



PRESSURIZZAZIONE DI FLUIDI IN MODELLI DI ROTTURA 3D DINAMICI E SPONTANEI

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Goals

- Comprendere l'evoluzione della trazione ed il meccanismo di weakening durante un evento sismico;
- Quantificare l'importanza di uno dei cosiddetti effetti del secondo ordine in una legge costitutiva realistica;
- Modellare il dynamic fault weakening includendo la pressurizzazione termica dei fluidi;
- Quantificare il calore prodotto per attrito durante il movimento dinamico su faglia;
- Enfatizzare le implicazioni sui valori inferiti della distanza di slip – weakening caratteristica d_0 e dell'energia di frattura E_G ;
- Esplorare come una porosità variabile nel tempo possa modificare i processi di breakdown;
- Inferire relazioni di scala tra le quantità fisicamente rilevanti ed i parametri della thermal pressurization.

Statement of the problem and methodology

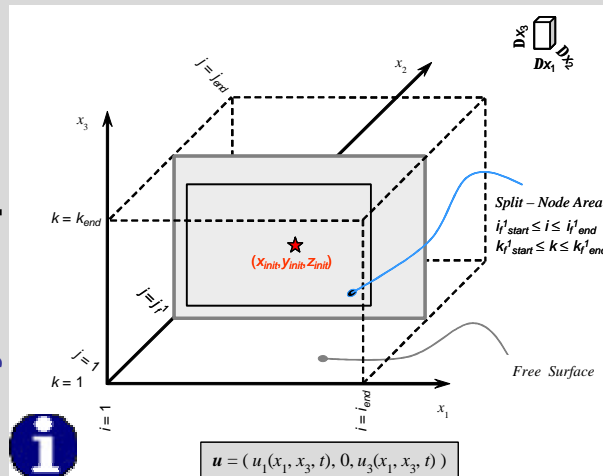
We solve *fully* dynamic, spontaneous problem (the fundamental elasto–dynamic equation), without body forces \mathbf{f}


$$\mathbf{r}\ddot{\mathbf{U}}_i = \mathbf{s}_{ij,j} + \mathbf{f}_i$$

We consider *truly 3–D* (not mixed – mode) problem, for which solutions (for instance the fault slip, i. e. the particle displacement discontinuity) are in the form: $\mathbf{u} = (u_1(x_1, x_3, t), 0, u_3(x_1, x_3, t))$

Numerical experiments refer to a vertical fault

truly 3 – D problem



The spatial computational domain is discretized using **cubic building blocks** in a conventional grid. The medium is linearly elastic except that in the fault plane ... 

... that obeys to the Fault Boundary Condition, i. e. to the governing law, that relates the fault friction t to some physical observables. In general is:


$$t = \mathbf{m}(u, v, Y, T, H, \mathbf{I}_c, h, g, C_e) s_n^{eff}(s_n, p_{fluid})$$

where \mathbf{m} is the friction coefficient and s_n^{eff} the effective normal stress that can change during time.

Slip weakening law (Baronblott)

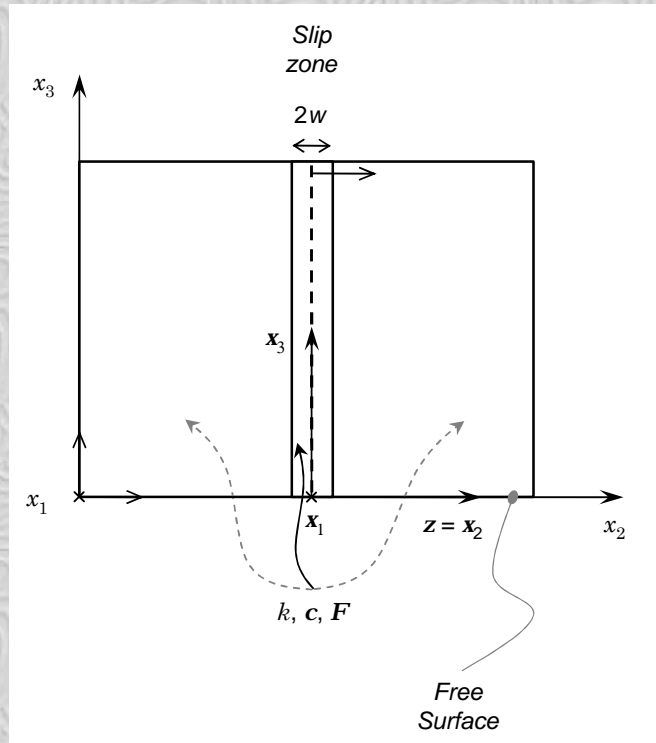
$$t = \begin{cases} \left[\mathbf{m}_u - (\mathbf{m}_u - \mathbf{m}_f) \frac{u}{d_0} \right] s_n^{eff} & , u < d_0 \\ \mathbf{m}_f s_n^{eff} & , u \geq d_0 \end{cases}$$

$$\begin{cases} t = \left[\mathbf{m}_* - a \ln\left(\frac{v^*}{v}\right) + b \ln\left(\frac{Y v^*}{L}\right) \right] s_n^{eff} \\ \frac{d}{dt} Y = 1 - \frac{Y v}{L} - \left(\frac{a_{LD} Y}{b s_n^{eff}} \right) \frac{d}{dt} s_n^{eff} \end{cases}$$

An explicit displacement discontinuity is assumed between the two sides of the fault: **Traction-at-Split-Nodes** scheme (Day, 1982a, 1982b; Andrews, 1999) 

The numerical details and the implementations of constitutive equations is given in **Bizzarri and Cocco (2005, Ann. Geophys, 48, 2, 279-299)**

Mathematical background



1 - D Fourier' s heat conduction equation:

$$\frac{\partial}{\partial t} T = c \frac{\partial^2}{\partial z^2} T + \frac{1}{c} q$$

Coupling of temperature T with pore fluid pressure p_{fluid} :

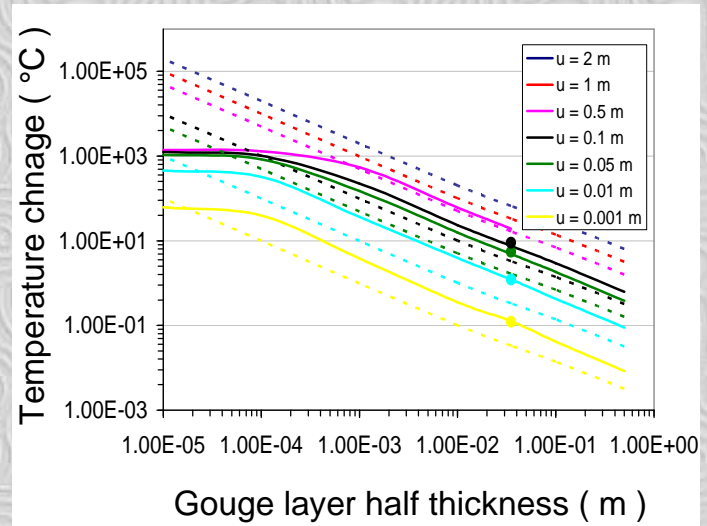
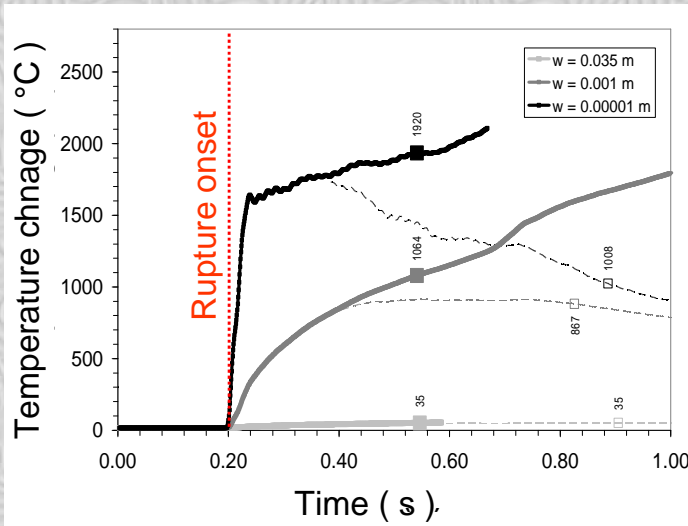
$$\frac{\partial}{\partial t} p_{fluid} = \frac{a_{fluid}}{b_{fluid}} \frac{\partial}{\partial t} T - \frac{1}{b_{fluid} F} \frac{\partial}{\partial t} F + w \frac{\partial^2}{\partial z^2} p_{fluid}$$

where c is the thermal diffusivity, c the heat capacity for unit volume, a_{fluid} the coefficient of thermal expansion, b_{fluid} the compressibility coefficient, F the porosity and $w = k/h_{fluid} b_{fluid} F$ the hydraulic diffusivity (being k the permeability of the medium and h_{fluid} the dynamic fluid viscosity). Analytical solutions at $z = 0$ are:

$$T^{wf}(\mathbf{x}_1, \mathbf{x}_3, t) = T_0^f + \frac{1}{2cw(\mathbf{x}_1, \mathbf{x}_3)} \int_0^{t-e} dt' \operatorname{erf}\left(\frac{w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{c(t-t')}}\right) \{ t(\mathbf{x}_1, \mathbf{x}_3, t') v(\mathbf{x}_1, \mathbf{x}_3, t') \}$$

$$\tilde{p}_{fluid}^{wf}(\mathbf{x}_1, \mathbf{x}_3, t) = p_{fluid_0}^f + \frac{g}{2w(\mathbf{x}_1, \mathbf{x}_3)} \int_0^{t-e} dt' \left\{ -\frac{c}{w-c} \operatorname{erf}\left(\frac{w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{c(t-t')}}\right) + \frac{w}{w-c} \operatorname{erf}\left(\frac{w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{w(t-t')}}\right) \right\} \\ \{ t(\mathbf{x}_1, \mathbf{x}_3, t') v(\mathbf{x}_1, \mathbf{x}_3, t') - \frac{2w(\mathbf{x}_1, \mathbf{x}_3)}{g} \frac{1}{b_{fluid} F(t')} \frac{\partial}{\partial t'} F(\mathbf{x}_1, 0, \mathbf{x}_3, t') \}$$

Temperature change on the fault



- Crack like models
- Pulse like models (with healing and finite slip duration)

- Numerical experiments
- Adiabatic prediction:

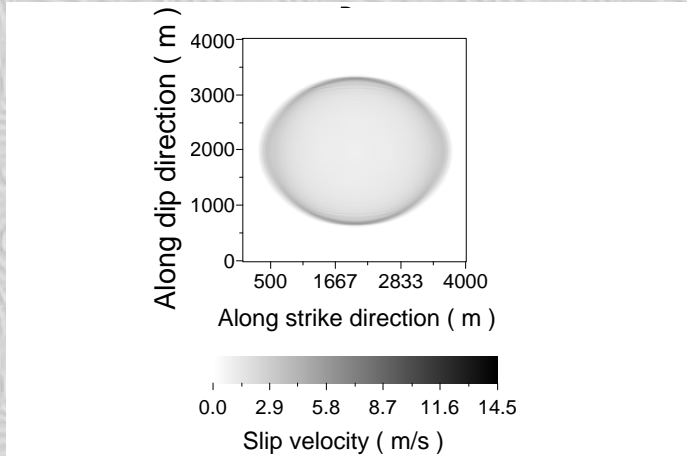
$$DT^{(adiab)} \cong \frac{t_f u}{2cw}$$

Bizzarri and Cocco (2006a, 2006b, *JGR*, 111, B05303 and B05304)

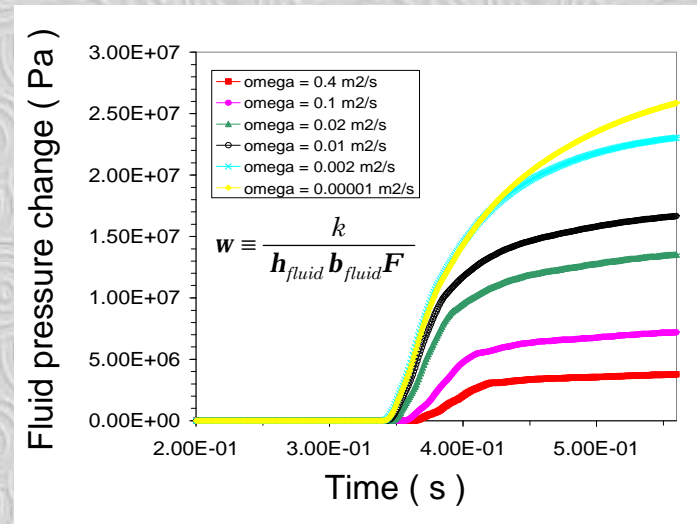
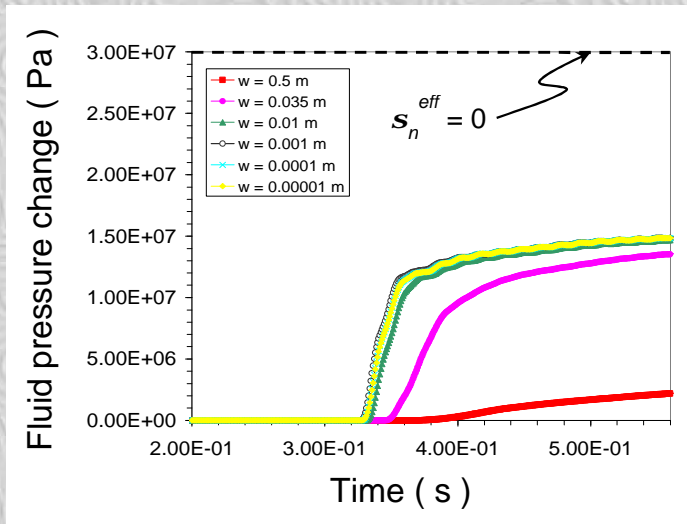
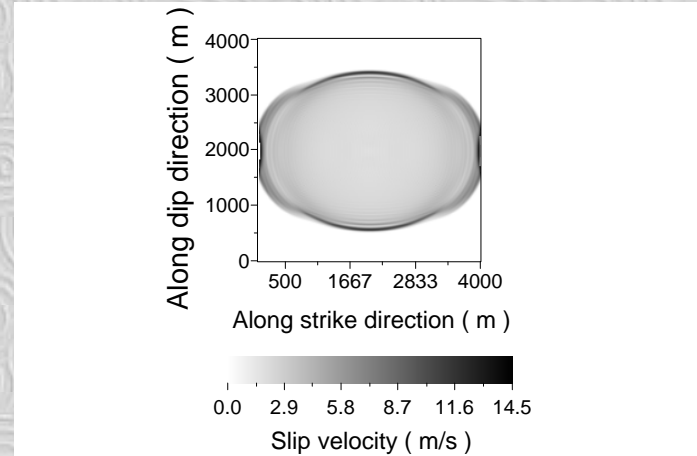


Results with SW law

Dry fault ($s_n^{eff} = \text{const}$)

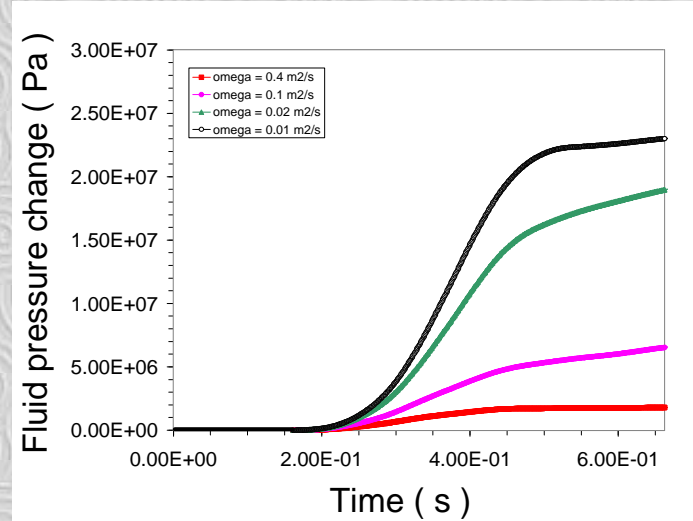
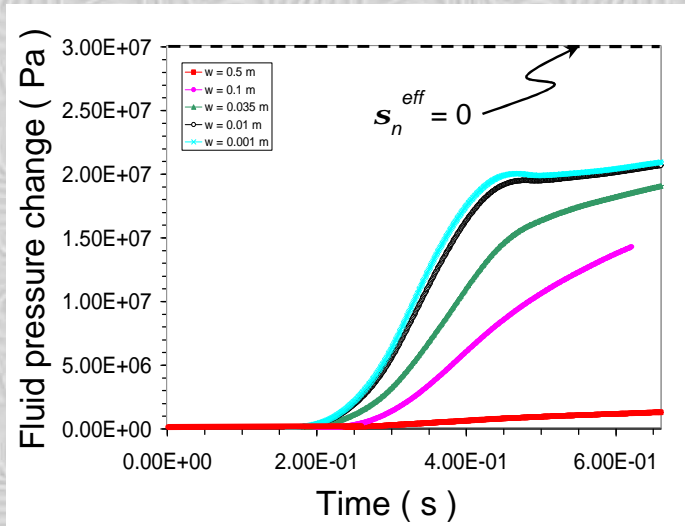


Wet fault (s_n^{eff} varies)





Results with DR law



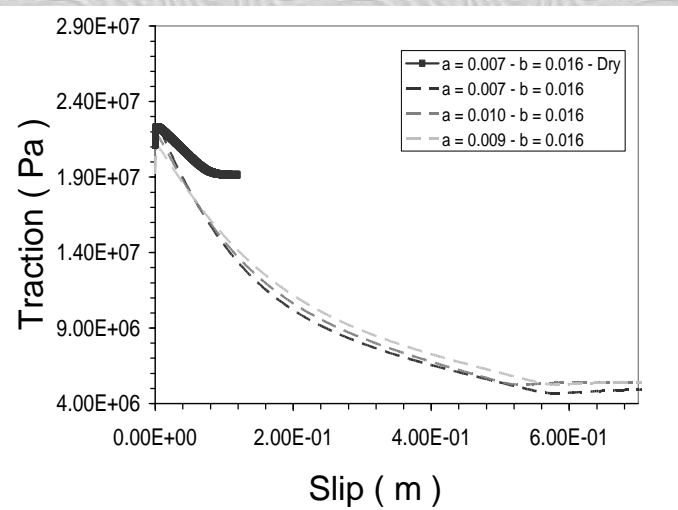
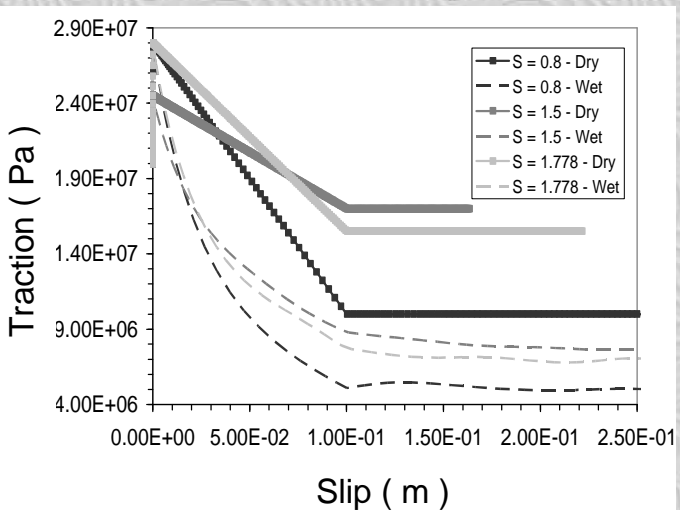
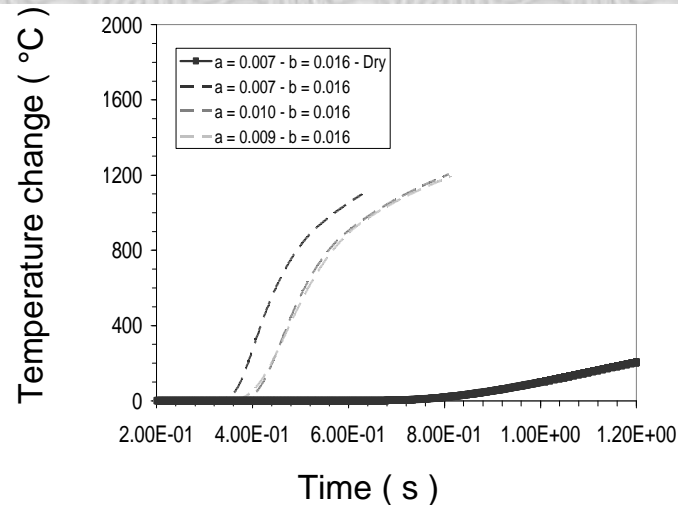
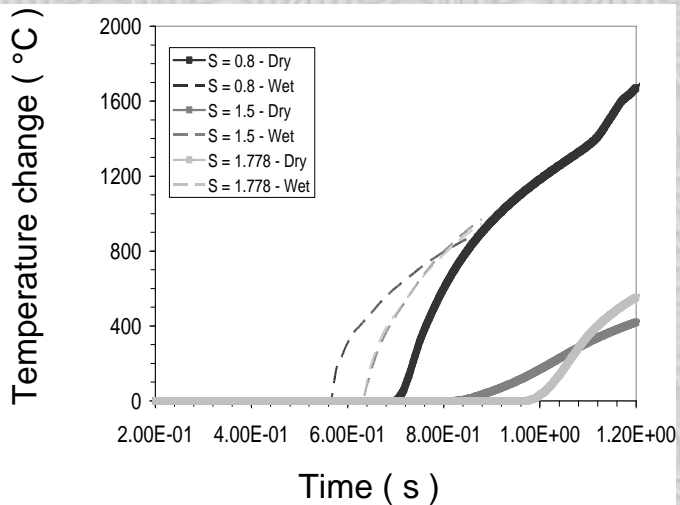
Bizzarri and Cocco (2006a, 2006b, *JGR*, 111, B05303 and B05304)

$$t = \left[m_* + a \ln \left(\frac{v}{v_*} \right) + b \ln \left(\frac{Y v_*}{L} \right) \right] s_n$$

$$\frac{d}{dt} Y = 1 - \frac{Y v}{L} \quad \text{Dry fault}$$

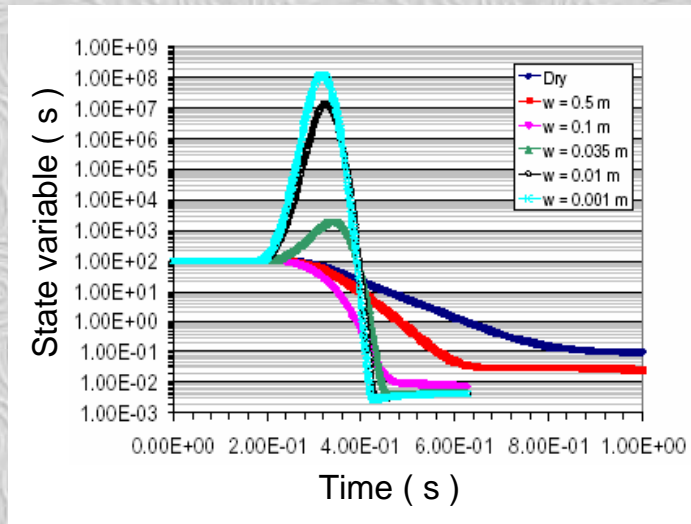
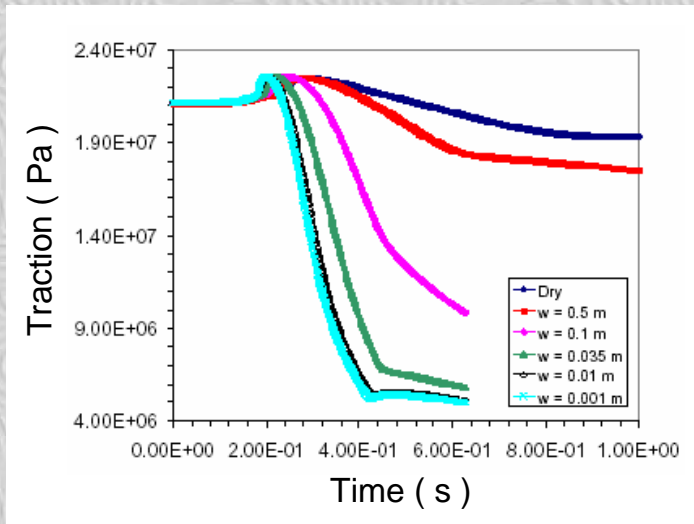
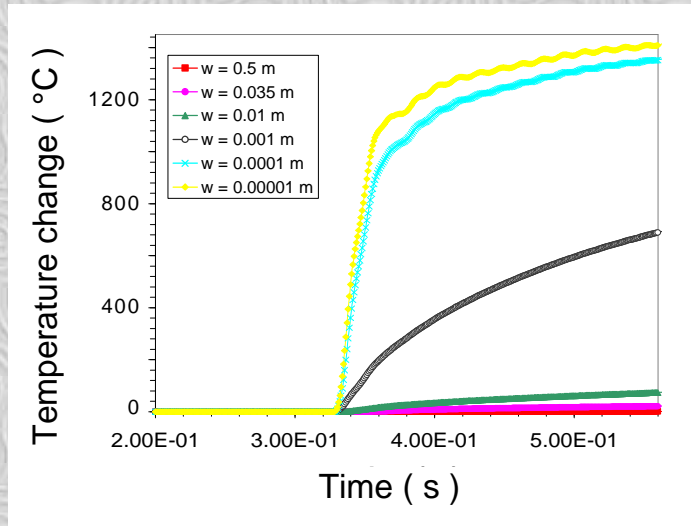
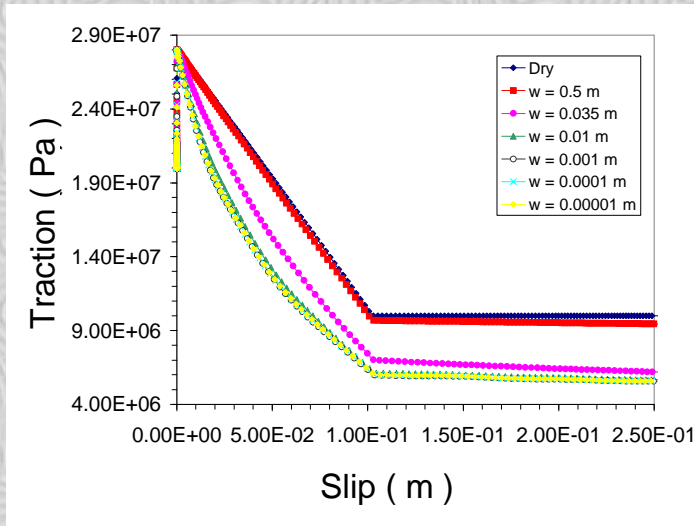


Dependence on friction parameters



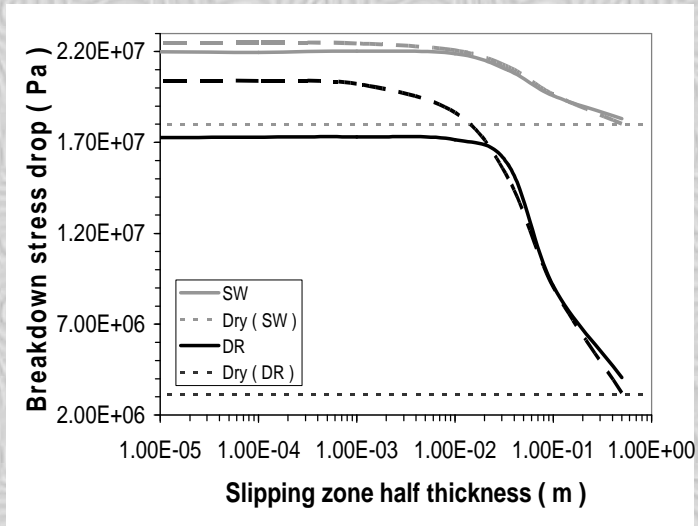


The breakdown zone





Scaling laws #1

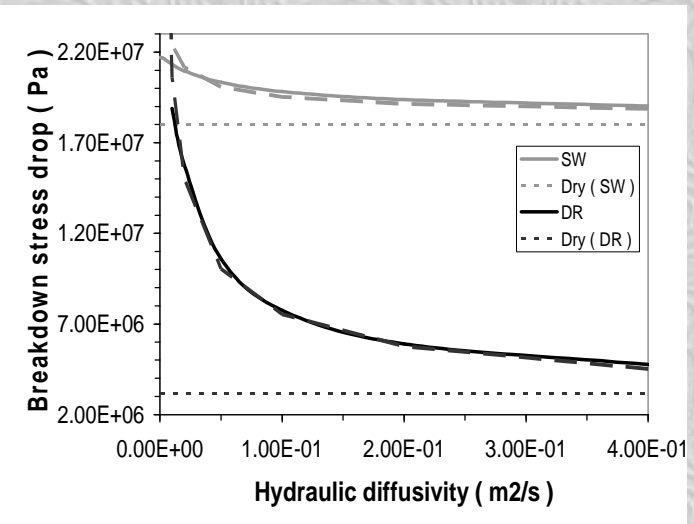


$$Dt_b = Dt_b^{(dry)} \left(1 + a_2 e^{-\frac{w}{d}} \right) \left(a_3 + a_4 \sqrt{\frac{w_*}{w}} \right)$$

where $w_* = 0.02 \text{ m}^2/\text{s}$ and

for SW: $a_2 = 0.25$, $a_3 = 0.86$,
 $a_4 = 0.14$, $d = d_0^{(dry)}$

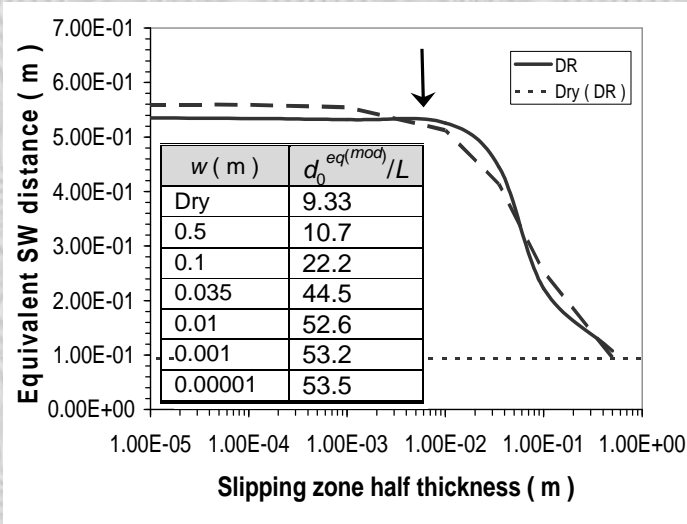
for RS: $a_2 = 5.5$, $a_3 = 0.1$,
 $a_4 = 0.9$, $d = d_0^{eq(dry)}$



Bizzarri and Cocco (2006a, 2006b, *JGR*, 111, B05303 and B05304)



Scaling laws #2

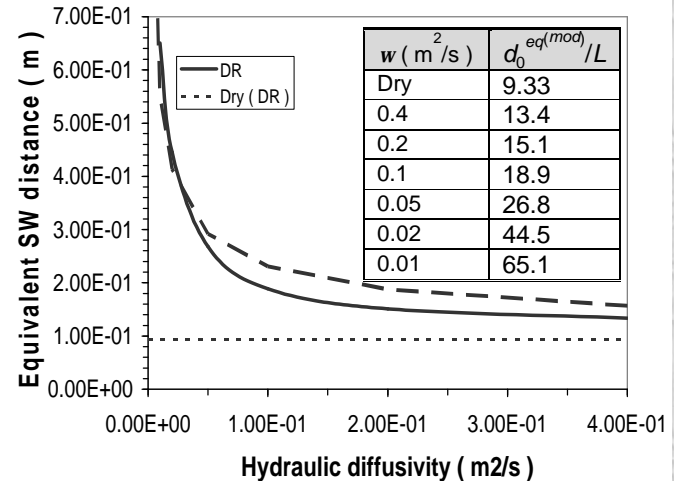


$$d_0^{eq} = d_0^{eq(dry)} \left(1 + b_2 e^{-\frac{w}{d}} \right) \left(b_3 + b_4 \sqrt{\frac{w_*}{w}} \right)$$

where $w_* = 0.02 \text{ m}^2/\text{s}$ and

$$b_2 = 5, \quad b_3 = 0.2, \quad b_4 = 0.8;$$

$$d = d_0^{eq(dry)}$$



Bizzarri and Cocco (2006a, 2006b, *JGR*, 111, B05303 and B05304)



Effects of porosity evolution #1

If porosity F evolves through time the heat source function for the elementary solution is:

$$\tilde{q}(\mathbf{x}_1, \mathbf{z}, \mathbf{x}_3, t) = \begin{cases} \frac{t(\mathbf{x}_1, \mathbf{x}_3, t)v(\mathbf{x}_1, \mathbf{x}_3, t)}{2w(\mathbf{x}_1, \mathbf{x}_3)} & \text{Constant } F, t > 0, |\mathbf{z}| \leq w(\mathbf{x}_1, \mathbf{x}_3) \\ 0 & , |\mathbf{z}| > w(\mathbf{x}_1, \mathbf{x}_3) \end{cases}$$

The solution for the pore fluid pressure is (erf(.) is the error function):

$$\begin{aligned} \tilde{p}_{fluid}^w(\mathbf{x}_1, \mathbf{z}, \mathbf{x}_3, t) = & p_{fluid_0} + \frac{g}{4w(\mathbf{x}_1, \mathbf{x}_3)} \int_0^{t-e} dt' \left\{ -\frac{c}{w-c} \left[\operatorname{erf}\left(\frac{z+w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{c(t-t')}}\right) - \operatorname{erf}\left(\frac{z-w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{c(t-t')}}\right) \right] + \right. \\ & \left. + \frac{w}{w-c} \left[\operatorname{erf}\left(\frac{z+w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{w(t-t')}}\right) - \operatorname{erf}\left(\frac{z-w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{w(t-t')}}\right) \right] \right\} \left\{ t(\mathbf{x}_1, \mathbf{x}_3, t')v(\mathbf{x}_1, \mathbf{x}_3, t') + \right. \\ & \left. \text{Constant } F \right\} \end{aligned}$$

On the fault plane (i. e. in the limit $z \rightarrow 0$) the pore fluid pressure change is

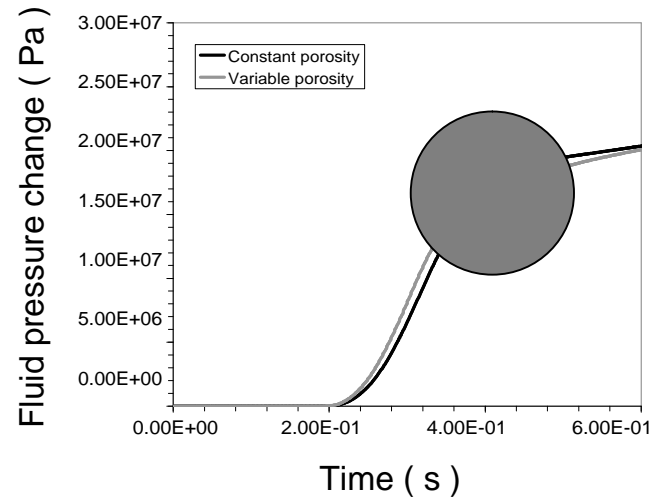
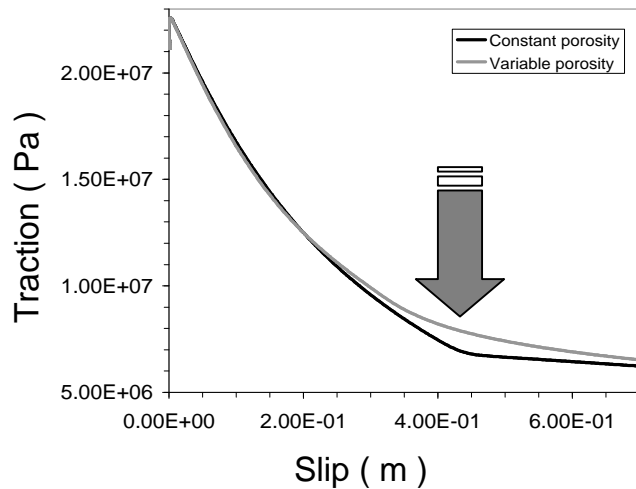
$$\begin{aligned} \tilde{p}_{fluid}^{wf}(\mathbf{x}_1, \mathbf{x}_3, t) = & p_{fluid_0}^f + \frac{g}{2w(\mathbf{x}_1, \mathbf{x}_3)} \int_0^{t-e} dt' \left\{ -\frac{c}{w-c} \operatorname{erf}\left(\frac{w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{c(t-t')}}\right) + \right. \\ & \left. + \frac{w}{w-c} \operatorname{erf}\left(\frac{w(\mathbf{x}_1, \mathbf{x}_3)}{2\sqrt{w(t-t')}}\right) \right\} \left\{ t(\mathbf{x}_1, \mathbf{x}_3, t')v(\mathbf{x}_1, \mathbf{x}_3, t') + \right. \\ & \left. \text{Constant } F \right\} \end{aligned}$$



Effects of porosity evolution #2

Assuming that porosity evolves accordingly to the law proposed by Segall and Rice (1995), and assuming $L_{SR} = L$

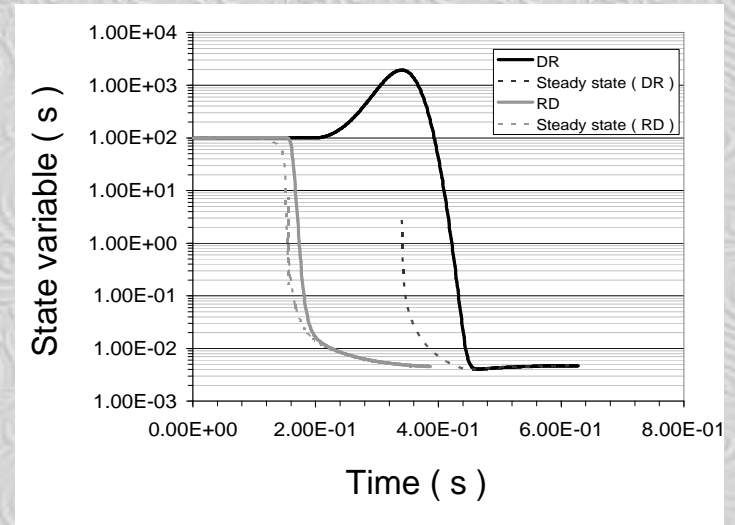
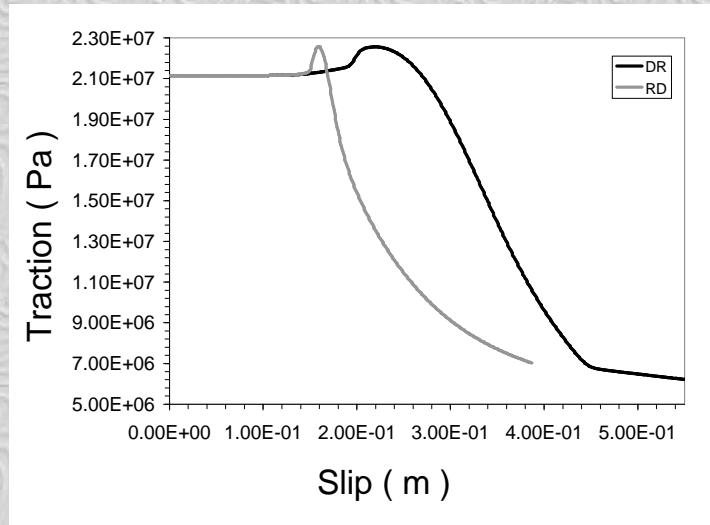
$$F(x_1, z, x_3, t) = F_* - e_{SR} \ln \left(\frac{Y v_*}{L_{SR}} \right)$$



Bizzarri and Cocco (2006a, 2006b, *JGR*, 111, B05303 and B05304)



Importance of the evolution law



DR (ageing evolution law)

$$t = \left[m_* + a \ln \left(\frac{v}{v_*} \right) + b \ln \left(\frac{Y v_*}{L} \right) \right] s_n$$

$$\frac{d}{dt} Y = 1 - \frac{Y v}{L} \quad \text{Dry fault}$$

RD (slip evolution law)

$$t = \left[m_* + a \ln \left(\frac{v}{v_*} \right) + b \ln \left(\frac{Y v_*}{L} \right) \right] s_n$$

$$\frac{d}{dt} Y = - \frac{Y v}{L} \ln \left(\frac{Y v}{L} \right) \quad \text{Dry fault}$$

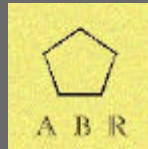
Conclusioni

- ✓ L' inclusione della pressurizzazione termica dei fluidi cambia la forma del fronte di rottura e l' evoluzione della trazione all' interno della zona coesiva;
- ✓ L' andamento della trazione mostra una continua diminuzione per crescenti valori dello slip su faglia;
- ✓ L' equazione evolutiva della variabile di stato influenza la dipendenza della trazione dallo slip e dal tempo;
- ✓ La lunghezza caratteristica di slip – weakening equivalente d_0^{eq} può perdere significato (e. g. per porosità temporalmente variabile);
- ✓ La caduta di sforzo di breakdown e d_0^{eq} sono inversamente proporzionali al fault thickness ed alla diffusività idraulica.

Grazie!

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Why “truly” 3 – D ?



Remembering the dimensionality of the problem:

2 – D Mode II (pure in – plane): $\mathbf{u} = (u_1(x_1, t), 0, 0)$

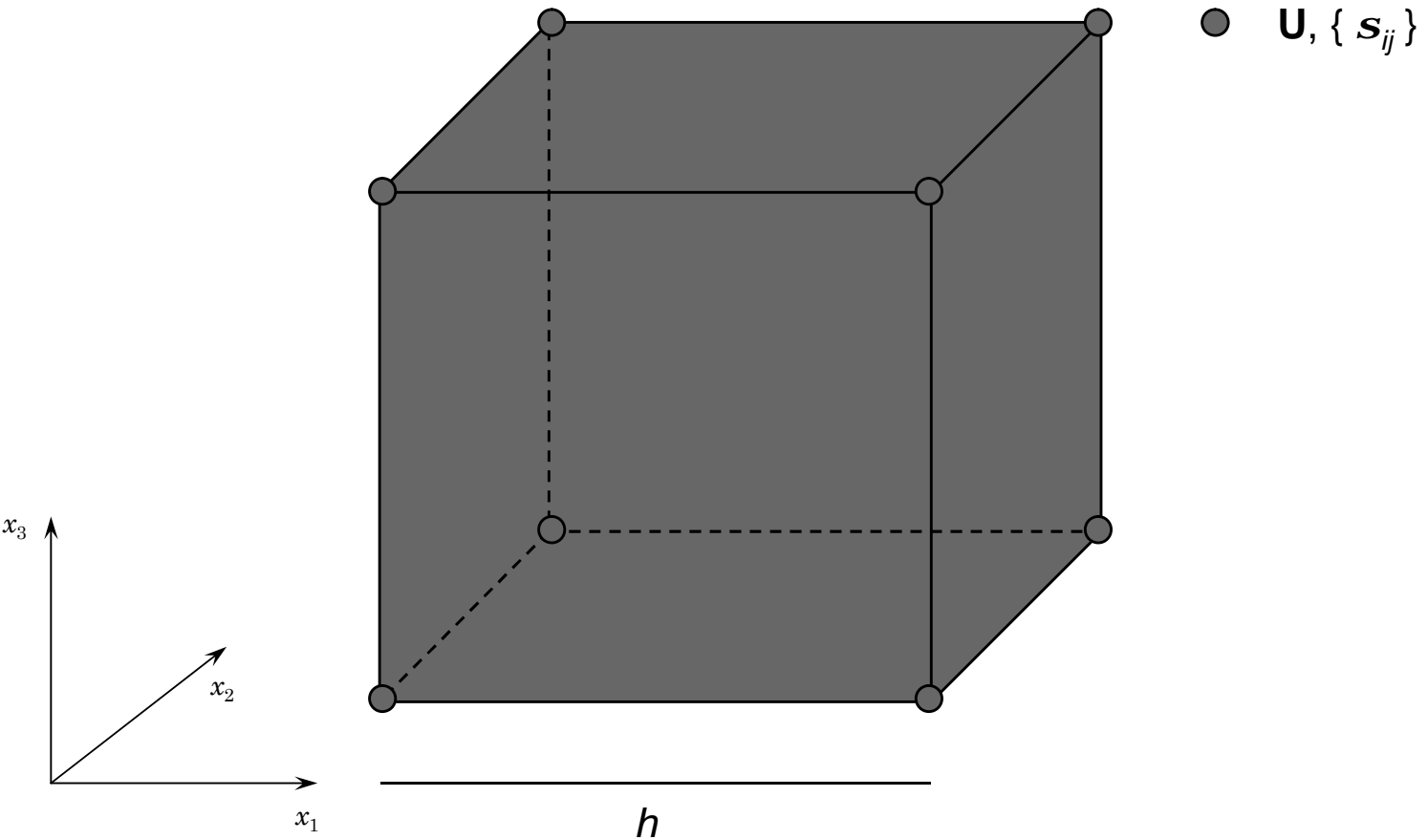
2 – D Mode III (pure anti – plane): $\mathbf{u} = (0, u_2(x_1, t), 0)$

3 – D Mixed mode: $\mathbf{u} = (u_1(x_1, t), u_2(x_1, t), 0)$

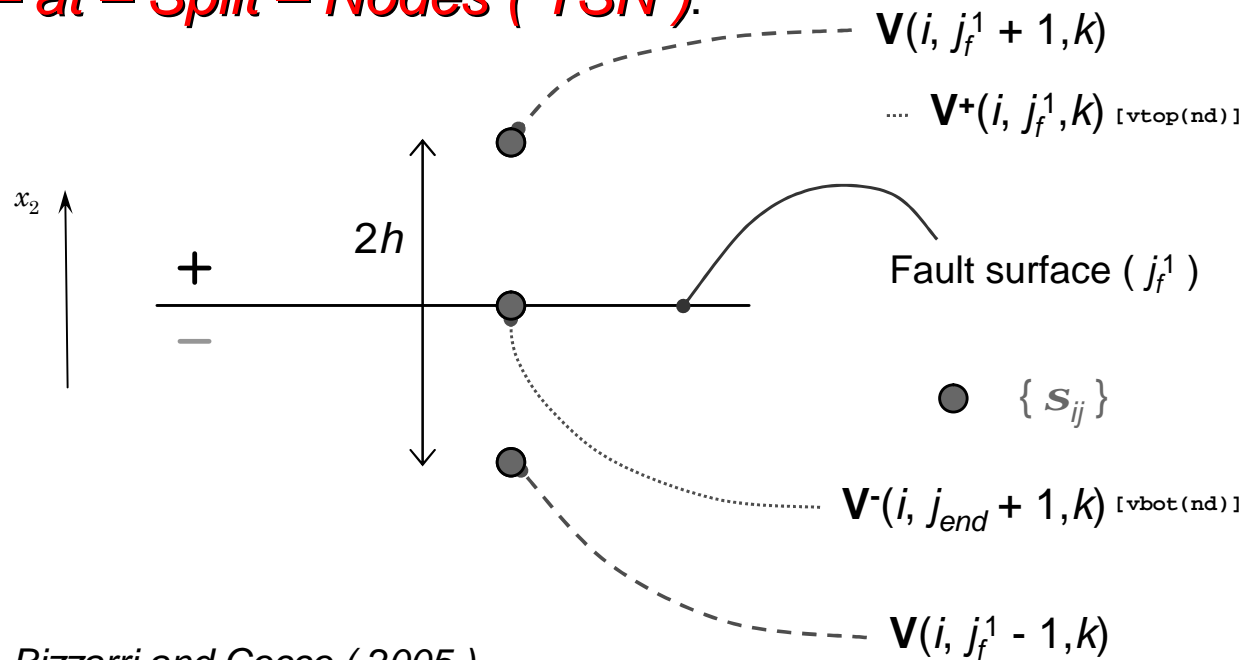
3 – D having only one non null component: $\mathbf{u} = (u_1(x_1, x_2, t), 0, 0)$

Truly 3 – D: $\mathbf{u} = (u_1(x_1, x_2, t), u_2(x_1, x_2, t), 0)$

- *Conventional - grid (CG):*



- **Traction – at – Split – Nodes (TSN):**



Andrews (1999), Bizzarri and Cocco (2005)

* **Discontinuum medium** (continuum mechanics equations of motion are applied to each half - space individually; the fault is an explicit discontinuity in displacement)

Non – laminar fault model

Internal Structure of Principal Faults of the North Branch San Gabriel Fault

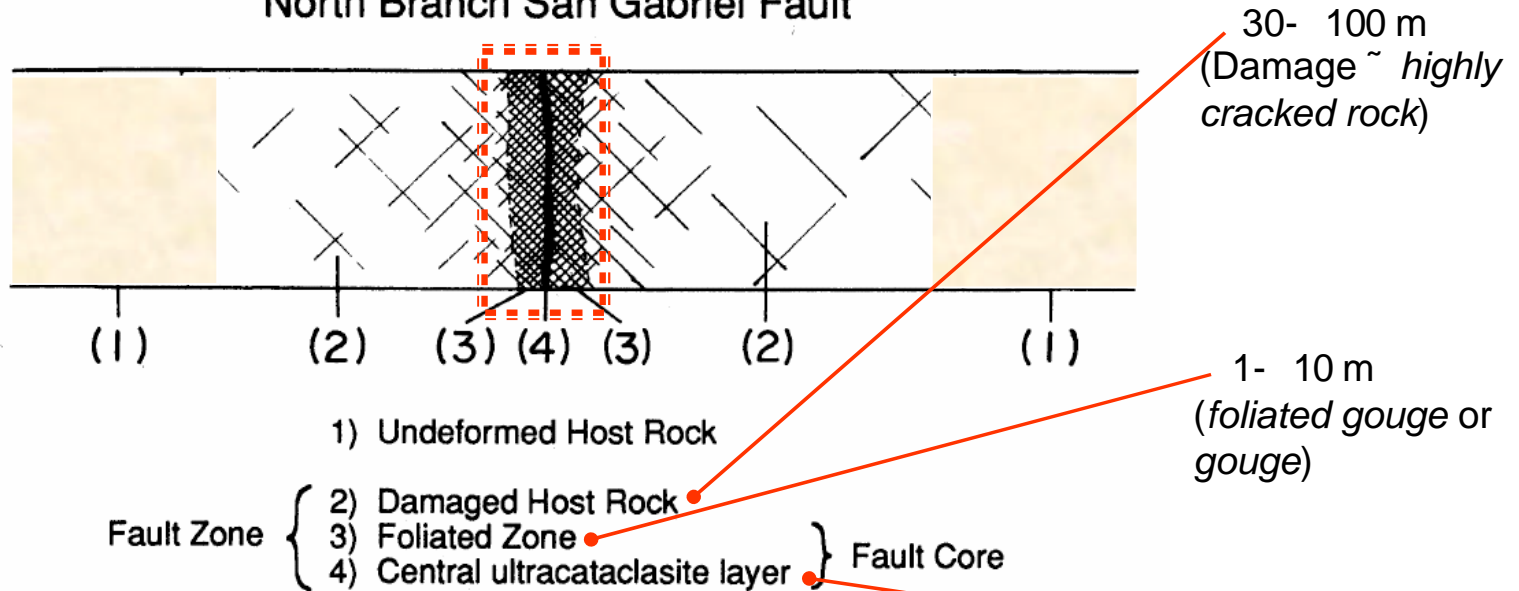


Fig. 2. Schematic section across the North Branch San Gabriel fault zone illustrating position of the structural zones of the fault. The diagram is not to scale.

Chester, Evans and Biegel, *JGR*, 1993
 Sibson, *BSSA*, 2003
 Chester and Chester, *SSA*, SCEC meetings 2004

Parameter	Value
<i>Medium and discretization parameters</i>	
$\lambda = G$	27 GPa
v_p	5196 m/s
v_s	3000 m/s
$\Delta x_1 = \Delta x_3$	25 m
Δx_2	100 m
Δt	0.83671×10^{-3} s
$\sigma_n - p_{fluid_0}$	30 MPa
<i>Slip-weakening model parameters</i>	
τ_0	20 MPa
μ_w	0.93333
μ_r	0.33333
S (at $t = 0$; reference)	0.8
d_0	0.1 m
<i>Rate- and state-dependent models parameters</i>	
τ_0	$t^{ss}(v_{init})$
a	0.007
b	0.016
L	0.01 m
v_{init}	1×10^{-4} m/s
μ_*	0.56
Ψ_{mocl}	1×10^{-4} s
$\Psi_{outside\ the\ nucleation}$	$\Psi^{ss}(v_{init})$
α_{LD} (reference)	0.53
<i>Thermal pressurization parameters</i>	
T_0^f	100 °C
k (reference)	5×10^{-17} m ²
η_{fluid}	1×10^{-4} Pa s
c	3×10^6 J/(m ³ °C)
χ	1×10^{-6} m ² /s
Φ_0	0.025
α_{fluid}	1.5×10^{-3} °C ⁻¹
β_{fluid}	1×10^{-9} Pa ⁻¹
w (reference)	0.035 m

