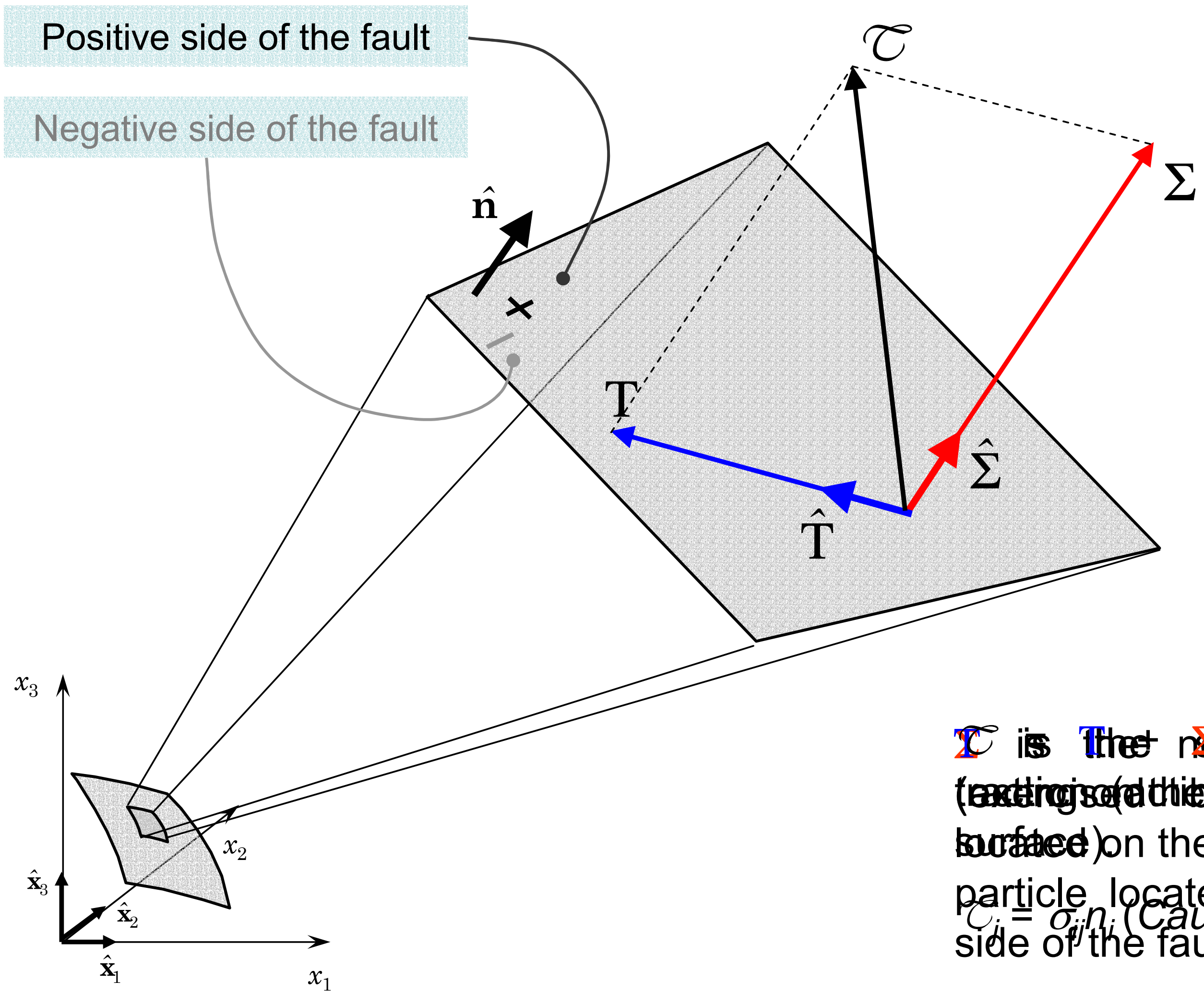
Fault governing laws
(constitutive equations)

Notations and symbols



$\mathcal{T}$ is the traction (exerted by the + particle located on the +ve side on a particle located on the −ve side of the fault surface)

T is the shear traction (tangential to the fault surface)

Σ is the normal traction (orthogonal to the fault surface)

$\mathcal{T}_i = \sigma_{ij} n_j$ (*Cauchy's formula*)

Fault models

Physical Phenomena in Faulting

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Fracture Criteria & Constitutive Laws

1. FRACTURE CRITERION

- **Condition that specify, at a given fault point and at a given time, if there is a rupture or not.**
- It can be expressed in terms of **energy**, in terms of **maximum frictional resistence**, and so on.
- It is based on (*i*) the *Benioff (1951)* hypothesis: The fracture occours when the stress in a volume reaches the rock strength
  or, analogoulsy,
  (*ii*) the *Reid (1910)* statement: The fracture takes place when the stress attains a value greater than the rock can endure.

2. CONSTITUTIVE LAW

- Analytical relation existing between the components of the stress tensor and physical observable(s), like the slip, the slip velocity, the state variable, etc.
- It is a **Fault Boundary Condition** (**FBC**) that controls earthquake dynamics and its complexity in space and in time
- Its simplest form consider only **two frictional levels**, τ_u and τ_f; it accounts for stress drop (or stress realease), but the process is instantaneous: there is a singularity at crack tip.
- **Cohesive zone models**: *Barenblatt (1959a, 1959b), Ida (1972), Andrews (1976a, 1976b)*. In these models the singularity is removed and the sress release occours over a breakdown zone distance X_b and in a breakdown zone time T_b .
- Friction laws (Rate and State dependent f. l.): *Dieterich (1976), Ruina (1980, 1983)*. They accounts for fault spontaneous nucleation, re – strengthening, healing, etc.

CONSTITUTIVE LAW (continues)

- In full of generality we can express the constitutive (or governing) as:

$$\tau = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\ \sigma_n^{eff}(\sigma_n, p_f)$$

where:

1st – order dependencies

u is the Slip (i. e. displ. disc.) modulus,

v is the Slip Velocity modulus (its time der.),

$\Psi = (\Psi_1, \ldots, \Psi_N)$ is the State Variable vector,

T is the Temperature (accounting for Ductility, Plastic Flow, Melting and Vaporization),

H is the Humidity,

λ_c is the Characteristic Length of surface (accounting for Roughness and Topography of asperity contacts),

h is the Hardness,

g is the Gouge (accounting for Surface Consumption and Gouge formation),

C_e is the Chemical Environment

Strength & Constitutive Laws

1. THE STRENGTH PARAMETER

- **Hystorically introduced by *Das and Aki (1977a, 1977b)* to have a quantitative extimate of the ability to fracture for a fault**
- Its expression can be generalized as:

$$S = (\mu_u \sigma_n^{eff} - \tau_0) / (\tau_0 - \mu_f \sigma_n^{eff})$$

where μ are the friction coefficient.

- We can also define

2. THE FAULT STRENGTH

- **as the parameter that quantify the Strenght in the more general case, in which a fault is described by a rhealistic friction laws**

$$S^{fault} = \mu(u, v, \Psi, T, H, \lambda_c, h, g, C_e)\ \sigma_n^{eff}(\sigma_n, p_{fluid})$$

Time - weakening Friction Law

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{(t - t_r)}{t_0} \right] \sigma_n^{eff} & , t - t_r < t_0 \\ \mu_f \sigma_n^{eff} & , t - t_r \geq t_0 \end{cases}$$

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ilaw = 11
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TW

t_r = $t_r(\xi)$ is the rupture onset time in every fault point ξ.

Andrews (1985), *Bizzarri et al. (2001)* and other following Bizzarri' s papers

t_0 is the characteristic time – weakening duration.

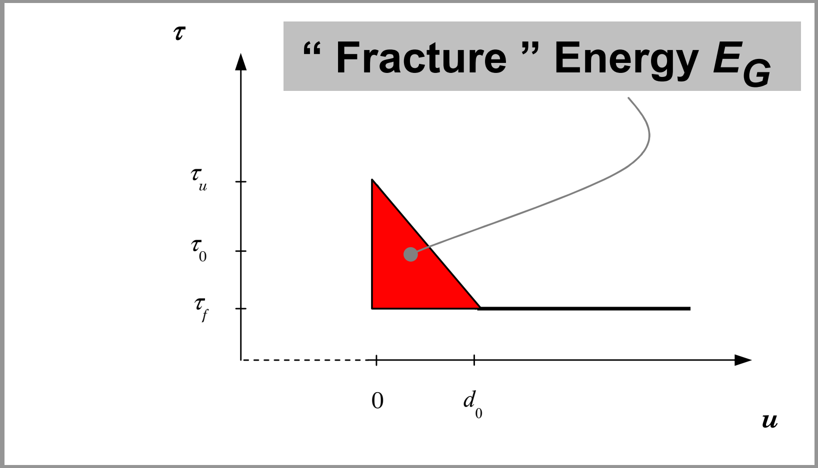
Slip - Dependent Friction Laws

1. LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - (\mu_u - \mu_f) \dfrac{u}{d_0} \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \geq d_0 \end{cases}$$

ilaw = 21

SW



Barenblatt (1959a, 1959b), Ida (1972), Andrews (1976a, 1976b), and many authors thereinafter

d_0 is the characteristic slip – weakening distance ilaw = 22

IW

2. NON LINEAR SLIP – WEAKEING LAW

$$\tau = \begin{cases} \left[\mu_u - \dfrac{\mu_u - \mu_f}{d_0} \left(u - \dfrac{(1-p)d_0}{2\pi} \sin\left(\dfrac{2\pi u}{d_0} \right) \right) \right] \sigma_n^{eff} & , u < d_0 \\ \mu_f \sigma_n^{eff} & , u \ge d_0 \end{cases}$$

Ionescu and Campillo (1999)

3. NON LINEAR SLIP – WEAKEING LAW WITH SLIP – HARDENING

$$\tau = \left\{ \left[\left(\tau_0 - \mu_f \right) \left(1 + \alpha \ln \left(1 + \frac{u}{\beta} \right) \right) \right] \mathrm{e}^{-\frac{u}{d_0}} + \mu_f \right\} \sigma_n^{\,eff}$$

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ilaw = 23
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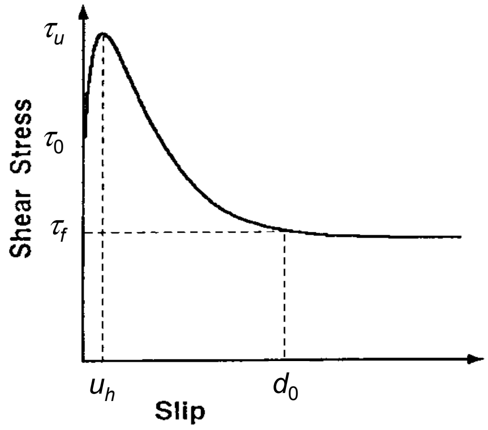
OW

$$u_h : \left. \frac{\mathrm{d}\tau}{\mathrm{d}u} \right|_{u_h} = 0 ; \qquad \begin{cases} u_h = r d_0 \quad (\mathrm{e.g.}\ \ r = 0.1) \\ \tau(u_h) = \tau_u \end{cases}$$



Ohnaka and Yamashita (1989) and the following papers by Ohnaka and coworkers

Rate - and State - Dependent Friction Laws

1. DIETERICH IN REDUCED FORMULATION

$$
\begin{cases}
\tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{eff} \\
\dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = 1 - \dfrac{\Psi\, v}{L}
\end{cases}
$$

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ilaw = 31
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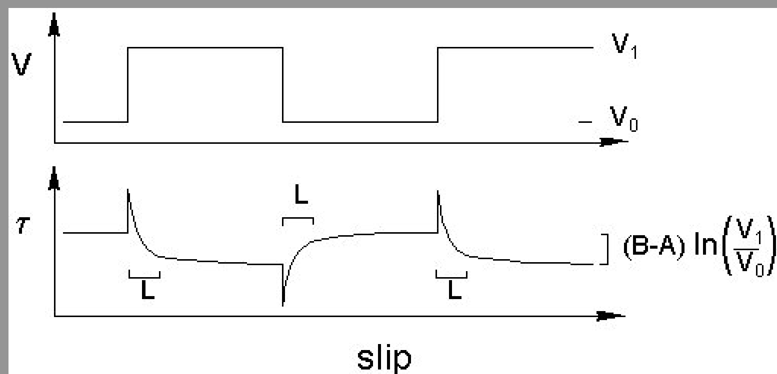
DR

However, while in velocity stepping experiments the traction response following the velocity variation is directly controlled by the parameter *L*, its effects are much less evident during the dynamic rupture propagation.

Bizzarri and Cocco (2005a)

Response to an abrupt jump in load



2. RUINA – DIETERICH

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{\Psi\, v}{L} \ln\left(\dfrac{\Psi\, v}{L} \right) \end{cases}$$

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ilaw = 32
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RD

Ruina (1980, 1983), Beeler et al. (1984), Roy and Marone (1996)

3. DIETERICH – RUINA WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\frac{v_*}{v}\right) + b \ln\left(\frac{\Psi v_*}{L}\right) \right] \sigma_n^{eff} \\ \frac{\mathrm{d}}{\mathrm{d}t}\Psi = 1 - \frac{\Psi v}{L} - \left(\frac{\alpha_{LD}\Psi}{b\,\sigma_n^{eff}}\right) \frac{\mathrm{d}}{\mathrm{d}t}\sigma_n^{eff} \end{cases}$$

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ilaw = 31
decis10=T
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DR

Linker and Dieterich (1992), Dieterich and Linker (1992), Bizzarri and Cocco (2005b, 2005c)

4. RUINA – DIETERICH WITH VARYING NORMAL STR.

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\frac{v_*}{v} \right) + b \ln\left(\frac{\Psi v_*}{L} \right) \right] \sigma_n^{eff} \\ \frac{\mathrm{d}}{\mathrm{d}t} \Psi = -\frac{\Psi v}{L} \ln\left(\frac{\Psi v}{L} \right) - \left(\frac{\alpha_{LD} \Psi}{b \sigma_n^{eff}} \right) \frac{\mathrm{d}}{\mathrm{d}t} \sigma_n^{eff} \end{cases}$$

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ilaw = 32
decis10=T
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RD

Linker and Dieterich (1992), Bizzarri and Cocco (2005b, 2005c)

5. DIETERICH IN REDUCED FORM REGULARIZED

$$
\begin{cases}
\tau = \left[\mu_* - a \ln\left(\dfrac{v + v_*}{v + v_p} \right) + b \ln\left(\dfrac{\Psi \left(v - v_p \right)}{L} + 1 \right) \right] \sigma_n^{\,eff} \\
\dfrac{\mathrm{d}}{\mathrm{d}\,t} \Psi = 1 - \dfrac{\Psi \left(v + v_p \right)}{L}
\end{cases}
$$

ilaw = 33

DE

v_p is a regularization fault slip velocity

Perrin et al. (1995), Cocco et al. (2004)

6. RUINA REGULARIZED

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_* - v_p}{v + v_p} \right) + \dfrac{\Psi}{\sigma_n^{eff}} \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = - \dfrac{v + v_p}{L} \left(\Psi + b \ln\left(\dfrac{v + v_p}{v_* - v_p} \right) \right) \end{cases}$$

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ilaw = 34
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RE

v_p is a regularization fault slip velocity

Bizzarri (2002, unpublished work)

7. DIETERICH IN REDUCED FORM WITH HEALING

$$\begin{cases} \tau = \left[\mu_* - a \ln\left(\dfrac{v_*}{v} + 1 \right) + b \ln\left(\dfrac{\Psi\, v_*}{L} + 1 \right) \right] \sigma_n^{eff} \\ \dfrac{\mathrm{d}}{\mathrm{d}t} \Psi = \dfrac{\gamma - \Psi}{t_{fh}} - \dfrac{\Psi\, v}{L} \end{cases}$$

ilaw = 35

DH

γ = 1 s

t_{fh} is the time for healing (slip duration)

Evolution law proposed by *Nielsen et al. (2000)* and by *Nielsen and Carlson (2000)*. Used in this form by *Cocco et al. (2004)*

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