Appendix G

Slip velocity behaviour after a stress perturbation

Using the quasi-static approximation of the spring slider equation of motion and the governing equation for friction we can derive the following equation

$$t + \Delta t(t) = t_* + A\ln(V/V_*) + \Theta \tag{G.1}$$

where Θ is the state variable for the slip evolution equation that corresponds to $\Theta = B\ln(xV_*/L)$ for a slowness evolution law. The factor $A = a\sigma_n$ accounts for the direct effect of friction, and σ_n is the normal stress. Then, from equation (G.1) we get

$$\mathbf{V} = \mathbf{V}_* \, exp\left(\frac{t + Dt(\mathbf{t}) - t_* - Q}{\mathbf{A}}\right) \tag{G.2}$$

G.ssuming $\Delta t(t) = 0$ for $t \le T_0$ and $V = V_i$, $t = t_i$ and $\Theta = \Theta_i$ for $t = T_0$, we can rewrite equation (G.2) as

$$\mathbf{V}_\mathrm{i} = \mathbf{V}_* \, exp\left(\frac{t_\mathrm{i} - t_* - Q_\mathrm{i}}{\mathbf{A}}\right) \quad , \; t = T_0 . \tag{G.3}$$

By dividing equation (G.2) by (G.3) we have

$$\mathbf{V} = \mathbf{V}_\mathrm{i}\, exp\left(\frac{t - t_\mathrm{i} - Q + Q_\mathrm{i} + Dt(\mathbf{t})}{\mathbf{A}}\right) \qquad \text{for } \ t \geq T_0 . \qquad \text{(G.4)}$$

We now indicate as $t = T_0^+$ the end of stress perturbation, corresponding to $\Delta\dot{t}(t) = 0$ for $t \geq T_0^+$. If we assume that for $T_0 \leq t \leq T_0^+$ the elastic traction variation $t - t_i$ and the state variable variation $\Theta - \Theta_i$ are negligible, then we have from (G.4) that

$$\mathbf{V} \cong \mathbf{V}_\mathrm{i}\, exp\left(\frac{Dt(\mathbf{t})}{\mathbf{A}}\right) \quad \text{for } T_0 \leq t \leq T_0^+ . \qquad \text{(G.5)}$$

The two assumptions leading to (G.5) are valid if the system at $t = T_0$ is close to the steady state value, if the system velocity and the loading point velocity (V_i and V_0) are relatively low (for instance, when they are comparable to the plate velocity value, ≤ 0.1m/yr) and if the duration of the perturbation $T_0^+ - T_0$ is of the order of several seconds, as is the time interval characterizing the stress perturbation variation in our study. Therefore, equation (G.5) shows that the velocity follows closely the applied stress perturbation in the time interval during which $\Delta t\,(t)$ varies.

If we now assume $\Delta\dot{t}(t) = 0$, which corresponds to unaltered conditions or to perturbed conditions for $t \geq T_0^+$, and we take the time derivative of equation (G.1), we obtain

$$\dot{\mathbf{V}} = \frac{\mathbf{V}}{\mathbf{A}}\left(\dot{t} - \dot{Q}\right) \qquad \text{for } t \geq T_0^+ . \qquad \text{(G.6)}$$

Considering $\dot{\Theta} \neq 0$ (we are not exactly at the steady state), we can write

$$\dot{\mathbf{V}} = -\frac{\mathbf{V}\dot{Q}}{\mathbf{A}}\left(1 - \frac{\dot{t}}{\dot{Q}}\right) \qquad \text{for } t \geq T_0^+ . \qquad \text{(G.7)}$$

Because V and A are positive, if $\dot{t}/\dot{\Theta} < 1$, then equation (G.7) shows that the slip acceleration $\dot{V}$ has the opposite sign of $\dot{\Theta}$. This means that after the end of the stress perturbation the system accelerates (or decelerates) if the state of the system is above (or below) the steady state line in the phase plane (V, t_f).