4. A realistic 3 – D fault model

4.1. Introduction to Chapter 4

In previous Chapters we have shown some simulations with different constitutive laws for a 2 – D fault model, in which all the solutions (slip, slip velocity, total dynamic traction and eventually state variable) have only one non – null component and they depend only on one spatial coordinate and on time. Even for more realistic, heterogeneous configuration discussed in Chapter 3, such a fault models represent an obvious idealization of real – world faults. In actual faults the crack tip is a close line, and we a mixture of the two modes of propagation (pure in – plane, showed in Chapters 2 and 3, and pure anti – plane).

In this Chapter we will present a new method, based of a Finite Difference code made by Andrews (unpublished work) and based on the Traction – at – Split – Nodes fault boundary condition, presented in Andrews (1999). We will show the main feature of this method, presenting some numerical results with the my implementation of slip – weakening law, even I had already implemented the whole set of constitutive models.

4.2. The numerical model

A large class of numerical method has been proposed to solve the dynamic problem and the wave propagation in continua (Day, 1977; Day, 1982a, 1982b; Madariaga, 1976 and their further modifications in Virieus and Madariaga, 1982; Madariaga et al., 1998). Some of them have the property, like Finite Element codes, that all components of displacement and slip velocity are defined in the sane set of points in spaces. In such a way a fault surface is identified by a class of split nodes, disjoint grids point in contact. This approach is called the Traction – at – Split – Nodes (TSN) fault boundary conditions, described in next Section. An alternative is to consider a fault as a yielding distributed through a region of finite thickness; if the independent variables are slip velocity and stress components, rather that displacement and slip velocity components, yielding can be treated with the Stress – Glut (SG) fault boundary condition. Andrews (1999) test this two methods for a 3 – D fault where no constitutive laws is assumed and a circular crack develops with an imposed and constant velocity (R – weave speed). The conclusion of that paper is that in the non – spontaneous case, refining adequately the grid size, both TSN and SG methods give the same results.

The numerical model presented here is based on a Finite Difference (FD) code made by Joe Andrews (unpublished work). This allow for the calculation and mutual interactions between a system of six faults, two vertical and parallel faults and four dipping faults. A grid of nodes is introduced in the tri – dimensional space and the solutions of the fundamental elasto – dynamic equation s obtained by stepping the solution at the previous time step through time by calculating the net force acting at all nodes. The discretization has been made by using quadrilateral isoparametric elements (Hughes, 1987); stress is not uniform inside an element and the fundamental variables are displacement and force at nodes. From displacement components at nodes we find strain tensor components at integration points, then stress tensor components and then net force components at nodes. For regular node points, at the actual time iteration n we know from previous iteration the old slip $\mathbf{u}^{(n-1)}$ and the old slip velocity $\mathbf{v}^{(n-1)}$, from the subroutine `sweep`. We also know form subroutine `swoop` the net forces acting on each nodes of cubic element $\mathbf{f}^{(n-1)}(ig(ix), jg(ix), kg(ix))$, where ix is the index (ranging from 1 to 8) of nodes of the cubic element. The actual force is determined as

$$\mathbf{f}^{(n)}(ig(ix), jg(ix), kg(ix)) = \mathbf{f}^{(n-1)}(ig(ix), jg(ix), kg(ix)) + \mathbf{fl}(ix) \qquad (4.1)$$

where $\mathbf{fl}(ix)$ are calculated in subroutine `box` for rectangular box elements (vertical faults) and in subroutine `wedge` for right prisms with rectangular cross section in the x_1–x_3 plane (dipping fault). The local forces $\mathbf{fl}(ix)$ are calculated using the Hooke' s law for an isotropic medium with the small displacement approximation, from strain and stress. From the actual net forces $\mathbf{f}^{(n)}$ we can obtain the actual slip acceleration, simply applying the second law of mechanics:

$$\mathbf{a}^{(n)}(ig(ix), jg(ix), kg(ix)) = \mathbf{f}^{(n)}(ig(ix), jg(ix), kg(ix)) \,/\, ssam(ig(ix), jg(ix), kg(ix)) \qquad (4.2)$$

where *ssam* is the mass matrix element. We explicitly update the slip and the slip velocity at the actual time step n as:

$$\mathbf{u}^{(n)} = \mathbf{u}^{(n-1)} + \mathbf{u}^{(n)}\, Dt \qquad (4.3)$$

$$\mathbf{v}^{(n)} = \mathbf{v}^{(n-1)} + \mathbf{a}^{(n)}\, Dt \qquad (4.4)$$

Such a formulation is equivalent to the local stiffness matrix. All components of slip velocity, displacement and force are defined at the same node point and the number of finite difference expressions that are calculated is 8 times larger than in a simple staggered grid scheme, in which velocities and stresses are defined at different points in space. In the latter way is easy to increase the order of accuracy to fourth order in space, while weeping it second order in time. There is not doubt that the fourth – order staggered – grid scheme is the most efficient and accurate method to propagate a wave to a distance that is large number of wavelength. However, to calculate earthquake sources, the fault boundary condition (FBC) is more important than propagation at large distance. We will describe such a FBC in the next Subsection.

The mass matrix is diagonal and the small displacement approximation is used: in such a way we can refer to the numerical algorithm either as a Finite Element (FE) scheme and Finite Difference (FD) scheme.

All the output of the code should be regarded as average over an area centered on the point that extends a half grid interval in each space dimension and extends in time a half time step before and after the nominal time of the output. Motion is calculated at all grid points, and we can observe effects of the rupture at the free surface (synthetic seismograms or ground motions), as well as effect of wave propagation in the medium.

4.2.1. The fault boundary conditions

The Traction – at – Split – Nodes (TSN) fault boundary condition assumes an explicit displacement discontinuity between the fault surface. The discussion that follow is focused on a vertical planar fault, perpendicular to the $x_3 = 0$ plane, as represented in Figure 2b. In this case the two non – null components are 1 and 3; all the solutions depends on x_1, x_2 and t. Let us write with index a the quantities referring to the superior (in the direction of increasing x_3) side of the fault plane and with index b those of inferior side. Let also t_0 be the reference value of dynamic traction. The code enter in the `flfslpfd` subroutine with the updated value of velocities and stresses at the actual time step n. We first find a trial value of traction on the fault plane that gives no differential acceleration (i. e. $\mathbf{a}^{\boldsymbol{a}} = \mathbf{a}^{\boldsymbol{b}}$, or D$\mathbf{a}$ = 0):

$$t^{t\,(n)} = t_0 + (\, m^a\, \mathbf{f}^{\boldsymbol{a}\,(n)} \;-\, m^b\, \mathbf{f}^{\boldsymbol{b}\,(n)} \;) \,/\, (\, A\, m^a + A\, m^b \,) \tag{4.5}$$

where A is the slip node area and m^a and m^b are the split node masses. We now specify the fault strength S_{fault} (equation (1.4)), that may incorporate, in full of generality, the adopted constitutive law. The value of S_{fault} is also known from previous iteration values. If the fracture criterion is not satisfied, i. e. if

$$\mathrm{D}\mathbf{v}^{(n-½)} = \mathbf{v}^{\boldsymbol{a}} - \mathbf{v}^{\boldsymbol{b}} = 0 \tag{4.6a}$$

$$3\; t^{t\,(n)}\; 3 < \; S_{fault} \tag{4.6b}$$

then the displacement is not changed ($\mathbf{u}$ = 0) and the traction s not updated ($t^{(n)} = t^{t\,(n)}$). The fault is not slipping in this case. On the contrary, if the fracture criterion is satisfied, i. e. if:

$$\mathrm{D}\mathbf{v}^{(n-½)} = \mathbf{v}^{\boldsymbol{a}} - \mathbf{v}^{\boldsymbol{b}} \neq 0 \tag{4.7a}$$

$$3\; t^{t\,(n)}\; 3 = \; S_{fault} \tag{4.7b}$$

the fault starts to slip or continues to slip. In this case we update the actual total dynamic traction by imposing the collinearity between traction and slip velocity:

$$t^{(n)} = c\; \mathbf{v}^{(n)} \tag{4.8}$$

where $\mathbf{v}^{*(n)} = \mathrm{D}\mathbf{v}^{(n\,-\,½)} + ½\; A\; Dt\; (\; 1/m^a + 1/m^b\;)\; t^{t\,(n)}$. From this equation we determine the director cosines:

$$g_i = \; v_i^{*(n)} \,/\, 3\; \mathbf{v}^{*(n)}\; 3 \tag{4.9}$$

Therefore the two actual components of total traction are expressed as:

$$t_i^{(n)} = g_i \, S_{fault} \tag{4.10}$$

By using (4.10) we can update the differential slip velocity:

$$Dv_i^{(n+½)} = Dv_i^{(n-½)} + A\, Dt\, (\, 1/m^a + 1/m^b\,)\, (\, \tau_i^{t\,(n)} - \tau_i^{t\,(n)}\,) \tag{4.11}$$

Then displacement **u** and slip velocity **v** are redefined and updated in subroutine `sweep` and depend on values of the velocities of center – of – mass $v^{av\,(n)}_i$ and on the fault differential slip velocity $Dv_i^{(n+½)}$:

$$v^{a\,(n)}_i = v^{av\,(n)}_i + m^b\, Dv_i^{(n+½)}\, (\, m^a + m^b\,) \tag{4.12}$$

$$v^{b\,(n)}_i = v^{av\,(n)}_i - m^b\, Dv_i^{(n+½)}\, (\, m^a + m^b\,) \tag{4.13}$$

where

$$v^{av\,(n)}_i = (\, m^b\, v^{b\,(n)}_i + m^a\, v^{a\,(n)}_i\,)\, /\, (\, m^a + m^b\,) + Dt\, (\, f^{a\,(n)}_i + f^{a\,(n)}_i\,)\, /\, (\, m^a + m^b\,) \tag{4.14}$$

The constitutive law is introduced by specifying the analytical expression of S_{fault}. In all cases (slip – weakening and rate – and state – constitutive laws) we use modulus of shear traction components and modulus of slip and slip velocity. For the numerical integration of equation with rate – and state – dependent friction laws we adopt the Rosembrok stiff integration (Press et al., 1992) that perform better than Runge – Kutta algorithm.

4.2.2. The domain boundary conditions

Within the code the $x_3 = 0$ plane is a free surface. This surface reflect waves and affect all calculations. If the initiation point of the rupture is deep enough that no reflection returned from the free surface, solutions are not affected by the presence of the free surface. On the contrary, the planes $x_1 = 0$ and

$x_1 = x_{1\ max}$ are cyclic boundary; the planes $x_2 = 0$ and $x_2 = x_{2\ max}$ are absorbing boundary, in the sense that no perturbation is reflected or transmitted.

4.3. The reference case

In this Section we present numerical results for a reference configuration in which parameters are listed in Table 4.1. We will show the prominent features of our numerical code with the adoption of the slip – weakening constitutive law. Such a presentation will be enriched with the comparison of the solution of the dynamic problem obtained for a 2 – D fault model (described in Chapter 2). In this way we can also understand what a 3 – D model add to a numerical model of faulting with respect to a simple 2 – D model. All the parameters are homogeneous and we will discuss results for heterogeneous configurations in next Sections.

Figure 2 show the slip behaviour as a function of distance along the fault and of the time. In panel (a) we report the corresponding slip for a 2 – D model, while in panel (b) and (c) the slip for the 3 – D model. Panel (b) is obtained fixing $x_1 = x_{init}$ (and varying x_3; formally the anti – plane mode); panel (c) is obtained fixing $x_3 = z_{init}$ (and varying x_1; formally the in – plane mode). We can see that also in the 3 – D model it is allowed a crack bifurcation, as in the 2 – D case, but we emphasize that the transition is less abrupt. The maximum value of the slip are in general lower that those obtained for a 2 – D fault.

We report in Figure 3 a more detailed comparison between 2 – D and 3 – D solution, by plotting the slip – weakening curves (panel (a)), the phase diagram (panel (b)), the slip velocity history in a log scale (panel (c)) and the traction history (panel (d)) in a fault point $x_1 = x_{init} + 18.0$. We observe that the constitutive law is well resolved in both cases and the imposed slip – weakening characteristic length is perfectly reproduced in both cases. However, the phase diagrams are quite different: even the maximum peak slip velocity is nearly the same the shape is more elliptical in the 3 – D case. We note that, as expected, no slip – hardening phase is present. Both models well resolve the maximum or yield stress and the final kinetic level.

In Figure 4 we superimpose the solutions (slip, slip velocity and traction), normalized to each maximum value. Fr the same configuration of Figures 2 and 3. We emphasize that in both cases the yield stress is reached before the maximum slip velocity, but that the maximum slip velocity does not coincide with the friction level. This is in agreement with the results of Mikumo et al. (2003), who propose a procedure for estimate the characteristic slip – weakening distance form recorded slip velocity. However, Bizzarri and Cocco (2003) showed that the same is not true if we adopt a Dieterich law. In the latter case v_{max} is reached before t_f.

We plot in Figure 5 the 3 – D trajectories of the dynamic traction, as a function of displacement and slip velocity. From these plots we can see that the weakening phase correspond to the acceleration phase only, while deceleration takes place only when the breakdown process is concluded. The final slip velocity is non zero in our simulation because the fault is homogeneous and in this case we expect a crack – like solution (Andrews, 1985; Bizzarri et al., 2001).

As noticed before, our codes allow for the consideration of the materials surrounding the fault line (in the 2 – D case) or fault (in the 3 – D one). In Figure 6 we have reported the time snapshots of the slip velocity vector distribution on the plane $x_3 = 0$ for the 2 – D model and $x_3 = z_{init}$ for the 3 – D one. In the latter model the line identified by $x_3 = z_{init}$ and $x_2 = y_{fault}$ correspond to the line $x_3 = 0$ and $x_2 = y_{fault}$ in the 2 – D model (see Figure 1). We can notice form Figure 6 that the 3 – D propagation propagate in the surrounding medium with lower values of slip velocity. At the tip we observe a higher variation of slip velocity, in correspondence of the crack tip.

4.4. Coupling of two modes of propagation

We have shown in previous Section that in the 3 – D model the mixture of the pure in – plane and pure anti – plane modes of propagation in general make the solution more smooth that those obtained for a 2 – D pure in – plane fault. In full of generality a real rupture develops in a direction and enlarges in the space with a shape that is characterized by a close contour and determined by frictional parameters (constitutive law and state of the stress). In Figure 7 we report a sketch, representing a general configuration in which the local slip velocity (at the time step at which the sketch has been drawn) is oriented in the fault plane with a rake angle different from zero. At the point chosen in the Figure the local crack speed enlargement vector (solid blue vector) determines the local in – plane (collinear to the local crack speed enlargement vector) and the local anti – plane (perpendicular to the local crack speed enlargement vector) vectors. We can see that, in full of generality, such two components does not coincide with the 1 and 3 components (solid green vectors) of the local crack speed enlargement vector.

We are interested in the coupling of two modes of propagation during crack enlargement. In Figure 8 we report snapshot vector plots of fault slip for five different configuration in which all the frictional parameters are identical to those used for previous Figures and the initial rake is varying: 0 (Figure 8a), 30 (Figure 8b), 45 (Figure 8c), 60 (Figure 8d) and 90 degrees (Figure 8e). We plot vectors only inside the cohesive zone, because outside the cohesive zone slip direction is the initial slip direction. We can observe a spatial variation of the rake near the secondary front. Such a variation is more appreciable in Figure 9, where we plot for the same cases and still at the final time step the rake variation with respect the initial value. All plots are done in the positive quadrant of the fault plane x_1–x_3. In the cracked region (the area inside the crack tip) the spatial variation of the rake is null (the slip is always collinear with the initial vector). The variation occurs only within the cohesive zone, in correspondence of the two front (we recall here that for the adopted frictional parameter a crack tip bifurcation occurs, with a primary front travelling at S – wave velocity and a secondary one travelling at P – wave velocity; see Figure 2). We emphasize that the general behaviour is identical and independent on the initial value of rake.

In Figure 10 we plots, for all the five cases of Figure 9, the slip – weakening curve (first column), the phase diagrams (traction vs. slip velocity, second column), the traction history (third column) and the slip velocity history (fourth column). Blue lines refer to the total value, while violet an yellow refer to the 1 and 3 components, respectively. All plot are calculated in a fault point indicated in the top of each panel. We can verify here the symmetry existing between different cases (compare, for instance, the first row with the last one, or the second row with the fourth).

We plot the history of the rake (formally the slip velocity azimuth) in Figure 11. Each panel correspond to a configuration of Figures 8, 9 and 10; each panel report the situation at a fault point located at a distance of 18.0 units from the nucleation point. Location #1 is in the x_1 axis, location #2 is in the direction of the initial rake and location #3 is on the x_3 axis. Under the horizontal grey line (corresponding to 45 degrees) component 1 is paramount with respect to the component 3; above that line is the opposite. Vertical lines represent, for each location, the time step of the start and the end of the breakdown process, i. e. the time at which the maximum yield stress is reached (first line) and the time at which the frictional revel is attained (second line). We emphasize that the temporal variation of the rake is concentrated in the correspondence of the cohesive zone, but also before a variation occurs. In particular, let we observe the location #1. With the exception of the case of 0 degrees of initial rake (Figure 11a), in which, by definition locations #1 and #2 coincide and of case of 90 degrees initial rake (Figure 11e), in which, by definition locations #2 and #3 coincide, the component 3 decreases before the breakdown process, then begin to increase inside the cohesive zone, reaches its maximum in correspondence of the end of breakdown process, then decreases again and reaches a final value corresponding to the initial value.

4.5. Dependence on the absolute stress level

In a paper of 1994 Andrews simulate with a Boundary integral Equation method a mixed – mode crack propagation in which the crack tip is an indefinite line (not close) propagating in the x_1 direction, the solutions of the dynamic problem are dependent only on the x_1 coordinate and have two non null components (1 and 3; see Figure 1). All the simulations have been done with an initial rake of 45 degrees. Even if the cohesive zone is not sufficiently resolved (about only three points within the breakdown zone time), he found that component 3 is predominant with the respect the component 1 and that the slip azimuth variation increases with increasing absolute level of friction parameters. We have shown in previous Figures a more detailed study, showing, with a proper resolution of the cohesive zone and of the breakdown process (see also Appendix E), that the mixture of components 1 and 3 is complex. The coupling of the two components is varying in space (Figure 8 and 9) and in time (Figures 10 and 11). What is more, the results presented here are more general, because we works with a fully 3 – D rupture propagation.

In this Figure 12 we plot the numerical results obtained for different configuration in which the strength parameter S is always 0.8, but the absolute value of stresses (initial, yield and kinetic stress) are different. The initial rake is 45 degree, as in Andrews (1994). From the top panels (a, b, c and d) we can study the spatial variation of rake (at the final time step), while in the inferior ones (e, f, g and h) we can see the temporal variation of the rake. In all panels we mark the cohesive zone. We emphasize that also varying the absolute levels of stress the spatial and temporal rake variation is concentrated in the cohesive zone, as noticed before (Figure 9). We noticed that as the absolute stress levels increases (see Table 4.2 for the details) the rake variation is lower. This confirm and generalize the results of Andrews (1994). This is more evident in Figure 13, where we report the rake variation history at a fault point located in the x_1 axis and at a distance of 18.0 units form the nucleation point. We can see from this Figure that component 3 (nominally the anti – plane mode) is paramount before the breakdown process, then tends to increase and inside the cohesive zone component 1 (nominally the in – plane mode) is predominant. At the end, after the frictional level, we observe a decrease of rake variation and two component are perfectly equally coupled (note that the final value of rake is 45 degree, as at the initial time step, indicating that two components are identical 9.

If we look at the shape of the crack for configuration #34 of Figure 13 (see

Table 4.2 for details). We can observe that the spatial symmetry is lost. In Figure 14 we plot the slip, the fracture energy (G) distribution and the rake variation on the fault plane, at the final time of calculation. As the absolute stresses increased with respect the reference configuration, reported for completeness in Figure 14d, the crack tip is not symmetric at all, as well as the fracture energy distribution. We can see a concentration of rake variation in the negative direction (we recall that the cracks initially start to propagate in the positive x_1 direction with an initial rake of 45 degrees); the crack tip bifurcation occurs later in the negative direction and G is quite asymmetric. This is an example of the effects of the free surface $x_1 = 0$.

4.6. Heterogeneous configurations

In all the Figures presented and discussed before all frictional and medium parameters were homogeneous. The crack enlargement is in general not complex, but we have shown in Figure 14 that free surface can ingenerate asymmetry in crack enlargement and in the distribution of the fracture energy. In this Section we will present two different configuration with heterogeneous distribution of the frictional parameters. This represent the logical end step in order to simulate a real – world fully 3 – D fault.

In Figure 15 we report the spatial variation of the rake distribution for three different time step for a fault in which a barrier (a zone with a higher value of the strength parameter S) is penetrated. We can observe a very high complexity in the rake distribution: also inside the crack tip the rake continues to varying, in contrast with the behaviour observed for homogeneous configuration (see Figure 9, for instance).

The same is true for a configuration in which the barrier is able to arrest the crack (Figure 16). In this case we have the so – called barrier healing phenomenon (discusses for 2 – D models in Chapter 3).

4.7. Discussion

In this Chapter we have presented a 3 – D Finite Difference (FD) numerical code able to solve the fully dynamic, spontaneous problem for a system of six faults. The adopted fault boundary condition is implemented by using the Traction – at – Split – Nodes (TSN), already used in our 3 – D FD code. We discuss here only the implementation of the classical slip – weakening law (Ida, 1972; Andrews, 1976a, 1976b), although the whole class of rate – and state – dependent friction law is actually implemented. The code is running on NEC XS-4 with 16 *GB* of RAM with vectorization and parallelization, on a AMD K7 @ 1200 *MHz* with 512 *MB* of RAM DIMM workstation and on a Intel Pentium 4 @ 1700 *MHz* with 1 *GB* RAM RIMM workstation.

In the second part of the Chapter we present some numerical results of one vertical fault only in order to show the capabilities and the general features of the code. We test our code comparing results obtained with the 2 – D FD numerical code. We also studied the coupling of two modes of propagation (nominally the pure in – plane and the pure anti – planes modes) and we have emphasized that in the realistic 3 – D case such a coupling is complex. We generalize the results obtained by Andrews (1994) for a mixed – mode fault.

At the end of the Chapter we report two heterogeneous configuration, to show that a high complexity arise from realistic frictional and rheological configurations.