2. Nucleation and dynamic propagation in 2 – D fault models

2.1. Introduction to Chapter 2

In this Chapter we present the results of the study the dynamic initiation, propagation and arrest of a 2–D in–plane shear rupture by solving the elastodynamic equation both by using a Boundary Integral Equation method and a Finite Difference approach. For both methods we adopt different constitutive laws: a slip–weakening (SW) law, with constant weakening rate, and rate– and state–dependent friction laws. Our numerical procedures allow the use of heterogeneous distribution of constitutive parameters along the fault for both formulations and we will present these results in Chapter 3. We first compare the two solution methods with a SW law emphasizing the required stability conditions to achieve a good resolution of the cohesive zone and to avoid artificial complexity in the solutions. Our modeling results show that the two methods provide very similar time histories of dynamic source parameters.

We point out that, if a careful control of resolution and stability is performed, the two methods yields identical solutions. We have also compared the rupture evolution resulting from a slip–weakening and a rate– and state–dependent friction law. This comparison shows that, despite the different constitutive formulations, similar behaviors are simulated during the rupture propagation and arrest. We observe a crack tip bifurcation and a jump in rupture velocity (approaching the P–wave speed) also with the Dieterich – Ruina (DR) law. We study the dynamic traction behaviour within the cohesive zone during the propagation of earthquake ruptures adopting rate– and state–dependent constitutive relations. The resulting slip–weakening curve displays an equivalent slip–weakening distance (d_0^{eq}), which is different from the parameter L controlling the state variable evolution. The adopted constitutive parameters (a, b, L) control the slip–weakening behaviour and the absorbed fracture energy. The dimension of the nucleation patch scales with L and not with d_0^{eq}. We propose a scaling relation between these two parameters which prescribes that $d_0^{eq}/L \sim 15$. We also study the nucleation phase duration T_n, defined as the time necessary for the crack to reach half–length l_c. We compare the T_n values resulting from distinct simulations calculated using different constitutive laws and different set of constitutive parameters. Our results confirm that the DR law provides a different description of the nucleation process than the SW law adopted in this study. We emphasize the DR law yields a complete description of the rupture process, which includes the most prominent features of SW.

2.2. Numerical models of 2 – D faults

In this work we solve the elastodynamic equation

$$r\, \ddot{u}_i = s_{ij,j} + f_i \qquad (2.1)$$

for a 2–D in–plane shear crack for which the displacement and the shear traction depend on time and one spatial coordinate and neglecting the body forces (f_i = 0). In (2.1) r is the mass density, u the displacement and $s_{ij,j}$ is the spatial derivative of the stress tensor. In particular, we assume that the crack propagates along x_1 in the plane perpendicular to the x_3 = 0 plane. All the solutions are independent on x_2 spatial coordinate. The medium is supposed to be infinite, homogeneous and elastic everywhere except along the fault plane. We solve equation (2.1) by using two different methods: a Boundary Integral Equation (BIE) method and a Finite Difference (FD) approach. We shortly summarize in the following the main features of our numerical solutions.

2.2.1. The Boundary Integral Equation (BIE) method

In order to solve equation (2.1) we have implemented the Boundary Integral Equation method originally proposed by Andrews (1985). We refer to that paper for a complete description of the numerical solution. We describe here only the main features of the numerical approach. In these first applications we use a slip–weakening constitutive law for a homogeneous fault, but in Chapter 3 we will also consider spatially heterogeneous distributions of constitutive parameters.

Let us denote with s_{ij}^{0} the initial stress tensor, and with s_{ij}^{p} the dynamic stress perturbation due to the dynamic propagation of the crack. The integral solution of the dynamic problem, in the general 3 – D case, is (see Kostrov and Das, 1988):

$$u_n(\mathbf{x},t) = \int_{-\infty}^{+\infty} \mathrm{d}t \int_{S_c(t)} G_{na}(\mathbf{x} - \mathrm{x}, t - t)\, s_{a3}^{\,p}(\mathrm{x},t)\, \mathrm{d}S_{\mathrm{x}} \;; \qquad (2.2)$$

$$n = 1, 2, 3;\ a = 1, 2;\ \mathbf{x}, \mathrm{x} \in \mathbb{R}^3$$

where $S_c(t)$ is the fractured region at the instant $t = t$ and G_{na} is the Green tensor. Considering the above described problem geometry, supposing that for

$t < 0$ the fault is in its static equilibrium ($T_1^p(x_1, t) = 0$) and limiting our analysis to the half–line $x_1 \geq 0$, the discretization form of (2.2) we obtain:

$$u_1(j,m) = \sum_{k=0}^{+\infty} \sum_{n=0}^{+\infty} F_{11}(k,n)\, T_1^{\,p}(j-k, m-n) \tag{2.3}$$

where the integers j and m sample the spatial and temporal coordinates, respectively and $F_{11}(k,n)$ is the discretized Green tensor. If the spatial step Dx is greater than or equal to the product between the P–wave velocity and the time step (i. e. if $Dx \geq v_P\, Dt$), there is no coupling between neighboring elements at the time step being calculated (Andrews, 1985, 1994). Therefore, it is possible to write equation (2.3) as

$$u_1(j,m) + C\, T_1^{\,p}(j,m) = L_1(j,m) \tag{2.4}$$

where u is the discretized slip function and T_1^p the instantaneous traction at the *jth* point at the *mth* time step; C is the local compliance, or impedance, over which the instantaneous traction $T_{11}^p(j,m)$ is averaged (Andrews, 1985):

$$C \equiv -F_{11}(0,0). \tag{2.5}$$

L_1 is a positive quantity, which expresses the dynamic load resulting from the convolution of the discretized Green function $F_{11}(k,n)$ with the past (known) values of the traction T_1^p. At each time step m, the total shear traction is given by the sum of the initial and the instantaneous traction ($t_0 + T_1^p$), as discussed in Bizzarri et al. (2001).

2.2.2. The Finite Difference (FD) approach

The Finite Difference (FD) numerical method used in this Chapter has been proposed by Andrews (1973) and it has been recently used by Andrews and Ben–Zion (1997), Ben–Zion and Andrews (1998), Bizzarri et al. (2001), Bizzarri and Cocco (2003), Cocco and Bizzarri (2002), Cocco et al. (2003). We have implemented in this work a numerical approach that allows to include spatially variable constitutive parameters and to consider different constitutive laws. We solve the discretized equations deriving from (2.1):

$$r \frac{\partial}{\partial t}\dot{u}_1 = \frac{\partial}{\partial x_1}\Sigma_{11} + \frac{\partial}{\partial x_2}\Sigma_{12} \qquad (2.6a)$$

$$r \frac{\partial}{\partial t}\dot{u}_2 = \frac{\partial}{\partial x_1}\Sigma_{12} + \frac{\partial}{\partial x_2}\Sigma_{22} \qquad (2.6b)$$

In (2.6) $\dot{u}_i$ indicate the time derivative of the slip and S_{ij} are the stress tensor components (the sum of an elastic contribution, s_{ij}, and a viscous contribution, the latter being added as a perturbative term, see Andrews, 1973 for further details). In all simulations presented in this Chapter, we do not consider any viscous term, and therefore the total stress tensor S_{ij} in (2.6) is simply equal to the elastic stress tensor s_{ij}.

The x_1–x_2 plane is linear elastic everywhere except along the fault line $x_2 = x_{fault}$, where the constitutive equation is introduced. This line represent the intersection of the fault plane with $x_3 = 0$. The fault is described by a number of split nodes coupled to each other by the constitutive relations. The solutions of (2.8) are stepped through time by calculating the net force acting on every node, by adjourning the velocities and the displacements and by recalculating the internal force that every element exercises on its nodes.

We emphasize the different strategy adopted to discretize the elastodynamic equation (2.1) in the two numerical procedures here described. While in the BIE approach described in the previous section only the " boundary "(i. e. the fault line) is discretized, in the FD method (as well as in all the so–called "domain type" numerical methods) the entire $x_3 = 0$ plane is discretized. In the latter approach a grid of nodes is introduced and each node is a vertex of an equilateral triangle; the choice of triangles, instead of other geometrical figures as rectangels for instance, has been made in order to increase the numerical efficiency (Petschek and Hanson, 1968; Trulio, 1964). All variables are defined in each node and therefore it is possible to consider the entire medium surrounding the fault line, possibly accounting for material heterogeneities.

2.2.3. Comparison between different numerical methods

In order to compare the two numerical approaches proposed here (BIE and FD), we have chosen to use a slip–weakening law with homogeneous constitutive parameters along the whole fracture line. We will discuss in the Chapter 3 further simulations with a heterogeneous distribution of constitutive parameters. The adopted SW parameters are the same for both methods. We consider a Poissonian medium, in which the elastic parameters (adimensional units) are: the density $r = 1$, the Lamé moduli $l = m = 1$, S – wave velocity $v_S = 1$ and P – wave velocity $v_P = 1.732$. In performing our simulations we often use the adimensional quantity S (Das and Aki, 1977a, b)

$$S \underset{\mathrm{df}}{=} \frac{t_u - t_0}{t_0 - t_f}$$

which is the ratio between the rise of the stress necessary to start the sliding and the dynamic stress drop.

We first discuss a numerical simulation which qualitatively reproduces the modeling results obtained by Andrews (1985) using a BIE method. We use for both adopted methods equivalent values for the SW constitutive parameters with the same spatial sampling. The rupture initiation is imposed in both approaches and the adopted numerical procedure is strictly the same: we initially impose a time–weakening. In this way the crack is initially non–spontaneous, with a rupture velocity which is a fraction of the Rayleigh wave speed v_R (v_{force} = 0.7 v_R). When the friction predicted by the imposed time–weakening becomes greater than the friction predicted by the slip–weakening law, the crack grows spontaneously. The time weakening duration is not constant, but it depends on the adopted parameters; for the simulations shown in Figure 2.1 it is equal to $7Dt$. This Figure shows the comparison between slip, slip velocity and total traction obtained by the BIE and FD methods, by using 129 × 160 and 257 × 198 grid points in the x_1–t plane, respectively. The difference between the number of grid points along x_1 is due to the fact that the BIE calculation is performed in the half–line $x_1 \geq 0$ only. The difference of the temporal grid points t is due to the different value of the Courant–Friedrichs–Lewy (CFL) ratio $w_{CFL} = v_S\ Dt/Dx$, which controls the convergence of the numerical algorithms (see, for instance, Fukuyama and Madariaga, 1998). Our stability tests have shown that the best choice is $w_{CFL}^{(BIE)} = v_S/v_P = 0.578$ and $w_{CFL}^{(FD)} = (0.975\ 3^{1/2} / 2)\ (v_S/v_P) = 0.475$.

The modeling results shown in Figure 2.1 point out that the two numerical methods provide similar solutions: we note a crack tip bifurcation that occurs slightly earlier for the FD method. The BIE solution is identical to that obtained by Andrews (1985). After the initiation the rupture front propagates at the Rayleigh velocity v_R, while the secondary front jumps to a super–shear rupture velocity and asymptotically tends to the P–wave speed v_P. We remind that only for anti–plane 2 – D cracks the limiting velocity is v_S. The time step at which the crack tip splits depends on the characteristic weakening distance d_0: for the same S values, the higher d_0, the later the tip splitting occurs. The two approaches yield similar slip velocity and total traction at the crack tip.

The cohesive (or breakdown) zone has been shown in Figure 2.1 (a and d) as the hatched area; this is the fault region where the fracture energy is released and the traction decreases from the value t_u to the frictional level t_f.

This is expected for a slip–weakening constitutive law (Andrews, 1976a, b; Ohnaka, 1993). The main difference between two solutions concerns the bifurcation, which occurs earlier in the FD case with respect to the BIE one. We point out that in both simulations the cohesive zone width decreases as the crack propagates far away from the nucleation patch: this is due to the increase of the dynamic load caused by the larger number of points in the region that already slipped (see equation 2.6). The shear stress falls out to the final level t_f over a smaller distance and during a smaller time (breakdown zone duration), although the weakening rate (determined by the stress versus slip) remains the same. This means that the slip velocity is increasing as the crack propagates far away from the nucleation zone. The analytical relation between slip velocity and breakdown duration is derived in Appendix A, where we show that, in order to have a constant d_0 (as prescribed for a homogeneous SW) slip velocity has to increase for decreasing duration of the breakdown process. Figure 2.1(g) shows the difference between the two slip functions resulting from the BIE and FD methods in the x_1–t plane, by introducing a misfit function defined in Appendix B. This Figure points out that the two methods provide nearly identical slip values. The main differences occur inside the breakdown zone and at the crack tip bifurcation.

Figures 2.2 and 2.3 depict the time histories of slip, slip velocity and traction (showed in Figure 2.1) computed at two grid points (x_1 = 3.9 and 5.3). For these calculations we have increased the spatial resolution (two times for the BIE and four times for the FD method). The BIE and the FD methods yield very similar time behaviors of source parameters. In particular, we remark that the two methods provide similar time evolutions of the total traction and the same peak values of slip velocity.

In Figures 2.2(d) and 2.3(d) we show the phase diagrams, that is the slip rate versus the shear traction, which exhibit characteristics analogous to those experimentally obtained by Ohnaka and Yamashita (1989). When the total traction reaches the yield stress value (t_u), the accelerating phase begins (see Figures 2.2d and 2.3d). This phase can be subdivided into an initial stage, where the traction is nearly constant both for the BIE and for the FD solution and a subsequent sharp reduction of total traction (analytically defined as $\mathrm{d}t/\mathrm{d}t < 0$ and $\mathrm{d}v/\mathrm{d}t > 0$). The accelerating phase resulting from the two numerical approaches is slightly different. Although the rate of traction decrease resulting from the two methods is almost the same, the FD provides a slightly more complex behavior when the slip velocity reaches its maximum. We observe a different behavior between the two solutions when the slip velocity is close to its maximum value and total traction is nearly at the kinetic stress level (see the shaded box in Figures 2.2d and 2.3d). Looking at Figures 2.2 and 2.3 it is possible to observe that the final slip velocity is non zero, that is the crack is not arrested, but it continues to enlarge in the time.

Our comparison indicates that, if the medium is homogeneous and if the solutions have a good resolution (see Appendix C), the two methods provide similar results and the general behaviors of slip, slip velocity and traction are quite similar, in particular before the crack bifurcation (as one can note looking at the Figure 2.1g). The phase diagrams shown in Figures 2.2(d) and 2.3(d) show that for a given slip velocity value there exists two values of dynamic traction; such observation is consistent with the stick–slip experiments of Ohnaka and Yamashita (1989). This implies that a pure velocity–weakening constitutive law is not supported by experimental observations. This is one of the motivations for introducing the state variable (Dieterich, 1986 and references therein; Ruina, 1980, 1983).

Figure 2.4 shows the total traction as a function of slip for the same grid points used in Figures 2.2 and 2.3 for the BIE and the FD solutions: both methods correctly reproduce the trend required by the SW law. We point out that the spatio–temporal sampling of our solutions provides a good resolution of the cohesive zone, as indicated by the large number of points appearing along such line. In Appendix C we discuss the conditions that has to be satisfied in order to have resolution and stability of the numerical solutions. We remark that these conditions have to be carefully verified since realistic spatio–temporal slip patterns can be also associated to unstable solutions. This is particularly true when complex dynamic process is investigated, as those resulting from heterogeneous distribution of constitutive parameters.

In order to provide some further information arising from our methodological comparison, we also compare the CPU time and the RAM requirements to run the numerical tests here discussed. We perform these simulations in two different 256 *MB* RAM processors: a Compaq Alpha Server 1000A at 433 *MHz* and an Intel Pentium III–based Personal Workstation at 450 *MHz*. In each hardware configuration the BIE code is faster with respect to the FD one, approximately of a factor 1.5. This is due to different factors: the most important is that while the BIE code calculates the solutions of the dynamic problem only in the half–space $x_1 \geq 0$, the FD one computes the solutions in the entire fault line $x_2 = x_{fault}$. Moreover, while in the BIE code only 5, real single precision arrays are necessary during the entire calculation (and therefore stored in memory), in the FD one 23 single precision arrays are stored. These includes the stresses S_{ij} and the two components of slip velocity in the $x_3 = 0$ plane, outside the intersection with the fault line. Such difference implies not only a larger calculation time, but also a quite larger RAM requirements.

2.3. Comparison between different constitutive models

In this section we present the results of several simulations calculated using the same numerical approach (the FD method) but with different constitutive laws. The choice of the FD approach is justified by the results of the comparison with the BIE solutions presented before. In these simulations we use the slip–weakening as well as the rate– and state–dependent friction laws described before (Chapter 1). In order to perform such comparison it is necessary to properly associate the physical parameters of the different constitutive laws (see also Mikumo, 1992). We will briefly explain this in the next section.

2.3.1. Theoretical comparison between constitutive parameters. A first hypothesis

All the governing equations previously described have a characteristic length d_0 or L, intrinsic to the dynamic problem. We derive the correspondence between the different constitutive parameters (d_0 and L, t_u and t_f with a and b) starting from the results of laboratory experiments (Gu et al., 1984) showing the response to an abrupt change in slip velocity (Chapter 1). From a physical point of view, the yield stress in the SW model t_u corresponds to the peak value of friction obtained when the slip velocity changes from an initial velocity v_{init} to the value v_2 (usually named the direct effect). Moreover, the kinetic friction level t_f in the SW law corresponds to the steady state friction $t^{ss}(v_2)$, in rate– and state–dependent friction laws. We propose the following association

$$t_u \leftrightarrow t^{ss}(v_{init}) + as_n \ln(v_2 / v_{init}) \tag{2.7a}$$

$$t_f \leftrightarrow t^{ss}(v_2) \tag{2.7b}$$

where $t^{ss}(v)$ represents the steady state friction. Rice (1993) describes the exponential decrease of friction observed in modeling the response of a spring–slider to a velocity change as:

$$t = t^{ss}(v_2) + e^{-\frac{u}{L}} \left[t^{ss}(v_{init}) + a\, s_n^{eff} \ln\left(\frac{v_2}{v_{init}} \right) - t^{ss}(v_2) \right]$$

This relation has been introduced to describe several common features of material response to a change in sliding velocity resulting from laboratory experiments; it has not been derived from theoretical constitutive laws. The most important feature arising from this relation is the dependence of friction on the ratio between the slip velocities. If we develop this expression limiting to the first order for $u << L$, we obtain:

$$t \cong t^{ss}(v_{init}) + a\, s_n^{\;eff} \ln\left(\frac{v_2}{v_{init}} \right) - \left[t^{ss}(v_{init}) + a\, s_n^{\;eff} \ln\left(\frac{v_2}{v_{init}} \right) - t^{ss}(v_2) \right] \frac{u}{L}$$

Taking into account relations (2.9a) and (2.9b), this expression becomes equivalent to (3.3) for the SW law when the following association is valid:

$$d_0 \;\leftrightarrow\; L \qquad (2.7c)$$

Such an analogy seems to be physically reasonable because in rate– and state-dependent laws, L represents the length in which the friction decreases to the stationary level (Ruina, 1983; Dieterich, 1992, 1994; Perrin et al., 1995). In Appendix D we also discuss the comparison concerning the fracture energy G resulting from the SW and the rate– and state–dependent friction laws. We only remark here that both constitutive formulations yield to a fracture energy proportional to the characteristic length and the difference between the yield stress (2.7a) and the kinetic friction level (2.7b). This implies that for both constitutive laws there exists a critical patch size, that is the minimum dimension for which a crack can propagate. In Appendix C we discuss the definitions of the critical patch size.

2.3.2. Dieterich-Ruina laws versus slip-weakening

In this section we use the FD method with different constitutive equations: we use both SW and DR law. In order to account for the continuous variations of u, F and t the solution resulting from rate– and state–dependent constitutive laws requires a sub–stepping in time, which is resolved by using a Rosenbrock stiff integration procedure (Press et al., 1992), preferred to the Runge–Kutta method for reasons of numerical efficiency.

We model a velocity weakening behavior where the parameters a and b are uniform on the fault line (equal to 0.75 and 1.6, respectively, see Table 2.1). According to Boatwrigth and Cocco (1996), this configuration corresponds to a very velocity weakening field ($B - A >> 0$); in the following we will refer to this field as a strong seismic regime. The adopted set of initial parameters at $t = 0$ along the whole fault line is listed in Table 2.1; the shear traction is always in the steady state at $t = 0$, while the state variable is in the steady state except in the nucleation region ($10Dx$ wide). In Figure 2.5 (a, b and c) we show the results of this simulation which is characterized by a large stress drop and considerable slip.

The simulation for a SW constitutive law is obtained by using the proper constitutive parameters resulting from the relations (2.7) previously derived. This is a first attempt to do simulations with comparable constitutive parameters. Looking at the results we will see that a more detailed set of correspondence formulae can be derived and we will present them in section 2.4. In particular, we have used $t_0 = t^{ss}(v_{init})$; v_2 is obtained from the state variable. We first run the FD code using the rate– and state– constitutive law. As we will describe in the following, we verify that after the stress release the state variable is in the steady state, that is $F^{ss}(v_2) = L/v_2$. Therefore, since we know F and L, we are able to derive the proper value of v_2. From (2.7) we have consequently determined the SW parameters t_u as well as t_f. Finally, we have chosen the same characteristic length for the two constitutive relations: $d_0 = L$. The resulting values are listed in Table 2.1 and they yield to a strength parameter S equal to 0.88. The solutions obtained using the SW constitutive law are shown in Figure 2.5 (d, e and f).

The comparison between these solutions points out that also with the rate– and state–dependent friction laws it is possible to model a highly unstable regime in which the dynamic load produces a bifurcation of the rupture front with a crack tip propagation velocity approaching v_P. Nevertheless, in the SW solution (Figures 2.5d, e and f) the tip splitting occurs much earlier than in the DR solution. This is due to the different behaviors of the constitutive laws during the nucleation phase. We remark that our SW law has a constant weakening rate. On the contrary, the rate– and state–dependent friction law has an initial evolution that is quasi–static, with very low slip velocities (compare Figures 2.5b and e). This effect is evident in the 3–D view of the shear traction (Figures 2.5c and f): with the DR law the stress gradually increases up to reach the maximum value and then decreases as far as the final level, whereas with the SW the dynamic process is more immediate. This feature is intrinsic of our SW law even when very low initial velocities (v_{init}) are used. As a consequence, the rupture acceleration is much faster for the SW law than for the DR one: worthy of note is the gradual variation of rupture velocity in Figures 2.5(a) and (b). In order to point out this feature, we have plotted in Figure 2.6 the crack tip velocity as a function of time. This Figure shows that the crack tip acceleration is higher for the SW solution than for the DR one, even if both methods provide a similar range of variability for the rupture velocity. Moreover, the initial velocity for the DR model is lower than that of the SW solution. Although in less general conditions, the crack tip bifurcation obtained with the DR law was obtained by Okubo (1989) using a BIE numerical approach.

The basic assumption we made to use equivalent constitutive parameters in both friction laws is that after the stress drop the friction is at the steady state. We have verified that in the final configuration the state variable F does not change with time: to show this we have plotted in Figure 2.7 the phase diagram (i. e., shear traction vs. slip velocity) for the DR law at the fault position $x_1 = 11.8$. The intersections with the steady state line $t^{ss}(v)$ show that at the end of the stress release t is in the steady state.

The temporal evolution of total traction and its behavior as a function of slip and slip velocity resulting from the two constitutive formulations are shown in Figure 2.8 for the same grid point as the one used for Figure 2.7. The time evolution of total traction (Figure 2.8a) points out that the two constitutive laws provide similar trends. It is important to consider that, in the selected position along the fracture line ($x_1 = 11.8$), the crack tip is split only for the SW solution. This explains the higher peak of the total traction resulting from the SW law and its earlier drop to the kinetic level t_f. The solution resulting from the DR law shows some oscillations before and after the traction drop. This behavior is also evident in the phase diagram shown in Figure 2.8(d). We emphasize that the two solutions yield the same stress drop

(t_0 – t_f), confirming that the association between constitutive parameters is correct.

Figures 2.8(b) and (c) shows the total traction as a function of slip: worthy of note is that the slip–weakening behavior is common to both solutions. However, while (as expected) the SW solution shows only a weakening phase, that is, traction always decreases for increasing slip, the DR solution has a slip–hardening phase preceding the weakening stage. We remark that this behavior is much more general than that resulting from the SW solution. According to Ionescu and Campillo (1999) the initial value of friction is a key parameter in controlling the nucleation process. We emphasize that the solution resulting from the rate– and state–dependent law may contain this initial increase of traction for increasing slip (direct effect); this implies more general behaviors during the nucleation stage.

In conclusion we believe that the rate– and state–dependent friction laws reproduce the results obtained with a slip–weakening constitutive relation, and that they allow to model an extensive class of phenomena concerning the frictional behavior of faults which includes aseismic slip, creep, as well as coupling and interactions between different frictional regimes (Rice, 1993; Boatwright and Cocco, 1996; Gomberg et al., 1998).

2.4. The dynamic propagation. The cohesive zone and the breakdown process

2.4.1. The reference model

In the previous sections we have shown a physically reasonable way to associate constitutive parameters of the slip – weakening model to those of the Dieterich – Ruina law. In this section, we present and discuss the main features of spontaneous crack propagation on a fault obeying a rate- and state-slowness law. We chose a reference set of parameters, which is now dimensional and typical of laboratory experiments: the medium surrounding the crack is linear elastic, homogeneous and Poissonian. Our goal is to study with high detail the behaviour of the total traction within the cohesive zone, in order to found a better way to associate the SW constitutive parameters to a, b and L. The total fault length is equal to 20 m and, after initiation, the crack propagates symmetrically with respect to the nucleation point $x_1 = 0$. At the initial stage, the fault is on steady state, except that in the nucleation region, which is 3 m wide. Starting from the steady state we simulate a spontaneous nucleation that is only a particular case of a general, natural fault state. However, as we are interested in breakdown process and stress release phenomena, we choose a configuration in which the nucleation stage is not too long. A very long nucleation stage will also require a lot of time step, and it may probably result computationally too challenging, regarding the convergency condition that we have to satisfy to correctly simulate and resolve the breakdown process. The whole set of model parameters are listed in Table 2.1 and we refer to Bizzarri et al. (2001) for a detailed description of the adopted nucleation strategy for the simulations.

Figure 2.9 shows the resulting total shear stress as a function of slip as well as the phase diagram (that is the traction as function of slip velocity) for a homogeneous fault where the spatial discretization is $Dx = 0.01$ m, and Dt is fixed from the Courant–Friedrichs–Levy ratio w_{CFL}, defined as $v_S\ Dt/Dx$. We emphasize that, adopting a rate- and state-dependent friction law, the dynamic traction clearly shows a slip–weakening behavior, as previously pointed out by Okubo (1989), Dieterich and Kilgore (1994) and Bizzarri et al. (2001). We identify the slip–weakening distance on this plot and, accordingly to Cocco and Bizzarri (2002), we named it an equivalent SW distance d_0^{eq}. The simulation shown in Figure 2.9 represents a reference model for our investigations. As pointed out by Cocco and Bizzarri (2002) the equivalent SW distance d_0^{eq} is not equal to the L value adopted in the simulations. We will examine and discuss this difference more in detail in following of the Chapter.

2.4.2. Interpreting the traction evolution within the cohesive zone

In order to understand the evolution of dynamic traction within the cohesive zone and to identify the physical quantities controlling the SW behavior, we compare the time histories of total traction, slip velocity and state variable (Figure 2.10a) calculated for the same model parameters used in Figure 2.9 (i. e. the reference case) in the same fault position ($x_1 = 3.0\ m$). By looking at Figure 2.10a it emerges that the resulting total dynamic traction reaches its peak value (the yield stress) earlier than slip velocity, and that the state variable evolves from its initial steady state value to a final one well before the other two. This first consideration allows us to suggest that is the state variable that drives the slip acceleration and the traction drop during the weakening phase. We aim to discuss this topic in detail in the following of this section. We subdivide the time window shown in Figure 2.10a in five distinct stages, which comprise the duration of the whole breakdown process. The first stage (indicated as I in the Figure) is characterized by a slight decrease of the state variable (which starts evolving from its steady state value) and a modest increase in traction; the slip velocity in nearly constant and equal to the initial value (v_{init}). Such an increase of traction is due to the contribution of neighbouring points, that are already slipping: the adjacent portion of the fault cause a stress transfer to the present point and this traction increase cause an increase in slip velocity. During the phase II the shear traction continues to increase and reaches its peak value, while the slip velocity shows a fast increase only when the state variable decreases more rapidly. We indicated the time interval starting with the sharp slip velocity increase and lasting until the traction has reached the yield stress value (τ_u^{eq}) as a phase II-b. Most of the state variable evolution occurs during this short stage, emphasizing that this temporal variation is extremely fast. We indicate with v_u the value of the slip velocity corresponding to the peak dynamic stress value; we emphasize again that the friction does not reach its maximum in correspondence to the peak of the slip velocity (v_{max}). This is in agreement with the numerical results of Olsen et al. (2001) and Mikumo et al. (2002). The two phases I and II represent the velocity–hardening phase depicted in Figure 2.9b.

The slip–weakening phase begins when the total dynamic traction has reached t_u^{eq}; we named this stage (whose duration defines the breakdown time T_b) as phase III during which the slip velocity rapidly increases, reaches the maximum value (v_{max}), and then decreases. As expected, the slip acceleration occurs during the breakdown time (phase III) which is associated with the slip–weakening behavior. Therefore, at the end of stage III, the slip is equal to d_0^{eq} and the corresponding slip velocity is v_0. It is interesting to note that for SW models (Olsen at. al, 2001; Mikumo at al., 2003) the friction reaches the kinetic level (and the fault displacement is d_0) when the slip velocity is at its maximum value and there is not any deceleration during the breakdown process. This can be intuited seeing the phase diagrams, that for the SW governing law are quite different (Bizzarri et al., 2001) from rate– and state–ones. Numerically, the end of phase III is determined as the time step in which the friction assumes its minimum value (see also Figure 2.10a). For a single degree of freedom spring – slider model, Gu and Wong (1991) and Beeler et al. (2002) estimate the equivalent slip – weakening distance calculating the radiated energy G and assuming that $d_0^{eq} = 2\ G\ /\ Dt_d$. We will show in a following that the analytical approximation for the dynamic traction is not only linearly dependent on the displacement (see equation (10) and therefore the expression for G is more complex (see for instance Bizzarri et al., 2001). At this point, and for the subsequent phase IV, the state variable is already close to its steady state curve $F^{ss}(v)$: this is clear by looking at the log–linear plot in Figure 2.910b, where we have represented a zoom of the state variable time history plotted in Figure 2.10a superimposed to the stationary state $F^{ss}(v) = L/v$. This Figure confirms that, when the SW phase (stage III) is ended (the traction drop is concluded), the state variable is at the steady state. At this time the friction is $m[v_0,F^{ss}(v_0)]\,s_n^{eff} = m^{ss}(v_0)\ s_n^{eff}$. This value corresponds to the kinetic traction t_f^{eq} in the SW model, as we will derive in a following section. This phase IV is characterized by a slip velocity decrease up to the final level v_2, the state remain in its steady state and therefore the friction is $m^{ss}(v_2)\ s_n^{eff}$. The slip velocity maintains its final value (v_2, in our simulations) if no healing of slip occurs (see Perrin et al., 1995).

These considerations can be easily summarized by looking at the 3–D phase trajectories shown in Figure 2.11: the state variable drives the velocity hardening phase and the slip acceleration (Figure 2.11a) and SW occurs when the state variable is close to its final steady state value (Figure 2.11b) and the accelerating phase is already started (Figure 2.11c). Several stimulating questions arise from these results: if the traction drop during the weakening phase is controlled by the state variable, does it mean that the SW behavior and the breakdown process depends on the properties of the contacts on the surface within the cohesive zone? And how much the analytical expression chosen for the evolution law controls the SW behavior? We aim to discuss these questions in the present study. To this goal we must investigate the temporal evolution of slip, slip velocity and dynamic traction within the cohesive zone (as done in Figure 2.10). This requires discussing the available resolution and the stability conditions before to present the results of numerous simulations performed with different constitutive parameters and initial conditions. We face this problem in the Appendix C.

2.5. A scaling law for the two characteristic length scale parameters

In this section we discuss the dependence of the equivalent characteristic distance on the parameter L inferred from numerical simulations. The constitutive parameters adopted for the simulations are those used in the previous Figures and listed in Table 2.2, and only the parameter L is varied. We adopt an appropriately selected spatial (and temporal) discretization, which allows to correctly resolve the dynamic traction evolution for all the cases considered here: we assume $Dx = 0.005\ m$ and the calculations are performed with 1401 points in space (x_1) and 5000 points in time (t). We show in Figure 2.12 five different SW curves obtained with different L, which shows quite different traction behaviors. This Figure suggests a direct dependence of d_0^{eq} on L and a weak inverse dependence of t_u^{eq} on L, implying that the weakening rate decreases for increasing L. We note however that, when L is large enough (as in the two simulations performed with the largest L values) to change the critical stiffness of the system, the dependence of the yield stress on L is more evident. It is important to point out that when L increases the fault undergoes to the instability more slowly, because the evolution of the state variable is slow. Because the critical patch size (l_c), that in turn controls the rupture nucleation, is analytically expressed as $hmL/(b - a)\ s_n^{eff}$ (h being a geometrical, dimensionless, numerical parameter), when L increase, l_c increases too, and therefore the fault had to spend a longer time and had to produce a larger slip to reach an extension equal to l_c.

The panel inserted in the upper right corner of Figure 2.12 shows the d_0^{eq} values resulting from the simulations and the ratio d_0^{eq}/L. We now derive an analytical relationship between these two length scale parameters (d_0^{eq} and L) accordingly to the results of Cocco and Bizzarri (2002). To this aim, we rewrite the evolution equation (3b, i. e. the slowness law) in the following way:

$$\frac{\mathrm{d}}{\mathrm{d}u}F = \frac{1}{v} - \frac{F}{L} \tag{2.8}$$

During the stage III (see Figure 2.10a), we have that $u \leq d_0^{eq}$ but v is large enough to allow the term $1/v$ in (2.8) to be neglected. The condition $1/v << F/L$ is always satisfied in all our simulation during the breakdown process. Integrating the approximated evolution equation we have:

$$F(u) \cong F(u=0)\,\mathrm{e}^{-\frac{u}{L}} = F_{init}\,\mathrm{e}^{-\frac{u}{L}}. \tag{2.9}$$

where F_{init} is the initial value of the state variable, that for the simulations presented in this Chapter is $F^{ss}(v_{init}) = L/v_{init}$. We have defined in Figure 2.10a that when the slip equals d_0^{eq} the slip velocity is v_0 and the state variable is at the steady state and it is equal to $F^{ss}(v_0)$. Thus, from (2.9) we have:

$$F_{init}\, e^{-\frac{d_0^{eq}}{L}} = \frac{L}{v_0} \tag{2.10}$$

which yields:

$$d_0^{eq} \cong L\, ln\left(\frac{F_{init}\, v_0}{L}\right) \tag{2.11}$$

and, for the particular case of initial conditions in steady state yields:

$$d_0^{eq} \cong L \, ln\left(\frac{v_0}{v_{init}} \right)$$

Cocco and Bizzarri (2002) have previously derived this relation, which states that the equivalent slip–weakening distance d_0^{eq} depends on L and the initial velocity. In order to estimate the theoretical value of the equivalent slip–weakening distance, we have to evaluate the slip velocity v_0 that is *a priori* unknown in the framework of rate– and state–constitutive formulation. We have derived an approximated expression in Appendix E that relates the slip velocity v_0 to the constitutive parameters, using the shear impedance relation (see, for instance, Scholz, 1990). By means of this approximation and of equation (2.11) we have calculated the theoretically derived values of the equivalent slip–weakening distance for the different parameters used in Figure 2.12 and we have inserted these values in the panel of that figure. We will discuss in a following section the problems related to the lack of knowledge of the slip velocity values associated with the different stages of the breakdown process represented in Figure 2.10.

The important result emerging from these numerical simulations is that the scaling law between d_0^{eq} and L is very close to a linear relation and the proportionality ratio is nearly 15.

We also note that as L increases t_u^{eq} decreases and on the contrary t_f^{eq} increases. We will further discuss these results in the next sections. We point out that, the velocity–hardening effects (phases I and II in Figures 2.9 and 2.10) are always present independently of the adopted constitutive parameters, but the slip–hardening distance (i. e. the amount of slip necessary for dynamic traction to reach its maximum value, the equivalent upper yield stress) is negligible unless the value of the parameter L becomes very large (see Figure 2.12). The former result is intrinsic in the rate and state constitutive formulation: the rate dependence implicitly involves the existence of both velocity–hardening and weakening effects (see Bizzarri et al., 2001). Slip–weakening is also a peculiarity of RandS laws (see Guatteri et al., 2001; Cocco and Bizzarri 2002), but the slip–hardening effects depends on the evolution of the state variable, that in this formulation is controlled by the value of the constitutive parameter L.

2.6. Theoretical interpretation

We aim to derive analytical relations to express the SW parameters (yield and kinetic stress values) as a function of the input constitutive parameters, consistent with the scaling law between the two length scale parameters, expressed in equation (2.11).

We derive an expression of the dynamic friction which is valid when the slip acceleration phase is already started and equation (2.9) holds. By inserting (2.9) in (1.7a), and neglecting the + 1 terms in the argument of logarithms, we can express fault friction as a function of slip velocity and slip:

$$t = \left[m_* + a \ln\left(\frac{v}{v_*}\right) + b \ln\left(\frac{v_*}{v_{init}}\right) - b\frac{u}{L} \right] s_n^{eff} \tag{2.12}$$

Equation (2.12) expresses the governing equation for friction during the breakdown process as a function of slip and slip velocity and it is analogous to (1.7a) when the state variable is expressed by equation (2.9). This equation allows to derive the analytical expression for the kinetic friction level: in fact, when the slip is equal to the d_0^{eq} and the slip velocity is v_0 we have:

$$t = \left[m_* + a \ln\left(\frac{v_o}{v_*}\right) + b \ln\left(\frac{v_*}{v_{init}}\right) - b\frac{\mathbf{d_o^{eq}}}{L} \right] s_n^{eff} \tag{2.13}$$

by substituting equation (2.11) in (2.13) and with simple algebra we can derive the following relation for the kinetic stress (τ_f^{eq}):

$$\tau_f^{eq} = \left[\mu_* + (b-a)\, ln\left(\frac{v_*}{v_o}\right) \right] \sigma_n^{eff} \tag{2.14}$$

It is interesting to observe that this expression coincides with the steady state value of fault friction for $v = v_0$ [$t_f^{eq} = t^{ss}(\mathbf{v}_o)$]. This is a further theoretical corroboration of the numerical results plotted in Figure 2.10b. Equation (2.14) confirms our interpretation of the numerical simulations that the kinetic friction only depends on the difference *b - a*.

The derivation of the analytical expression for the yield stress is slightly more complex. Figures 2.10 and 2.11 clearly show that when the total dynamic traction increases to the peak yield stress the state variable is evolving and driving the slip acceleration. Fault friction (i.e., the total dynamic traction) reaches its peak value (i.e. the yield stress) well before than slip velocity. In other words, the friction increase, leading to the yield value, is still driven by the state variable evolution but it occurs when the state variable is not at the steady state. We therefore re-write the governing equation for fault friction as defined in equation (1.7a) as follows:

$$t_u^{eq} = \left[m_* + a\,ln\left(\frac{v_u}{v_*}\right) + b\,ln\left(\frac{F_u\, v_*}{L}\right)\right] s_n^{eff} \qquad (2.15)$$

indicating with v_u and F_u (see Figure 2.10) the slip velocity and the state variable when traction is at the yield stress. It has to be remarked that the values of slip velocity (v_u) and state variable (F_u) at the peak stress value are unknown *a priori*. The same is true for the dependence on the value of slip velocity v_0 of the kinetic stress level and the equivalent slip–weakening distance defined in (2.11) and (2.13). This explains from an analytical point of view the concept that the traction behavior within the cohesive zone cannot be assigned *a priori* in the framework of rate- and state- constitutive formulation. In our opinion, this might represent a limitation in using rate– and state–dependent constitutive laws in dynamic model of earthquake rupture propagation during a single earthquake. For such purposes, the adoption of a classical slip–weakening law (as defined in Chapter 1)) is more convenient for numerical purposes because it allows to prescribe the traction evolution and therefore to control the fracture energy absorbed at the crack tip. In the framework of a rate- and state- formulation, the slip velocity values controlling the traction evolution within the cohesive zone are not specified or assigned (see equations 2.11, 2.13 and 2.14): therefore the yield stress, the kinetic friction and the characteristic slip–weakening distance cannot be prescribed *a priori*. These parameters, which are commonly assigned as input values in SW models, depends in the rate- and state- formalism on the constitutive parameters and initial conditions.

Equation (2.13) shows a dependence of the equivalent slip–weakening distance on the initial slip velocity v_{init}. This parameter influences the initial value of the state variable, which starts from a steady state $F^{ss}(v_{init}) = L/v_{init}$, and consequently the initial stress value. This means that the (equivalent) strength parameter S (as defined by Das and Aki, 1977a and b; see also Bizzarri et al., 2001) changes varying the initial slip velocity. This is well represented in Figure 2.13, where we plot the SW curve and the phase diagram for different values of the initial slip velocity, which shows that the fault response varies by changing v_{init}. This observation corroborates our interpretation that the evolution of the state variable from its initial value drives slip acceleration and the unstable response (i. e. the weakening effects). Figure 2.13 also shows that for small value of v_{init} we have high values of the yield stress and larger equivalent SW distances. The correspondence between the modeled and the theoretical (resulting from equation (2.13)) values of d_0^{eq} is shown in the panels of Figures 2.12 and 2.13a. In Figure 2.13b we plot the traction as a function of slip rate (in a log scale) to emphasize the contribution of the direct effect of friction and the different steady state friction values (for $v = v_0$) in each configuration.

In conclusion, we note that the slip velocity and the state variable values (v_u and F_u) appearing in equation (2.15) depend on the adopted a, b and L values as well as on the initial slip velocity. Therefore, the dependence of the yield stress on the constitutive parameters is quite complex. The same is true for the slip velocity value v_0. However, while we are able to derive an analytical expression for the equivalent SW distance and for the kinetic friction expressing the dependence on the constitutive parameters found in numerical simulations, the same cannot be easily done for the yield stress.

2.7. The nucleation phase

In this section we discuss the effects of using different sets of constitutive parameters on the nucleation process. The duration of the nucleation phase has been experimentally defined by using ground motion recordings (Ellsworth and Beroza, 1995; Beroza and Ellsworth, 1996), laboratory experiments on fault friction (Dieterich, 1992; Dieterich and Kilgore, 1996; Ohnaka et al., 1993 and references therein) and theoretically explained both using slip–weakening behavior (Ohnaka and Yamashita, 1989; Ionescu and Campillo, 1999) and rate– and state–dependent friction laws (Okubo and Dieterich, 1986; Dieterich, 1994). Shibazaki and Matsu'ura (1998) and Ohnaka (1993) describe the phase, preceding the dynamic propagation at very high speed ($v_{crack} \geq 2\ Km/s$), as a very slow quasi–static process ($v_{crack} \cong 1\ cm/s$) followed by a quasi–dynamic but still slow ($v_{crack} \cong 10\ m/s$) enlargement which still does not radiate seismic waves (see Figure 2.11 in Shibazaki and Matsu'ura, 1998). This implies that the quasi–dynamic phase proposed by these authors is distinct from that identified by Beroza and Ellsworth (1995) on recorded seismograms. The beginning of the first slow quasi–static phase is unknown, therefore the definition of nucleation duration accounts only for the two latter phases of quasi–dynamic and fully dynamic rupture growth.

Umeda (1992), Ellsworth and Beroza (1995) and Shibazaki and Matsu'ura (1998) proposed a relation between the duration of the nucleation phase and the total seismic moment of the final event. Roy and Marone (1996) use a spring slider model and define the duration as the time during which the system evolution is quasi–static and is not yet inertia–dominated. This definition is quite well in agreement with the previous ones. Dieterich (1986) identifies the duration of the nucleation process as a first phase in which the crack half–length l is less than the critical half–length l_c (see the definition in Appendix D) and the slip is intrinsically stable, and a second phase, in which $l > l_c$, characterized by a progressive acceleration up to dynamic instability. This definition is also used by Ionescu and Campillo (1999). Ohnaka and Shen (1999) point out the correlation existing between the propagation velocity fields and the rate l/l_c.

According to Dieterich (1986), we define in this Chapter the duration T_n of the nucleation phase as the time necessary for the crack half–length to reach l_c. The analytical expression for the critical length l_c is given in Appendix D both for the SW model and for the Dieterich–Ruina formulation. Therefore, because l_c depends on the constitutive parameters, T_n also depends on the frictional properties of the fault.

In this section we aim to understand how such different constitutive laws and the variability of their parameters influence the nucleation phase. By using the FD approach, we have performed a large number of numerical tests which satisfy all the stability conditions (Appendix C). In particular, we have studied some relations between the duration T_n and the nucleation seismic moment M_{0_n}, the critical half–length l_c, the constitutive distance L or d_0 (Figure 2.14). The nucleation seismic moment M_{0_n} is defined by accounting for the amount of slip in the nucleation region. We have performed several simulations using the DR law and choosing different values of the constitutive parameters based on the nondimensional set reported in Table 2.1: a and b range between 0.75 and 1.12 and between 1.41 and 1.625, respectively, and L between 1. and 1.6. In this way we obtain different values of l_c and therefore different T_n. In the simulations performed using the SW we have varied the constitutive parameters t_u (between 1.05 and 1.8) and d_0 (between 0.6 and 2.8). The initial shear stress t_0 and the kinetic friction t_f are constant for all the simulations and they are 1. and 0, respectively. In this way we change the strength parameter S and/or the characteristic length. We first investigate the dependence of M_{0_n} on T_n (Figure 2.14a): both methods exhibit a nearly linear scaling of these parameters. However, the DR law predicts larger values of M_{0_n} than the SW law, and, more interesting, the DR simulations shows an evident variability and dependence on the constitutive parameters. In Figure 2.14(a) we have indicated with solid circles the values corresponding to those simulations where a and b are constants and only L is changed; this yields an evident linear scaling between nucleation time and seismic moment. The open circles in Figure 2.14(a) show the values resulting from those simulations in which L is constant, while a and b are varying: the upper values correspond to those cases where the difference $b - a$ is large (b = 1.6 and a in the range [0.75, 0.9]: very strong unstable behavior), while the lower values correspond to those where $b - a$ is smaller (weak seismic behavior). This Figure points out that the nucleation seismic moment depends on the difference $b - a$. A linear relation

between of M_{0_n} and T_n as a function of the critical distance has been found both using the SW (solid triangles) and the DR (solid circles) laws. The latter yields larger values of the nucleation seismic moment. The variations of b and a in the DR law yields a different slope in the scaling law than that obtained by changing L. These results can be explained by considering that the different frictional parameters (a, b and L) have a different effect on the frictional behavior of faulting.
A linear correlation is also found between T_n and l_c (Figure 2.14b): in the SW case it results $T_n \cong 3\ l_c/2\ v_S$, consistently with the relation derived by Shibazaki and Matsu'ura (1998), while with the DR we obtain $T_n \cong 3.3\ l_c/v_S$. In the latter case we found a greater dispersion for the DR law. The difference between the slopes in the two trends shown in Figure 2.14(b) points out that the SW law predicts a propagation velocity in the quasi–static nucleation phase (roughly expressed by $v_{crack} \cong l_c/T_n$) greater than that resulting from a DR law. By maintaining constant a and b in the DR law as well as S in the SW relation, we have shown in Figure 2.14(c) the link existing between T_n and L or d_0. The correlation is still linear in both constitutive models, although with different slopes.

These modeling results clearly show that slip–weakening and rate– and state–dependent constitutive laws yield different behaviors during the nucleation phase. Nevertheless, it is important to remind that the SW law used in this study has a constant weakening rate depending on the assumed constitutive initial parameters. According to Ionescu and Campillo (1999), if variable weakening rates and/or different values for the initial slope of the slip-weakening curve are used, more complex behaviors for the nucleation phase are expected. However, we emphasize that such more complex trends of the slip–weakening curves are well predicted by the rate– and state–dependent friction laws. Therefore, the variability of the nucleation parameters resulting from this constitutive formalism should include and agree with the results of Ionescu and Campillo (1999).

2.8. Discussion

The simulations discussed above have been performed using a fault parameterization at the laboratory scale dimension adopting values of constitutive parameters derived from laboratory experiments. A major problem that we have therefore to face is the scaling of our results from laboratory to actual fault dimensions. We have used in our calculations values of the L parameter ranging between 1 and 10 *mm* and, accordingly to the derived scaling law (equation 2.13), we found values of the equivalent slip–weakening distance comprised between 0.02 and 0.2 *mm*. These latter values are much smaller than those recently proposed for actual faults and constrained by modeling ground motion waveforms recorded during large earthquakes (see Guatteri et al., 2001, and reference therein). In fact, recent investigations have proposed d_0 values larger than 0.2 *m* (Ide and Takeo, 1997; Olsen et al., 1997). A first solution to this problem consists in assuming the scaling from laboratory–derived values to those appropriate for earthquake fault dimension: we thus consider L to be of the order of 1 *cm* (as obtained by scaling laboratory results with gauge materials of Mair and Marone, 1999 to real faults). This would yield critical slip–weakening values of d_0 = 0.2 *m*, in agreement with observations. However, it must be pointed out that there exist several concerns about the reliability of such large values of the critical slip distance: Guatteri and Spudich (2000) discussed this topic and concluded that estimates of SW distance inferred from kinematic inversions of seismograms are biased because of the trade–off between strength excess and slip–weakening distance and because of the effects of smoothing constraints used in inversion algorithms. Marone and Kilgore (1993) suggested that the critical slip distance is controlled by the thickness of the fault zone of localized shear strain. Therefore, the discrepancy between laboratory measurements and values obtained from modelling earthquakes might be attributed both to the differences in roughness between laboratory surfaces and natural faults (Scholz, 1988) as well as in the thickness of the localized shear strain zone (Marone and Kilgore, 1993).

The numerical results of our study demonstrate that the state variable evolution law controls the absorbed fracture energy and the traction drop within the cohesive zone. The proposed analytical relations and the scaling of the characteristic parameters from the laboratory to actual fault dimensions allows to associate the breakdown process to the state variable and to its evolution. Therefore, the explanation of the traction evolution within the cohesive zone relies on the physical interpretation of the state variable, which might lead to the association between the shear stress degradation and the properties and the roughness of the contact surface. On the contrary, it might be a likely explanation to assume that the breakdown process, occurring at high slip rates, does not depend on the properties of the contact surface, which indeed can control the rupture nucleation (interpreted as a quasi static process) and the long-term fault restrengthening. Several authors in fact proposed alternative interpretations of the state variable: Segal and Rice (1995) and Sleep (1997) suggest a relation between the state variable and the porosity of the fault zone. Thus, we might still use a constitutive law based on the rate and state formulation, but with a different interpretation of the state variable and its evolution. However, we should also consider alternative explanations, which does not require to scale laboratory to earthquake fault values, consisting to assume that slip–weakening and breakdown processes depend on other weakening mechanisms, such as thermal–weakening (Sleep, 1997), which can be independent of the state variable evolution. Our opinion is that different phenomena can contribute to the traction evolution and the weakening mechanisms associated with the rupture growth, which affect both the friction coefficient and the effective normal stress. Brodsky and Kanamori (2001) and Andrews (2002) proposed different mechanisms to explain the raise of pore pressure and the consequent reduction of frictional resistance to slip due to mechanical lubrication and thermal pressurization, respectively. The increase of fluid pressure reduces the effective normal stress, thus affecting the friction law. These phenomena may coexist, since the friction coefficient depends on the slip, slip rate and state variables (see Chapter 1) and pore pressure affects the effective normal stress. In this contest, it is important to note that the rate- and state- constitutive formulation with an assigned evolution law provides a good description of the traction evolution within the cohesive zone. The solution of this problem is well above the goal of the present study. We only aim to contribute to the debate concerning the constitutive properties in the achievable perspective to find a unified constitutive law to describe the earthquake dynamic rupture growth.

In order to understand how the analytical expression adopted for the evolution law in the rate– and state–dependent formulation affects the traction evolution within the cohesive zone, we have performed a set of simulations using slowness and slip evolution laws (equations 3 and 4). In particular, we use the rate– and state–dependent slip law (see Ruina, 1983; Beeler et al., 1994) in the formulation proposed by Roy and Marone (1996) as defined in (4).

We show in Figure 2.15 the comparison between the SW curve resulting from a slip and a slowness evolution law, taking fixed all the constitutive and initial parameters. In these simulations only the evolution law differs, while the governing equation for fault friction is nearly the same. Figure 2.15 shows that the SW behavior and the equivalent slip–weakening distance depend on the analytical expression adopted for the evolution law. The characteristic SW distance is strongly reduced (roughly by a factor 3) when a slip evolution law is used. This further confirms that the evolution law and the state variable controls the traction drop and the finite fracture energy absorption at the crack tip. The weakening rate for a slip evolution law is not constant, which results in a non-linear decay, the opposite of what observed with a slowness evolution law. An evident traction roll–off characterizes the traction evolution for slip values equal to the equivalent slip–weakening distance. The kinetic friction level is exactly the same, confirming that the steady state value is unchanged, but the yield stress is different. This result suggests that the choice of the analytical relation of the evolution law and the physical meaning of the state variable are crucial to model and interpret the breakdown process in the framework of a rate– and state–dependent formulation.

All the simulations discussed above have been obtained modeling the rupture propagation on a homogeneous fault, where the constitutive parameters are constant along the fault line. In such a configuration we always obtain crack–like rupture propagation (Zheng and Rice, 1998) and the resulting slip–weakening curve does not depend on the fault position. In a more realistic situation the frictional parameters will be heterogeneous along the fault, as shown, for instance, by Boatwright and Cocco (1996) and Bizzarri et al. (2001). In these cases the slip – weakening curves will be varying along the fault, because other physical mechanisms, like healing and arrest phases, taking places and consequently the scaling relations derived in this Chapter may change. We will not present here the details of these simulations because it will be the topic of a future work.

The results presented here further support the importance and the benefit of using rate – and state – dependent constitutive laws to model fault and earthquake mechanics. This constitutive formulation implies a SW behavior during the fully dynamic rupture propagation, which is not assigned "a priori" and spontaneously evolves depending on the adopted constitutive parameters. There is no need to assume that friction must become independent of slip velocity at high speeds to resemble SW. Our results show that SW is a characteristic behavior of rate- and state- constitutive laws during the dynamic rupture growth and that this constitutive formulation contains a physical control of the breakdown processes occurring within the cohesive zone. This is in agreement with the results of stick–slip laboratory experiments, which have been interpreted either as rate and state (Dieterich, 1979; Okubo and Dieterich, 1986) or slip – dependent friction (Ohnaka et al., 1987). However, the friction behavior at high slip rates affects the fracture energy absorbed within the cohesive zone. Our results could also be interpreted in the perspective of a unified constitutive formulation, at least for the dynamic slip episodes. While this can be a likely expectation for the dynamic rupture growth, this is certainly not true for the nucleation process. Earthquake nucleation is described in a different way by these two constitutive formulations (Dieterich, 1992; Shibazaki and Matsu'ura, 1998), which have both been proposed to model stick – slip episodes.