip – weakening curves

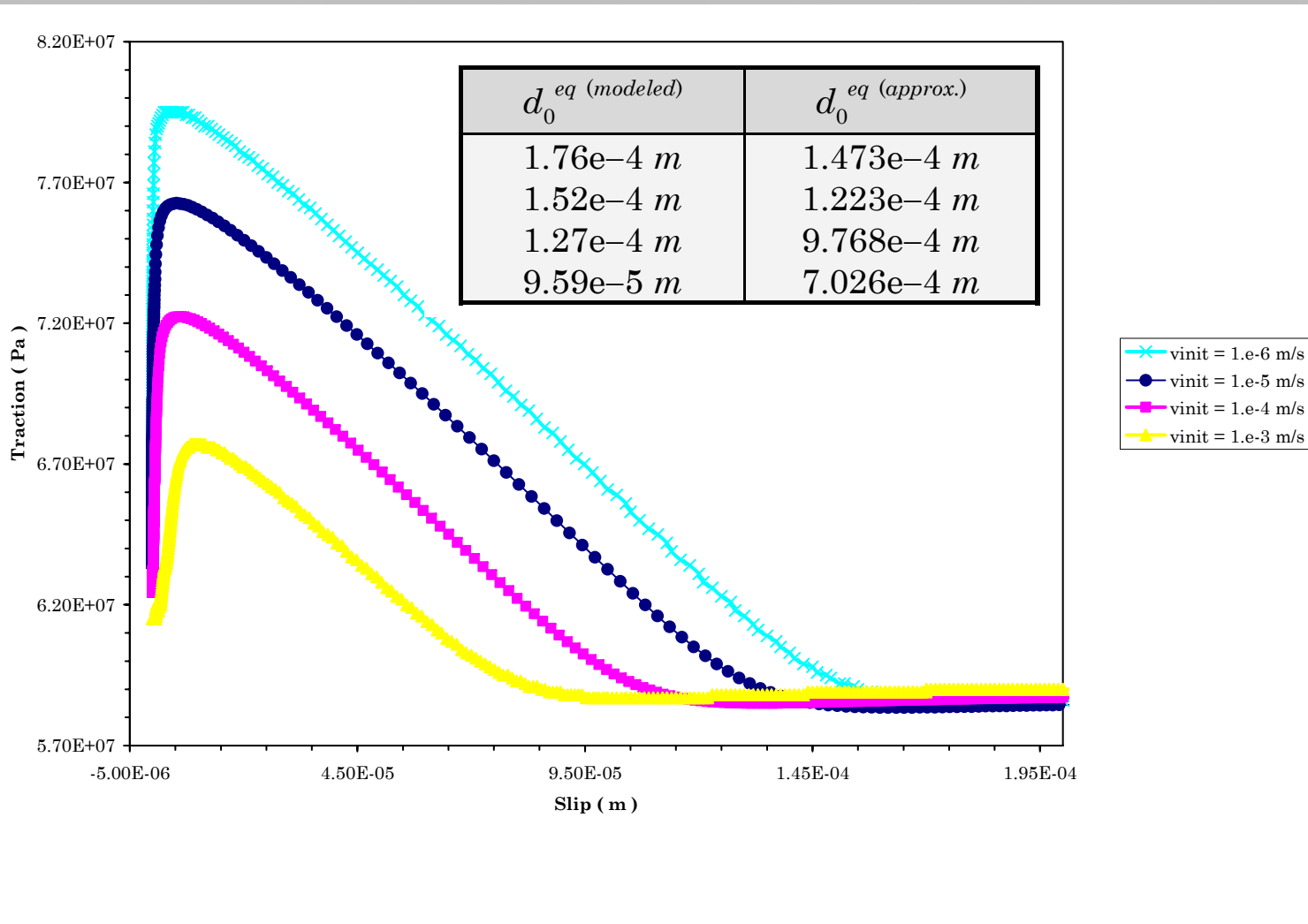


Phase portraits





Dependence on the initial velocity



Limitation for modeling dynamic Rupture and seismic wave generation

- Because the initial slip velocity is totally arbitrary, it is difficult in the framework of R&S formulation to prescribe the traction evolution and the **SW** behavior within the cohesive zone.
- We can only infer an approximated value of the equivalent slip – weakening distance from the proposed scaling law. Moreover, the difference between d_0^{eq} and L depends on the adoption of a slowness (ageing) evolution equation.

Theoretical interpretations

➤ We derived analytical expressions that relate the yield and the kinetic frictional stresses to the constitutive parameters and to slip:

$$\tau = \left[\mu_* + a \ln\left(\frac{v}{v_*}\right) + b \ln\left(\frac{v_*}{v_{init}}\right) - b \frac{u}{L} \right] \sigma_n^{eff}$$

$$\tau_f^{eq} = \left[\mu_* + (b - a) \ln\left(\frac{v_*}{v_2}\right) \right] \sigma_n^{eff}$$


$$D_0^{eq} = L \ln\left(\frac{V_0}{V_i}\right) \approx \frac{\tau_u^{eq} - \tau_f^{eq}}{b \sigma_n} L$$

➤ These relations hold under the assumptions that and that slip velocity is large enough to neglect the term $1/v$. This yields

$$\phi(u) = \frac{L}{v_{init}} e^{-\frac{u}{L}}$$

Numerical estimates of characteristic length

- Laboratory experiments:

Laboratory scale  fault dimension $\sim 20 \text{ m}$ $\Delta x \sim 0.01 \text{ m}$ $L \sim 10^{-5} \text{ m}$
 $L \sim 10^{-5} \text{ m}$ $d_0^{eq} \sim 10^{-4} - 10^{-3} \text{ m}$


- Extending our calculations to real faults

✓ Estimates of D_0 from ground motions or kinematic source models range within $0.5 \leq d_0 \leq 1 \text{ m}$.

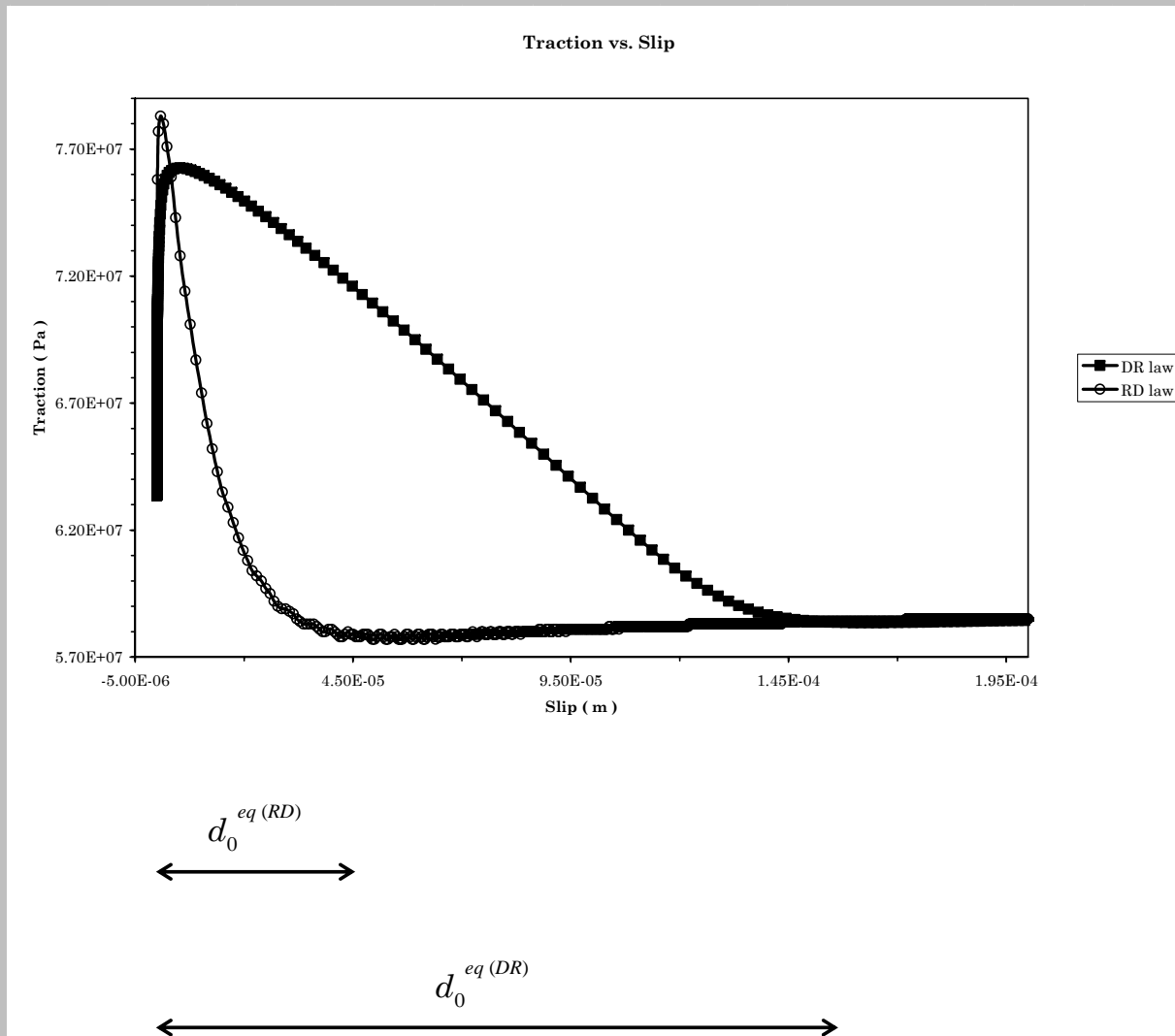
[*Ide and Takeo, 1997; Olsen et al., 1997; Guatteri and Spudich, 2000*]

✓ There is a trade - off between strength - excess and the slip weakening distance d_0 [$d_0 < 0.3 \text{ m}$ are not resolved]

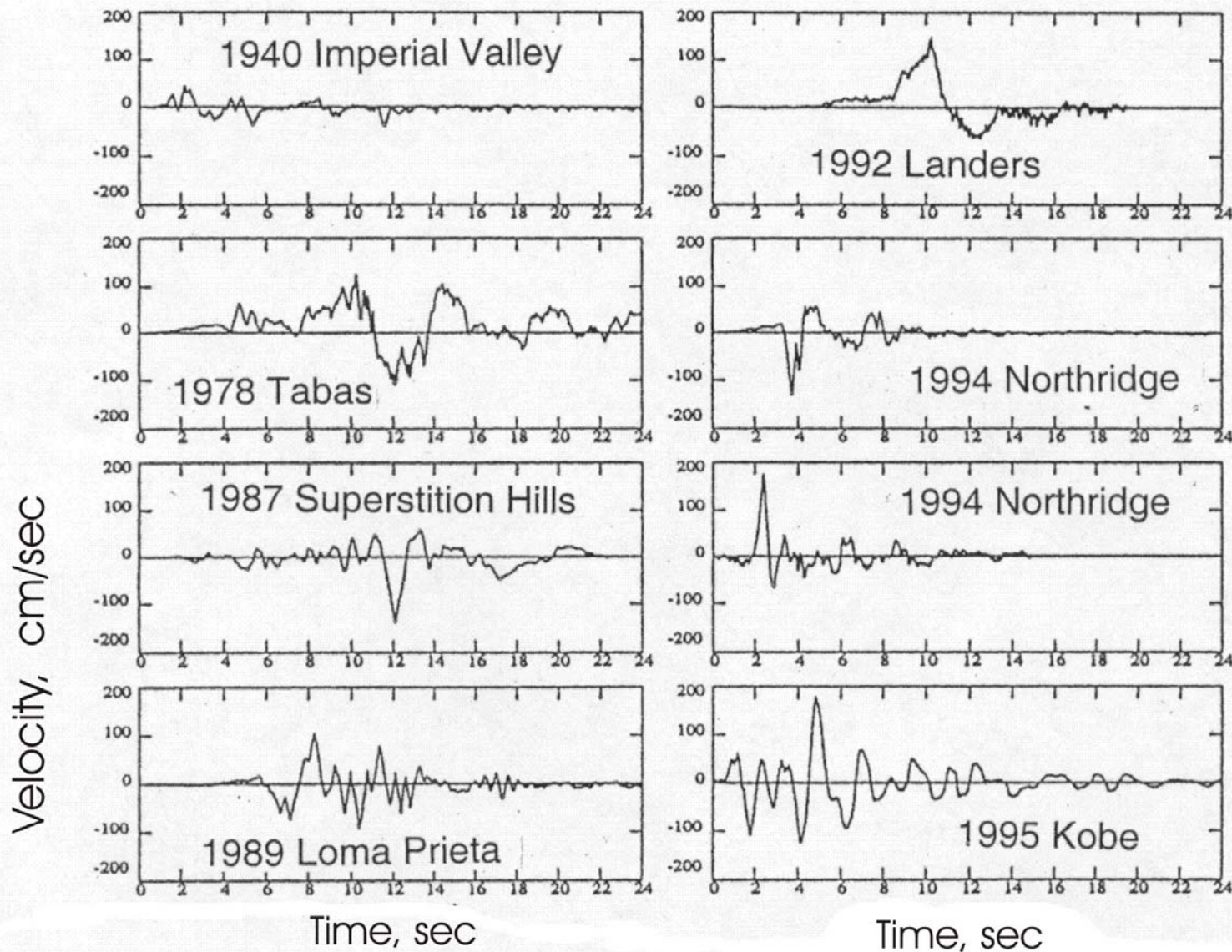
✓ Estimate of d_0 inferred from kinematic inversion models are biased due to smoothing constraints used in the inverse - problems formulation [*Guatteri and Spudich, 2000*]

Fault scale  fault dimension $\sim 20 \text{ km}$ $\Delta x \sim 10 \text{ m}$ $L \sim 10^{-2} \text{ m}$
 $L \sim 10^{-2} \text{ m} = 1 \text{ cm}$ $d_0^{eq} \sim 10^{-1} \text{ m}$

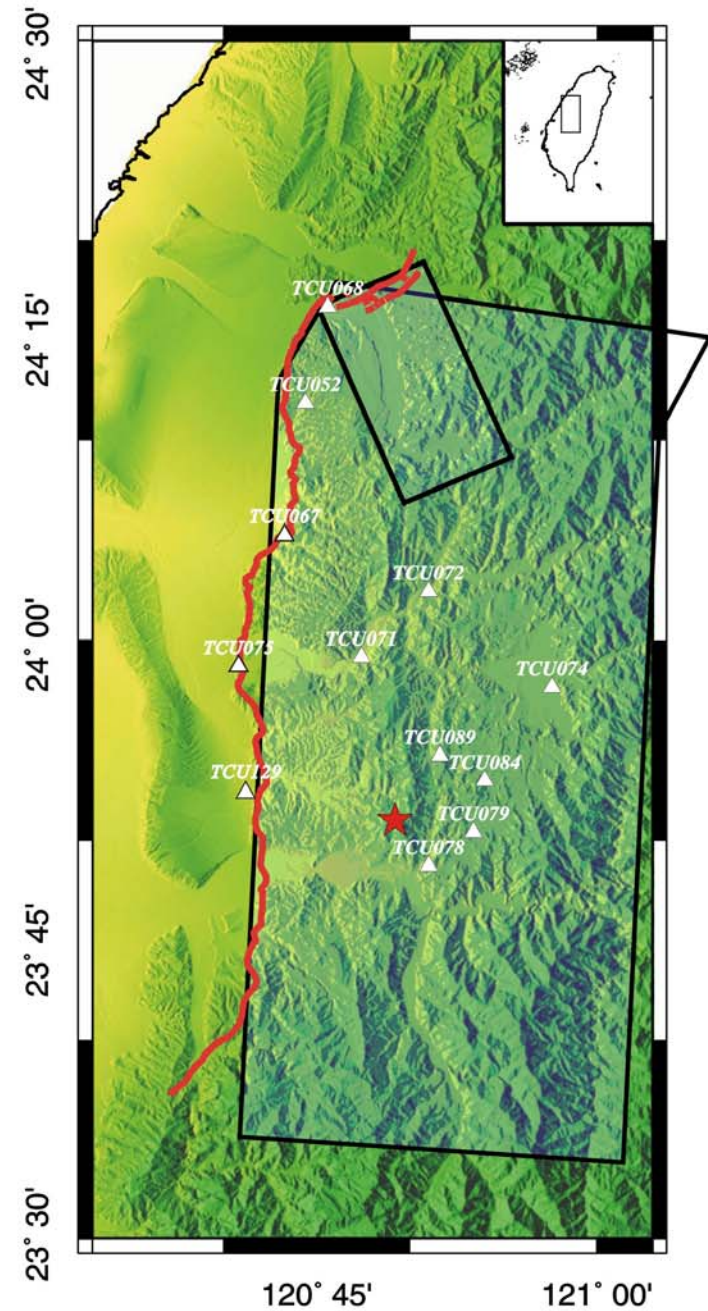
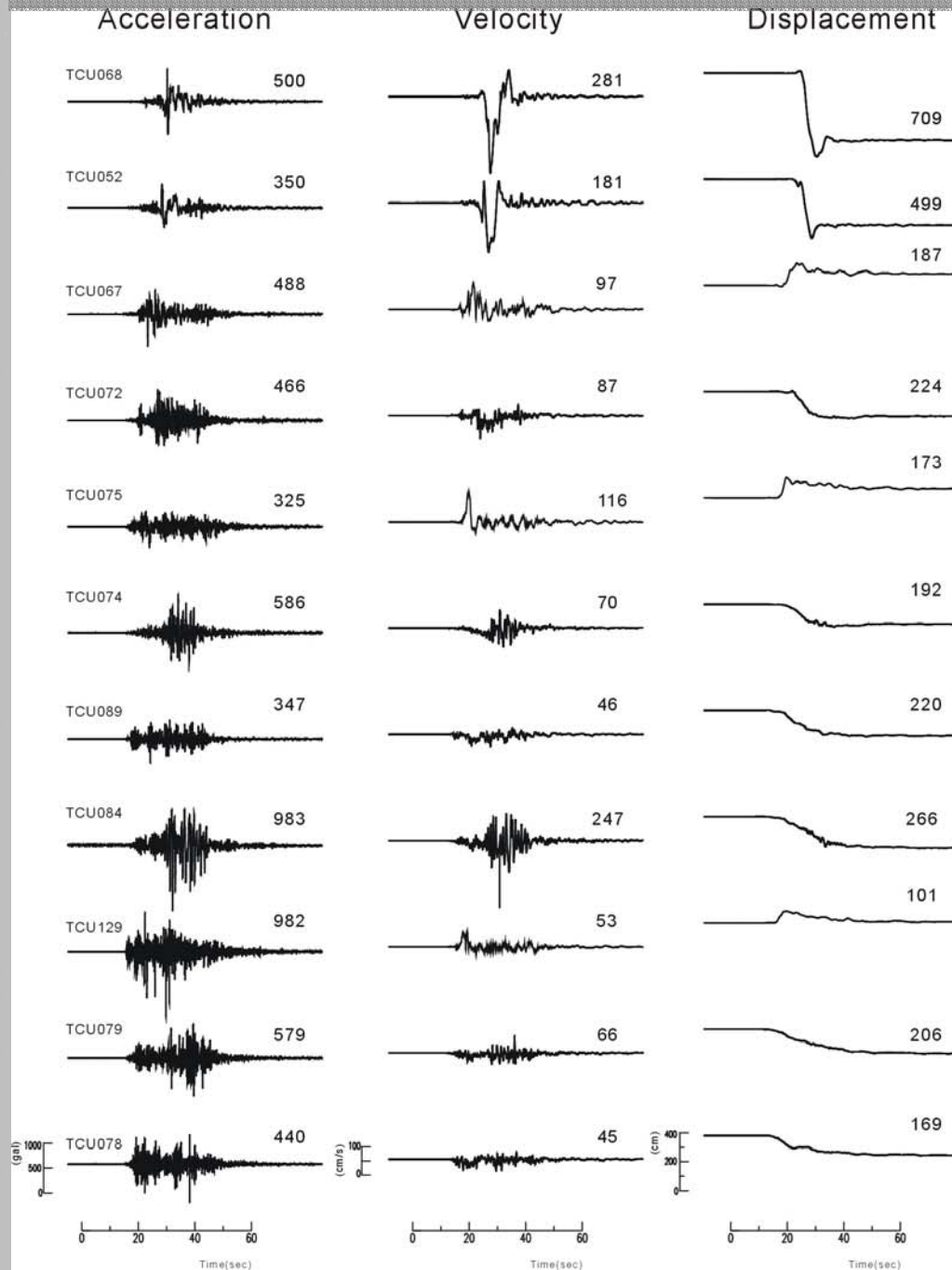
Differences between DR and RD



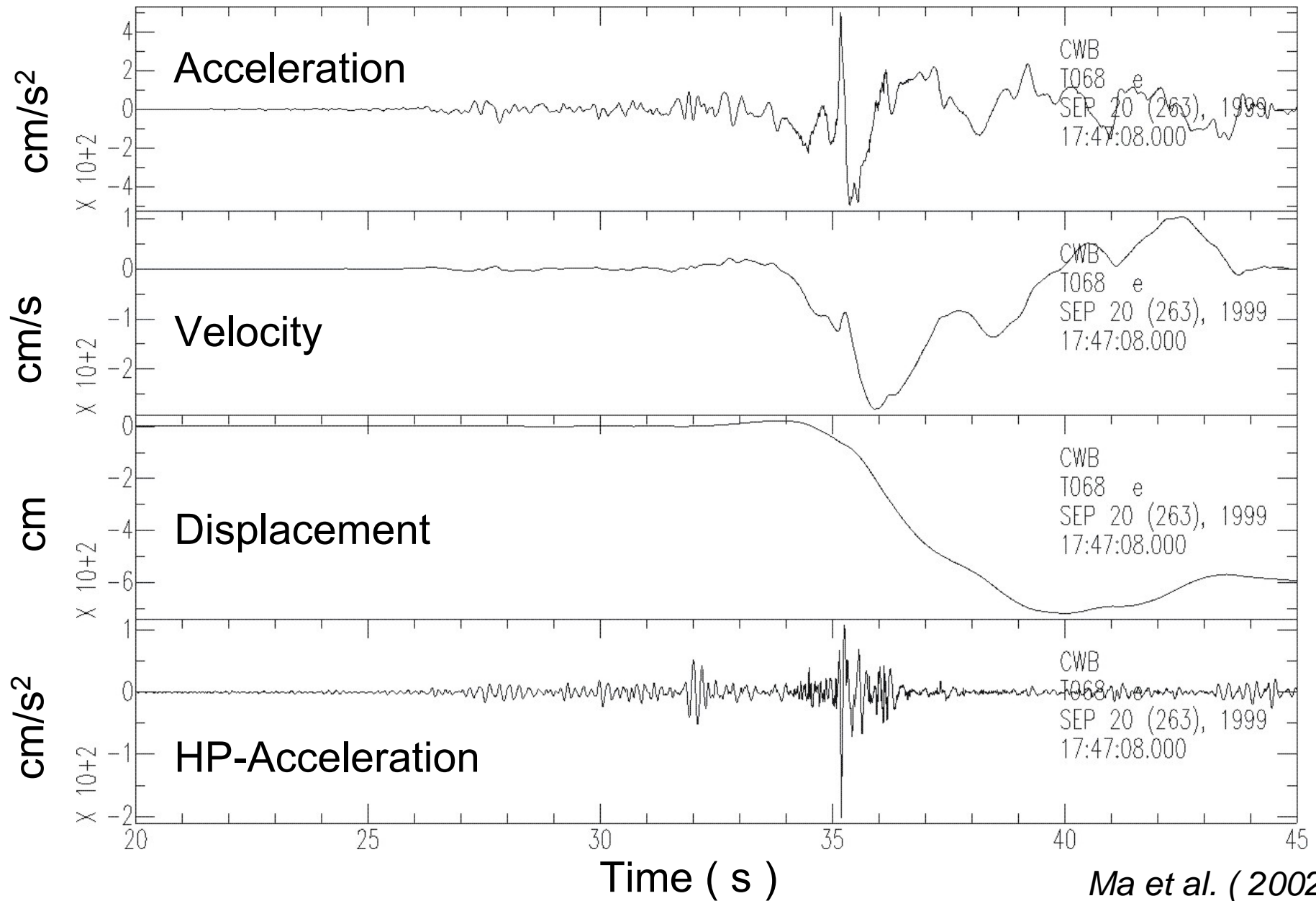
How to relate model results to physical observables



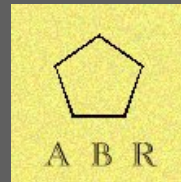
Ground motion from Chi – Chi, Taiwan, EQ



Chi – Chi, Taiwan, EQ; CWBT068



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Support Slides: Parameters, Notes, etc.

To not be displayed directly. Referenced above.

Remembering constitutive law...



LINEAR SLIP – WEAKENING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \frac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

Remembering constitutive law...



DIETERICH IN REDUCED FORMULATION

$$\left\{ \begin{array}{l} \tau = \left[\mu_* - a \ln \left(\frac{v_*}{v} + 1 \right) + b \ln \left(\frac{\Psi v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = 1 - \frac{\Psi v}{L} \end{array} \right.$$

Remembering constitutive law...



RUINA – DIETERICH

$$\left\{ \begin{array}{l} \tau = \left[\mu_* - a \ln\left(\frac{v_*}{v}\right) + b \ln\left(\frac{\Psi v_*}{L}\right) \right] \sigma_n^{eff} \\ \frac{d}{dt} \Psi = -\frac{\Psi v}{L} \ln\left(\frac{\Psi v}{L}\right) \end{array} \right.$$