Appendix A

Relations between the slip – weakening characteristic length and the breakdown zone time

In this section we derive an expression that connect the slip–weakening characteristic distance d_0 and the breakdown time T_b.

Let us consider, in the $x_1 t$ plane, an arbitrary point of the crack tip (x_1^*, t^*); by definition, the slip in this point is zero: $u(x_1^*, t^*) = 0$. Let us consider also the point $(x_1^*, t^* + T_b)$; by definition of breakdown time it results $u(x_1^*, t^* + T_b) = d_0$. Therefore we have:

$$\frac{u\left(x_1^*, t^* + T_b\right) - u\left(x_1^*, t^*\right)}{T_b} = \frac{d_0}{T_b} \tag{A.1}$$

In full of generality the breakdown time is a whole multiple of the time step:

$$T_b = n\,Dt \qquad , n \geq 1$$

and thus it is possible to write the first member of (A.1) as follow:

$$\frac{u\left(x_1{}^*, t^* + T_b\right) - u\left(x_1{}^*, t^*\right)}{n\,Dt} = \frac{1}{n}$$

$$\frac{u\left(x_1{}^*, t^* + T_b\right) - u\left(x_1{}^*, t^* + T_b - Dt\right) + u\left(x_1{}^*, t^* + T_b - Dt\right) + \ldots +}{Dt} \qquad \text{(A.2)}$$

$$\frac{-u\left(x_1{}^*, t^* + T_b - (n-1)\,Dt\right) + u\left(x_1{}^*, t^* + T_b \ \ (n-1)\,Dt\right) - u\left(x_1{}^*, t^*\right)}{Dt}$$

In (A.2) we have add and subtract couples of values of displacement u in the time steps between $t^* + T_b - Dt$ and $t^* + T_b - (n - 1)\,Dt$. In this way in (A.2) we recognize the definition of the slip velocity, and therefore we rewrite (A.2) as:

$$\frac{1}{n}\sum_{k=1}^{n} \dot{u}\left(x_1{}^*, t^* + T_b - (k-1)\,Dt\right) \qquad \text{(A.3)}$$

Inserting (A.3) in (A.1) we have:

$$d_0 = T_b\left(x_1{}^*\right)\frac{\sum_{k=1}^{n} \dot{u}\left(x_1{}^*, t^* + T_b - (k-1)\,Dt\right)}{n} \qquad \text{(A.4)}$$

In this expression we have indicated the dependence of T_b on x_1^* to emphasize the fact that the breakdown time is variable along the fault line, as we have discussed in section 4.1. If we denote with the symbol $< \ldots >_{t_b}$ the average of the slip velocity over the times t_b within the cohesive zone (where the displacement is between 0 and d_0), we can briefly rewrite the formula (A.4) for d_0 as

$$d_0 = T_b\left(x_1{}^*\right)\left\langle \dot{u}\left(x_1{}^*, t_b\right)\right\rangle_{t_b} \qquad \text{(A.5)}$$